HYDRO-ELECTRIC POWER

VOLUME I

HYDRAULIC DEVELOPMENT AND EQUIPMENT

BOOKS BY

LAMAR LYNDON

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PREFACE TO VOLUME I

In writing this book it has been the intent of the Author to produce a work for the guidance of engineers in the practical design of hydro-electric plants, which would have the characteristics of accuracy, clearness and completeness. Scientific discussions of various hypotheses and theories have been omitted except in cases where their incorporation in the text has been essential to the understanding of the subjects treated.

Where a divergence of views exists among engineers, the only wise procedure is to follow safe, current practice, and this fact has guided the preparation of certain portions of this work. However, the Author has been compelled to take issue with a few conclusions which are, generally, accepted by the engineering profession, as in the theory of uplift pressure under solid dams.

Certain of the methods of treatment are new. In every case, a consistent adherence to the physics of the problem has been maintained, and in practically every instance, the physical phenomena are apparent in the equations and mathematical discussions. Seldom have abstract mathematics been employed and only under stress of absolute necessity.

A number of new and original formulæ appear for the first time, here. Among these may be mentioned the exact formulæ for solid dams, and for the magnitude and location of the resultants of forces acting on dams.

Occasional repetitions, both of statements and conclusions, will be found. The object of these duplications is for the purpose of making complete any single chapter or section of the book. Treatises of this character are seldom read through consecutively, but are used for reference, and it is both an annoyance and a waste of time to search through every part of a book for data on some single subject. The avoidance of this necessity has been one of the objects that has been attempted in this book. Likewise, and for the same reason, frequent repetitions of the meaning of the symbols used in the formulæ will be found. Engineers who, in impatience and annoyance, have been accustomed to

hunt back through many pages to determine what the symbols mean in a much needed formula, will appreciate this feature.

This work was written as a single volume, but its magnitude has compelled its separation into two volumes, and the division has been made as well as the conditions and the judgment of the Author would allow. Some of the material in Volume I belongs, in a measure, to the subjects which comprise Volume II, and *vice versa.*

An effort has been made to eliminate all errors, particularly in the formulæ. The Author, and publishers, will be truly grateful to any reader who will advise them of any incorrect formulæ and, also, of statements or discussions which appear obscure and wanting in clearness. Obscurity is as great a fault as a wrong formula. Criticisms from engineers who are new in the art are of particular value.

The Author here expresses his appreciation of, and sincere thanks for, the assistance he has received from several eminent engineers and scientists in the preparation of this book. Among others who have contributed to give it any merit it may possess are: Dr. A. S. Chessin, Mr. F. S. Taylor and Mr. A. G. Hillberg.

Many of the manufacturing companies have been courteous in furnishing photographs and data, and their names are mentioned in appropriate places in the text.

LAMAR LYNDON.

NEW YORK, *October*, 1916.

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HYDRO-ELECTRIC POWER

CHAPTER I

GENERAL CONDITIONS

When any substance having weight passes from one elevation to a lower elevation, energy is released and under certain conditions this energy may be converted into mechanical power and made useful for industrial purposes. Obviously, the energy is proportional to the weight of the body or substance, and it is also proportional to the vertical distance through which it travels in passing from the higher to the lower elevation.

Algebraically, the energy released by any downward moving body is

$$\text{Energy} = \text{Weight} \times \text{Height}.$$

If the weight be given in pounds, and the height in feet, the energy is in units of foot-pounds.

If a body pass from one elevation to a lower one, the energy is given out only during the time of movement of the body, and it ceases as soon as the body reaches the lower level toward which it travels. Hence, for a continuous delivery of energy there must be a continuous supply of bodies or substance having weight.

A stream of falling water fulfils all the conditions necessary to produce a continuous supply of power, having weight, fall and continuity.

The power—which is the rate at which energy is delivered—depends on the quantity of water flowing continuously and the height through which it falls.

It must be noted that the height of fall is the difference in elevation between the surface of the water at its upper position and the surface at its lower position, *measured vertically.* No matter what path the water follows in passing from the upper to the lower level, and no matter how long this path may be,

the vertical height of the upper surface above the lower level is the useful fall and is called the "head."

The gross horsepower delivered by falling water is

$$\frac{62.5QH}{550} = \frac{QH}{8.8} = 0.1135\,QH \tag{1}$$

Q = cubic feet per second of water used through the water wheel

H = is the head or vertical distance, in feet, through which the water falls in passing through the water wheel

62.5 = weight of water per cubic foot

550 = foot-pounds per second to give 1 hp.

Since the energy is the product of quantity of water, times height of fall, it is obvious that 1 cu. ft. of water per second falling through 8.8 ft. will produce 1 gross hp.

In the preceding formula and the others which follow, the weight of water is taken at 62.5 lb. per cubic foot. This is the usual practical figure used, but it is greater than the actual value, which for the standard temperature of 60°F. is 62.366. The difference between the usually assumed and actual values is, therefore, 0.134 in 62.5 or about 0.2 per cent.

At higher temperatures, the weight diminishes appreciably, as shown by the following table.

TABLE 1.—WEIGHT PER CUBIC FOOT OF WATER AT VARIOUS TEMPERATURES

Deg. F.	Weight, lb.	Deg. F.	Weight, lb.
32.0	62.416	80.0	62.217
39.3	62.424	100.0	62.061
50.0	62.408	110.0	61.933
60.0	62.366	120.0	61.719
70.0	62.300		

From this it may be seen that in making efficiency tests in tropical countries, with the temperature of the water at 100°F., the error in the assumption of 62.5 lb. per cubic foot, would amount to

$$\left(\frac{62.5 - 62.061}{62.061}\right)100 = 0.7 \text{ per cent.}$$

There is a loss in the conversion of the energy of the falling water into mechanical energy at the water-wheel shaft. This

loss averages 20 per cent., though, as will be shown later, the loss varies with many conditions.

The general practice is to take the average loss as 20 per cent. so that the formula for delivered horsepower at the water-wheel shaft is $\frac{0.80QH}{8.8} = \frac{QH}{11}$ hp.

In the same manner the constants in Table 2 are derived. They are based on the following efficiencies:

Water wheel at full load, 80 per cent.

Dynamo, at full load, 94 per cent.

Transmission line, at full load, 90 per cent.

TABLE 2.—POWER CONSTANTS

Horsepower, gross	$= \frac{QH}{8.8}$	(1)
Horsepower at water-wheel shaft	$= \frac{QH}{11}$	(2)
Horsepower at dynamo	$= \frac{QH}{11.7}$	(3)
Horsepower, delivered at end of transmission line	$= \frac{QH}{13}$	(4)
Kilowatts, gross	$= \frac{QH}{11.8}$	(5)
Kilowatts at water-wheel shaft	$= \frac{QH}{14.74}$	(6)
Kilowatts at dynamo	$= \frac{QH}{15.68}$	(7)
Kilowatts, delivered at end of transmission line	$= \frac{QH}{17.4}$	(8)

The important factors in the consideration of a water-power development are:

1. Stream flow { Absolute minimum. Absolute maximum. Average minimum.
2. Available head.
3. Character of geological formation at site of dam.
4. Market { Load factor. Diversity factor. Value of power. Distance of transmission.
5. Total cost of development.
6. Cost of operation and maintenance.
7. Income and profit.

From these can be computed all the other quantities necessary

to reach a proper engineering and financial conclusion as to the advisability of developing a power site.

The most important factor in the development of a water power is to determine, in advance, the actual amount of power that may be obtained continuously, over a long period of years. Failure to give this subject the attention and careful investigation which its importance deserves has resulted in financial disaster in many instances.

In the United States, the Government has long maintained gauges at different points on most of the large rivers, and their records are available and may be used in computing the available power without making any additional observations on the stream itself. In many countries, however, the engineer is dependent on his own observations, and as these can not be carried over a long number of years, he must resort to the methods of computation from the rainfall, the drainage of the stream, the local conditions as to the character of the country, its vegetable growths, and whether its geological formation is such that underground storage reservoirs exist which supply springs that continue to feed the streams during dry weather. With these data, reinforced by experience, an approximate determination of the minimum stream flow may be arrived at.

The character and extent of the underbrush, shrubbery and trees; the proportion of wooded area to that denuded of trees: the proportion under cultivation; all have an influence on the variation in the flow. Trees and shrubs tend to hold the rain water and make it move slowly toward the stream—so slowly that much of it is absorbed into the earth and then reaches the river or its tributary creeks only by percolation which greatly retards its movement. These effects combine to equalize the amount of water which is given to the stream by each rainfall. Rains come intermittently and are of varying volume. The flow of streams would be equally intermittent and variable as to volume if it were not for these retarding influences. Where springs are numerous, they tend to keep the stream flow up in dry weather and these are valuable when they discharge enough water to be of real assistance.

The minimum flow sometimes may be increased by means of storage. When a dam is built across a stream and a lake of considerable area is formed, the water thus accumulated may be partially drawn off during the dry season, the total water passed

through the water wheels being that furnished by the stream plus that taken from the lake. Generally, the power is not used throughout the full 24 hr. per day, and if the storage area is sufficiently great, the water which flows during the night is accumulated in the lake, and on the following day the water available for power is that supplied by the stream flow plus that impounded during the previous night. In this way, the power furnished by a given stream may be greatly increased during a part of the day, as is more definitely pointed out in the paragraph on "Load Factor" in Chapter IV.

For the purpose of forming extensive storage lakes, dams of great height and length are often constructed, instead of small dams, further up the stream, with canals or flumes leading to the foot of the falls, which would cost much less and would serve just as well, if the question of storage were not involved.

The quantity of stream flow and its variation are arrived at in one of the following ways:

(*a*) From observations of the stream extending over a number of years.

(*b*) From records of rainfall and drainage area of the stream down to location of power house.

(*c*) From observations made at the time of known low water.

Where possible, all of these means should be used to check the final result.

From (*a*) and (*b*) the maximum as well as the minimum flows are obtained, and either (*a*) or (*b*) is, therefore, preferable to (*c*) alone. Neither (*b*) nor (*c*) alone should ever be accepted as final, but the two always used to check each other.

The maximum flow must be known, so that the dam may be designed to withstand it, and the spillway—that is, the crest of the dam over which the water flows—made long enough to allow the maximum volume of water to pass over it without an excessive rise in the height of the water over the dam.

Abnormal increase in the height of water above the spillway endangers the dam and may result in it being swept away.

The fall is found by starting at the head of the falls with an engineer's level, the lower end of the level rod being against the surface of the water for the first observation. The second observation is made with the level rod on the bank and succeeding observations are made with the level rod on the ground, working

down to the foot of the shoals. When this point is reached th last observation is taken, and thus the difference in level betwee the head and foot of the shoals is determined.

Generally, the rod should be moved over to the water a intervals so that the drop at various points may be taken a well as the total difference in head.

Having determined the power obtainable at the turbin shaft at times of lowest water, if this is ample for all possibl needs, the development may be made in the most inexpensiv manner practicable, for the particular conditions. If, how ever, the power is insufficient when the water is low, it become necessary to make the development so that ample storage will b provided.

After fixing the position of the dam and its height, a survey must then be made to locate the contour of the lake which will be formed by the impounded water. From this survey will be found the lands which will be overflowed, the area of the lake, and hence, the volume of the stored water useful for power purposes.

In computing the volume of storage water available for power, it must be remembered that the level of the reservoir can be lowered only a comparatively small amount. If the storage lake be drawn off too much, and its level sinks too far, the head acting on the water wheels will be diminished by an amount that will impair the operation of the plant. The drop in level of the reservoir should never exceed 25 per cent. of the effective head. In cases of extreme necessity this drop may be exceeded, but all calculations as to the amount of power obtainable from a given stream with storage, should be based on a drop in head not exceeding 25 per cent.

The amount of storage is more often regulated by financial considerations than engineering possibilities, as will be set forth later.

Power companies usually have two different forms of power contracts. One provides that the company shall supply the customer with continuous power throughout the year regardless of the fluctuations in stream flow. This continuous, year-round supply is called primary power.

The other form of contract is with users who agree to take certain amounts of power as long as the company has sufficient water to furnish the required power, and the supply may be

cut off when the water in the stream falls below a certain value. This class of service is called secondary power.

Secondary power is usually guaranteed for 9 months in each year, and furnished longer if the flow of water in the stream keeps up to the value necessary to supply it. It is always sold at a lower price than primary power and is purchased by manufactories which have steam plants that were installed prior to the development of the water power. During periods of low water, when the power supply is discontinued, the factory steam plant is put into service and the load carried by it until the power service is resumed.

Where developments are made on streams with a minimum flow that is considerably less than the average and lasts a comparatively short time—20 to 40 days—in each year, it is frequently desirable to install a steam- or gas-engine plant to assist the hydraulic generating equipment. The capacity of this auxiliary plant is equal to the difference between the maximum capacity of the hydraulic generating equipment and the actual power available from the stream flow during the periods of low water. With this arrangement, the total plant can supply a load continuously throughout the year which is equal to that which the hydraulic power yields at ordinary stages of flow, the auxiliary equipment being operated during the periods of low water only. Although the production of power by the engine plant may cost more than the price received for it during the time of engine operation, it must be remembered that the amount of power thus produced brings an income for the whole year, while its cost continues only from 20 to 40 days.

These "general conditions" are the usual, well-known elements of water-power developments, and will be fully discussed under their several respective captions.

The finance of water-power developments is not one of the subjects covered in this treatise, and no values can be laid down for it. A whole special work on the financial aspects of water-power development would be required to do adequate justice to the subject.

In general, the fundamental points are to see that the minimum income will be ample to pay: interest on the securities, depreciation, maintenance, general operating costs, insurance, taxes, accidents and damages, and leave a reasonable margin, in addition, for unforeseen contingencies.

These factors are in many cases, not subject to computation, and the financial predictions concerning a proposed development must be based on wide experience together with a special study of the success or failure of the principal plants in operation and the factors which contributed to their prosperity or to their bankruptcy. A few points that must be given consideration and which are important are:

1. Make sure, in advance, of the market for power, not only of the immediate probable demand, but what the future market may be; this requires a consideration of all possible sources of power supply for a given district. Note particularly whether any other water-power site is available for development within a reasonable distance from the district which the projected plant will supply. If so, it should be brought under the control of the owner of the development under consideration, or competition may be expected at some future time. Nearness of undeveloped coal fields and other power sites that may become actual power producers and competitors, must be included in the study of the situation.

2. Be sure of the total cost of development. The particular items which inexperienced engineers usually fail to estimate correctly, in advance, are:

(*a*) Foundation work, under water.

(*b*) Cofferdams, and water control.

(*c*) Flood damages.

(*d*) Delays, due to floods and washed-out cofferdams.

(*e*) General accident and damage account.

(*f*) Extras paid to contractor because of the first five items just given.

(*g*) Machinery erection costs.

(*h*) Details of construction and equipment. These mount up very fast and are frequently overlooked as unimportant.

(*i*) Engineering costs. The fee paid the consulting engineer does not cover the cost to the company of much necessary expense; 7 to 7½ per cent. is a reasonable allowance.

(*k*) Damages to apparatus during erection and testing, and losses occasioned by them.

(*l*) Delays of all kinds. These mean increase in cost because of the fixed charges which continue until the plant is completed.

(*m*) Interest charge. This is usually figured on the basis

of 5 per cent. per annum. As a matter of fact the actual cash for a water-power development costs about 7 per cent.

3. Always be ready to pay, even unreasonable prices, for guaranteed, quick completion. The extra charges for speedy work are, usually, small compared with loss of income, interest charges, possible flood losses and other costs that arise from slow processes of construction.

4. Always investigate, fully, the laws relating to streams of the State in which the development is to be made, taking due note of special irrigation, placer mining or other statutes which have a bearing on the proposed use and storage of water.

5. Note whether existing irrigation or mining projects have preëmpted any portion of the stream flow.

6. Note whether the stream is used for floating lumber, and if so, be prepared to include the cost of log chutes in the development.

CHAPTER II

FLOW IN STREAMS

The determination of the quantity of stream flow, and the drop in level of the water surface along the length of the stream, are the important observations to be made to fix the available power. The quantity of water delivered by a stream may be computed or measured. The methods of computation are given, in this chapter, in their proper places. The formulæ all depend on the determination of the drop in level of the water surface per unit length of stream, the form and area of the stream cross-section, and the character of the stream bed with respect to roughness. For the flow in a stream and its variation, it is usual to rely on the records of stream flow as published in the *Bulletins* of the United States Geological Survey. These are available gratis and may be obtained on request to the Director of the United States Geological Survey at Washington. They are numerous and, in making requests for them, it is necessary to state either the *Bulletin* number or the name of the stream on which information is desired, in order to obtain the proper pamphlets.

The records are based on daily gauge readings of every important stream in the United States and, in most cases, they cover a number of years, so that these records are indicative of the maximum and minimum flows that ever may be expected.

Usually, the records are simply daily gauge heights, or the elevation of the surface of the water each day. A few values of stream flow, in cubic feet per second, for different gauge heights are also given. In order to find the flow for any gauge height it is necessary to plot a *rating curve.*

This is done by laying out, on cross-section paper, the given gauge heights as ordinates, and the corresponding flow for each as abscissæ, and drawing a curve through these points. It is to be noted that the zero of the gauge never corresponds to zero water, or in other words, a dry stream. Usually, a value of flow is given for the zero gauge reading.

As an example, take the following data on the flow of the Colorado River at Austin, Tex., made in 1908.

A survey of the cross-section of the stream and observations of its velocity, at different times and with varying conditions of water level, give the following results:

TABLE 3.—DISCHARGE OF COLORADO RIVER, AT AUSTIN, TEXAS

Gauge height	Discharge, sec. ft.	Gauge height	Discharge, sec. ft.
0.8	200	2.9	2,100
1.3	375	3.0	2,300
1.7	620	3.7	3,800
1.9	800	4.0	4,600
2.0	920	4.9	7,600
2.4	1,390	5.8	11,400

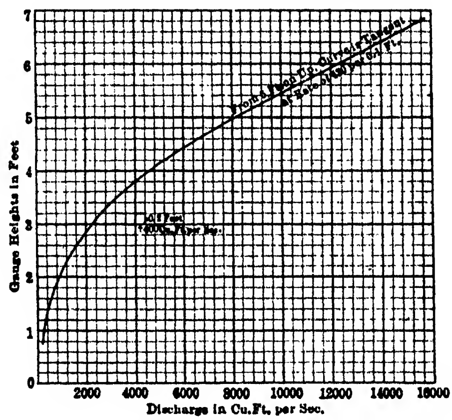


FIG. 1.—Rating curve of Colorado River at Austin, Tex.

From these values the rating curve shown in Fig. 1 is plotted, and, from this curve, the flow corresponding to any intermediate gauge height may be taken.

If no gaugings are available for the particular stream under consideration, the stream flow may be approximately predicted by comparing it with streams in the same section of country the flows of which latter streams have been recorded for a number of years. The flows of the streams will be approximately, proportional to their respective drainage areas, provided that

none of the streams possess any unusual features, such as very precipitous watershed, or many large springs supplying a continuous flow from underground storage reservoirs, and thus augmenting the water supply during periods of drouth.

Also, if a power site is located at a point on a stream at a considerable distance from a gauging station on that stream, the flow at the power site may be taken as proportional to that at the gauging station in the ratio of their respective drainage areas.

In some sections of country, the minimum and maximum stream flows are based on a certain flow per square mile of drainage area. Thus, the streams in South Carolina and Georgia average 0.40 cu. ft. per second, per square mile of drainage area for the minimum flow, and about 20 cu. ft. per second per square mile, for the maximum flow. In that section, the rainfall is about 50 in. per annum.

In the Southwest, where the rainfall is small, such figures will not apply. For instance, the Colorado River at Austin, Tex., has about 30,000 sq. miles of drainage area. Its minimum flow is 175 cu. ft. per second while its maximum flow is about 1300 times this or 230,000 cu. ft. per second. These figures correspond to 0.00583 and 7.76 cu. ft. per second, per square mile of drainage area as the minimum and maximum respectively.

The drainage area of a stream is easily computed from any reliable map. The calculations are more accurate if contours are marked on the map as in the United States Government topographic maps. However, an ordinary geographical map is quite sufficient for all practical purposes if it be to a scale not smaller than 20 miles to the inch.

Figure 2 is a portion of a map of Georgia on which are shown the rivers and their tributaries. If the power site is on the Oconee River at Milledgeville, the drainage area is found by marking out the boundary of this area and then integrating the figure in any convenient manner. The boundary is drawn to include all the tributaries that lie upstream above Milledgeville and to exclude all streams that flow away from the Oconee to other rivers. The ridge of the watershed is, obviously, at or near the upstream end of any creek or river. Where the sources of two streams are shown near each other, but the streams flow in different directions and empty into different main trunk streams, the backbone of the watershed lies between the sources

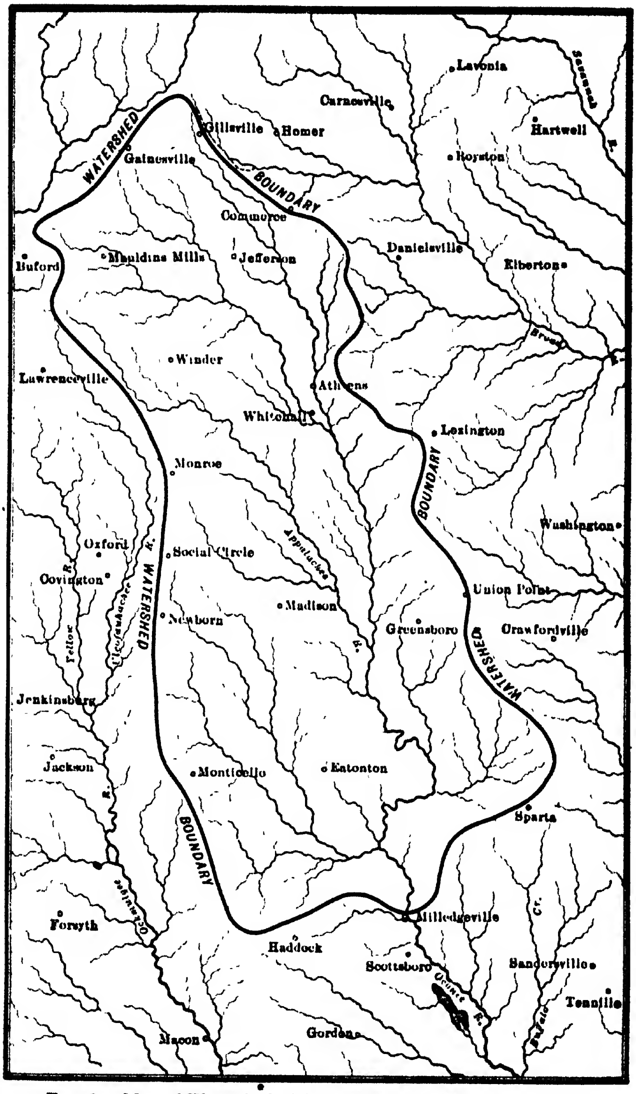


FIG. 2.—Map of Watershed of Oconee River at Milledgeville, Ga.

of these two streams. By following the heavy irregular line in Fig. 2 marked "Watershed Boundary" and noting how it includes the area of all streams emptying into the Oconee and excludes all those which empty into some other river, and also, how it closes in at the power site, the method of locating the boundary is made clear.

The area may be taken by a planimeter in the usual manner. It is the general practice, however, to draw cross-sectional lines over the face of the map, these lines being two series of equidistant parallel lines at right angles to each other, forming a number of squares, exactly as if the map were drawn on cross-section paper. In fact, it is often convenient to trace the watershed boundary on thin, semi-transparent, cross-section paper. Each square on the map represents, to scale, a definite area. Therefore, by counting the squares and portions of squares and multiplying the number of squares by the area of each square, the area of the watershed is found. By this method, the area of the watershed shown in Fig. 2 is found to be 2923 sq. miles. Obviously, the larger the scale of the map, the more accurate will be the computation.

After finding the drainage area, the average rainfall over this area and its distribution with the seasons must be studied. It will be found that the rainfall corresponds approximately to that in some territory where the flow of a stream has been recorded for a number of years. Considering the similarity of rainfall, of drainage area and of topography, soil and vegetation, the minimum and maximum flows of the stream may be established with reasonable accuracy. In any case the most useful data are available in the *Bulletins* of the United States Geological Survey, and these should always be referred to in computing the flow of any stream.

Most investigators who have written on the subject of stream flow give considerable importance to the questions of rainfall and run-off. Since the total stream flow is due to run-off and this in turn comes from the rainfall, it would seem logical that some relationship between these factors and stream flow would exist. While this is in a measure true, there are so many variable conditions which influence the amount of run-off for a given rainfall that no definite flow of water in any stream can be determined from the rainfall. Thus, a rainfall of 1 in. in 2 hr. will deliver a greater quantity of water to a stream than a slow

rain that gives an inch of water in 24 hr. Also, the run-off from a rain following closely on a preceding rain will be greater than the run-off from an exactly similar rain which falls a month or more after a previous rain. A high annual or seasonal rainfall, if concentrated within short periods, may result in a lower minimum rate of stream flow than that produced by less rainfall well distributed over the season. In the investigation of this subject the author has never been able to get satisfactory data on stream flow from any rainfall records and run-off computations unless the run-off is reduced to the form of cubic feet per second, per square mile of drainage area, for maximum and minimum flows, and the data are put in this form by reference to stream flows in the same district, which have been measured as has been herein mentioned.

The only real practical facts, to be deduced from rainfall and run-off, are that long periods of drouth over the drainage area of a stream will result in low water, and if the time of the drouth be the longest known, the stream will have its lowest flow. Also, if many heavy rains fall, closely succeeding each other, the stream flow will be correspondingly great.

The maximum and minimum flows of a stream may be determined by a survey if two conditions can be obtained. One is that the survey be made at a period when the lowest flow for many years is passing, and the other is that a heavy flood has occurred within a length of time prior to the survey such as to leave well-defined traces of the height of the water.

People who live in the neighborhood of rivers and creeks have rather clear ideas as to low and high water in them and the general dictum of a community that a stream is lower than it has been for many years is usually a reliable datum. A measurement of flow during such a period may be safely accepted as showing the minimum rate of flow. The method of measuring stream flow is given elsewhere in this chapter.

Also, extraordinarily high floods will leave definite marks of the flood level that may last for several years. These indications are familiar and well known. They are simply discolorations of tree trunks, rocks and marks on the stony banks of the stream where they are precipitous enough to have confined the flood waters. If a line of levels be run which shows the slope of the level of the water surface as indicated by the marks left by the flood, and several cross-sections be taken, sufficient data will be

obtained to compute, within practical limits, the flow at the time of the flood.

The cross-sections should be surveyed from the level of the flood marks on one side of the stream to the flood marks on the other side, so that they will represent the actual cross-section of the stream at the time of high water, in the same manner as described in the "Measurement of Streams." The line of levels should extend at least half a mile along the stream. A full mile is a good, practical length. The cross-sections should be taken at intervals of one-fourth the length of the survey, one section being taken at the beginning and one at the end of the line of levels, making five cross-sectional measurements. From these data, the flood flow may be computed by Chezy's formula, given elsewhere in this chapter.

If possible, the line of levels should be begun at a point above which the stream bed has little or no slope or fall, and the levels run downstream from this point. In this way, the velocity of approach due to slope will be practically eliminated. Also, the survey should be madé several miles away from dams or other obstructions in the stream which would produce flood levels, higher than those which would result from a given quantity of water moving in the unobstructed channel. To make a survey of the cross-section of the stream, it is usual to select a time of low water, and, by means of a surveyor's level, take the differences in level from the surface of the water out to either side of the stream to such a distance that the maximum high-water point is reached, care being taken to move outward from the stream at right angles to its direction of flow. Observations are made at intervals of from 2 to 20 ft., depending on the variation in the contour of the banks, and the distance from the water surface outward to the maximum high-water level.

The cross-section of the stream itself is then determined. The best way to do this is to stretch an iron cable 1/8 to 1/4 in. in diameter, across the stream, this wire having been previously marked by metal or wooden tags spaced along it at equal intervals. The distance apart of the tags should be not more than 10 per cent. of the width of the stream. With a steel tape, weighted at one end by a heavy plumb bob, measure the depth of the water at each marking on the transversely stretched wire, using a small rowboat when necessary.

In swift flowing streams the weight must be heavy and the

thin edge of the tape turned toward the direction of flow. Otherwise, the tape will be swept downstream, and the measurements will be inclined instead of vertical. The usual weight required for a velocity up to 2 ft. per second is 3 lb. while for a velocity of 10 ft. per second the weight should be not less than 25 lb.

From these measurements the cross-section may be mapped and computed. This is done by assuming some scale on the paper, say $\frac{1}{16}$ in., as equal to 1 ft. of horizontal distance, and some other greater scale, say 1 in., as equal to 1 ft. of vertical measurement.

The area of the cross-section of the water may be computed by any method of integrating irregular surfaces. A simple, approximate way is to add all the observed depths and divide the sum by the number of observations. This gives the average depth in feet. Multiply this average depth by the width of the stream in feet, and the product will equal the cross-section of the stream in square feet.

In the formulæ for computation of stream flow, later given, a conversion from feet head to velocity is necessary.

The velocity acquired by any falling body is (theoretically)

$$V = \sqrt{2gh} \tag{9}$$

in which

V = the velocity in feet per second of the body when it has fallen a distance = h ft.

g = acceleration due to gravity which is (practically) 32.2 ft. per second.

Putting $\sqrt{2g}$ outside the radical, the formula becomes

$$V = 8.02\sqrt{h} \tag{10}$$

This law applies equally to falling water.

Flowing water, having a certain velocity can be equated to still water having a pressure head

$$h = \frac{V^2}{64.4} = 0.01555V^2 \tag{11}$$

so that velocity and head are interchangeable in a definite ratio.

In making computations based on the flow of water one of the principal factors is the "*hydraulic radius.*" This quantity is designated by "r" and is numerically equal to the area of cross-

section of the stream divided by the length of the line bounding the cross-section where it is in contact with the stream bed, which length is called the "*wetted perimeter.*"

Thus in Fig. 3 the area of the cross-section of the stream is equal to "A." The wetted perimeter is the length of the irregular curve $ABCDEFG$. Calling this p, the hydraulic radius, is

$$r = \frac{A}{p} \tag{12}$$

Another factor in computing rate of flow is the *slope*, which is the rate of fall of the stream bed, or expressed in another way, it is

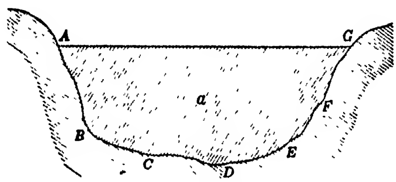


FIG. 3.—Typical cross section of a stream.

the drop in elevation of the stream bed per unit of length of the stream.

$$s = \frac{h}{l} \tag{13}$$

in which

s = slope
l = any length taken along the stream
h = fall of the stream in the length l.

In making measurements to determine the slope, l should be as great as convenient, and the longer it is the more accurate will be the result.

Having determined r and s, the velocity of flow is determined by the formula of Chezy, or Hazen & Williams, or Johnson, or others.

Chezy's formula is

$$V = C\sqrt{rs} \tag{14}$$

in which C is a constant which is computed from the empirical formula of Basin or Kutter.

Bazin's formula for determining the value of C is

$$C = \frac{87}{0.552 + \frac{m}{\sqrt{r}}} \tag{15}$$

The values of m are taken from the following table:

TABLE 4.—VALUES OF m FOR BAZIN'S FORMULA

For smooth cement or planed boards	$m = 0.06$
For unplaned planks or bricks.	$m = 0.16$
For masonry.........	$m = 0.46$
For smooth earth or clay	$m = 0.85$
For open streams, ordinary beds	$m = 1.30$
For open streams with vegetable growths..... ..	$m = 1.75$

As an example of the use of this formula, take a stream having the following characteristics:

$$A = \text{area cross-section} = 120 \text{ sq. ft.}$$
$$p = \text{wetted perimeter} = 48 \text{ ft.}$$

Drop in level of stream bed in 6000 ft. length along stream = 9 ft. Then

$$r = \frac{A}{p} = \frac{120}{48} = 2.5$$
$$\sqrt{r} = 1.581$$
$$s = \frac{h}{l} = \frac{9}{6000} = 0.0015$$
$$\sqrt{s} = 0.03873$$
$$C = \frac{87}{0.552 + \frac{1.30}{\sqrt{2.5}}} = 63.3$$
$$\sqrt{rs} = 1.581 \times 0.03873 = 0.0612$$
$$V = C\sqrt{rs} = 63.3 \times 0.0612 = 3.874 \text{ ft. per second}$$
$$Q = AV = 3.874 \times 120 = 464.8 \text{ cu. ft. per second.}$$

Kutter's formula for the determination of C is

$$C = \frac{\frac{1.811}{r} + 41.65 + \frac{0.00281}{s}}{\frac{n}{\sqrt{r}}\left[41.65 + \frac{0.00281}{s}\right] + 1} \tag{16}$$

In order to make use of this formula the following table of values of n must be used.

TABLE 5.—VALUES OF n FOR KUTTER'S FORMULA

For planed planks	$n = 0.009$
For neat cement	$n = 0.010$
For unplaned planks	$n = 0.012$
For smooth masonry and brickwork	$n = 0.013$
For rubble masonry	$n = 0.017$
For canals in firm gravel	$n = 0.020$
For canals and rivers free from stone and weeds	$n = 0.025$
For canals and rivers with rough stony beds and weeds	$n = 0.030$
For canals and rivers with extremely bad beds	$n = 0.035$

Considering the same conditions as before given and taking n as 0.025, the value of C by Kutter's formula is

$$C = \frac{\frac{1.811}{0.025} + 41.65 + \frac{0.00281}{0.0015}}{\frac{0.025}{\sqrt{2.5}}\left[41.65 + \frac{0.00281}{0.0015}\right] + 1}$$

$$= \frac{72.44 + 41.65 + 1.866}{0.01581(41.65 + 1.866) + 1} = 68.7$$

$V = 68.7\sqrt{rs} = 68.7 \times 0.0612 = 4.20$ ft. per second

$Q = 4.2 \times 120 = 504$ cu. ft. per second.

This result compares fairly well with the flow of 464.8 cu. ft. per second as found by the Bazin formula.

In general, the use of Bazin's formula gives more accurate results for steep slopes and high velocities while Kutter's formula is the more accurate for small slopes and low velocities. Kutter's formula is most accurate in the limits of r not exceeding 10, V not exceeding 5 ft. per second, and s between 1 and 10 in 10,000.

The Hazen & Williams formula is

$$V = Cr^{0.63}s^{0.54} \times 0.001^{-0.04}$$
$$= 1.32Cr^{0.63}s^{0.54} \qquad (17)$$

in which, C varies according to the following table:

TABLE 6.—VALUES OF C FOR WILLIAMS & HAZEN FORMULA

Open Channels

For smooth plank	$C = 110$ to 140
For unplaned plank	$C = 100$ to 120
For good masonry	$C = 80$ to 120
For rough masonry	$C = 65$ to 70
For gravel	$C = 50$ to 80
For rough earth	$C = 65$ to 75
For earth with vegetable growths	$C = 35$ to 70

Considering again the stream having the same hydraulic radius of 2.5, slope of 0.0015, area of cross-section of 120 sq. ft. and taking C as 70 for rough earth and using the Williams & Hazen formula:

$$V = 1.32 \times 70 \times (2.5)^{0.63} \times (0.0015)^{0.54}$$

$$\log 2.5 = 0.3979$$
$$\log 0.0015 = \bar{3}.1761$$
$$0.3979 \times 0.63 = 0.250677$$
$$\bar{3}.1761 \times 0.54 = \bar{2}.475090$$
$$\text{sum of logs} = \bar{2}.725767$$

Number corresponding to the log $\bar{2}.725767$ is 0.05318. Hence $r^{0.63}s^{0.54} = 0.05318$

$$V = 1.32 \times 70 \times 0.05317 = 4.913 \text{ ft. per second}$$
$$Q = 4.913 \times 120 = 549.5 \text{ cu. ft. per second.}$$

Of the three formulæ this last one gives the highest value for the flow. The differences probably arise from the selection of value of the constants.

Kutter's formula, while originally based on erroneous assumptions, has been made reasonably accurate by judicious assignment of values of n and is still regarded as the most accurate formula for flow in open channels. Bazin's is sufficiently accurate for all practical purposes and is the easiest to solve by means of the slide rule.

Professor Clarence T. Johnston of the University of Michigan, has conducted a series of experiments on a large number of ditches and canals of various conditions of slope and of materials. As a result of these investigations he has deduced a formula of the same exponential character as the Williams & Hazen formula. *Johnston's formula* is: $V = Cr^ps^q$.

By assigning various values to the exponent p, and taking the different values he has found for the coefficient C, he has produced a series of curves from which the velocity of flow may be taken directly. By plotting these curves on logarithmic cross-section paper, they become straight lines. This diagram is given in Fig. 4 herewith.

The table shown in Fig. 4 gives the values of C and p for different characters of stream beds. In order to find the velocity of flow for a given stream from the diagram, first, take the values of C and p from the table corresponding to the character of the

stream bed; then compute the hydraulic radius. Beginning on the scale of the lower left-hand portion, marked "Hydraulic Radius in Feet," at a point corresponding to the computed hydraulic radius of the stream, follow horizontally to the left until the diagonal line having the proper value of p is reached; then from this point of intersection pass vertically upward until the diagonal line having the proper value of C is reached. From this point of intersection, move horizontally to the right until the diagonal line corresponding to the slope is reached. From this intersection follow vertically down to the lower scale marked "Velocity in Feet per Second" where the desired answer will be found. Two examples are indicated on the diagram; one is for a stream having a hydraulic radius of 0.72; value of p = 83; value of C = 59.8, and slope 2.4 ft. per thousand, the resultant

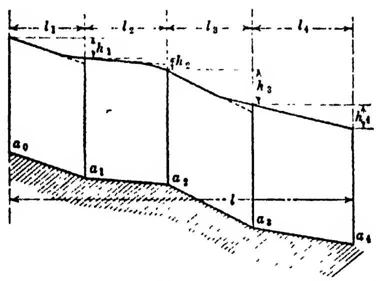


FIG. 5.—Profile of stream bed and corresponding water surface.

velocity being 2 ft. per second. The other example is for a stream having a hydraulic radius of 7.2; value of p = 80, value of C = 59.8 and slope = 0.16 per thousand feet, the resultant velocity being 3.2 ft. per second.

There is a middle scale giving the slope in feet per 5000 ft. which distance, being, approximately, a mile, may be used without serious error where the slope per mile is known.

The computation of the flow of rivers, presents some complications owing to the fact that the cross-section is variable and hence r, A, and V change correspondingly. However, the quantity of water, Q, remaining constant, AV is constant.

Consider that the slope of the water surface is found by measurement. Divide a selected length l, of the stream into a number of parts, say 5, in a distance of 2000 ft., although if the

stream changes its cross-section very greatly, a greater number will increase the accuracy of the computation.

Measure the cross-section of the stream at each division point, and from this compute A and r.

Let the length of the sections be l_1, l_2, l_3, l_4, etc., and the drop in head from the upper to the lower end of each section be h_1, h_2, h_3, h_4, etc.

These quantities are shown in Fig. 5.

From the formula $V = C\sqrt{rs}$, compute the velocity for each section and multiply this by the mean area of the section which is equal to one-half the sum of the cross-sectional areas at the two ends of the section. Thus the mean area of division No. 1 is $\frac{A_0 + A_1}{2}$.

These several values of the product of velocities by areas should be nearly the same. The average of these products is the rate of stream flow, in cubic feet per second.

As an example take a stream in which the survey gives the following data.

Cross-sectional Areas

A_0	=	212.5 sq. ft.	
			Average = 267.25 sq. ft.
A_1	=	322.0 sq. ft.	
			Average = 364.5 sq. ft.
A_2	=	407.0 sq. ft.	
			Average = 344.5 sq. ft.
A_3	=	282.0 sq. ft.	
			Average = 254.8 sq. ft.
A_4	=	227.6 sq. ft.	

Perimeters of Stream Bed at Different Cross-sections

p_0	=	66 ft.	
			Average = 130 ft.
p_1	=	194 ft.	
			Average = 201 ft.
p_2	=	208 ft.	
			Average = 154 ft.
p_3	=	100 ft.	
			Average = 95 ft.
p_4	=	90 ft.	

Drops in Water Surface between Sections

From h_0 to h_1	=	0.30 ft.
From h_1 to h_2	=	0.18 ft.
From h_2 to h_3	=	0.23 ft.
From h_3 to h_4	=	0.41 ft.

Lengths of Sections

l_1	=	500 ft.
l_2	=	500 ft.
l_3	=	750 ft.
l_4	=	1000 ft.

Then the hydraulic radii $\frac{A}{p}$ are (using average values of A and p) as follows:

HYDRAULIC RADII OF SECTIONS AND VALUES OF $\sqrt{r}$

$$r_1 = 2.05, \quad \sqrt{r_1} = 1.43$$
$$r_2 = 1.81, \quad \sqrt{r_2} = 1.345$$
$$r_3 = 2.24, \quad \sqrt{r_3} = 1.496$$
$$r_4 = 2.68, \quad \sqrt{r_4} = 1.64$$

THE SLOPES $s = \frac{h}{l}$ ARE:

$$s_1 = 0.0006, \quad \sqrt{s_1} = 0.0245$$
$$s_2 = 0.00036, \quad \sqrt{s_2} = 0.01895$$
$$s_3 = 0.000306, \quad \sqrt{s_3} = 0.0175$$
$$s_4 = 0.00041, \quad \sqrt{s_4} = 0.0203$$

USING BAZIN'S FORMULA AND TAKING $m = 1.30$,

$$C = \frac{87}{0.552 + \frac{1.30}{\sqrt{r}}}$$

VALUES OF C ARE:

$$C_1 = 59.6$$
$$C_2 = 57.3$$
$$C_3 = 61.0$$
$$C_4 = 64.6$$

THE VELOCITIES ARE $C\sqrt{r} \times \sqrt{s}$, and are as follows:

$$V_1 = 59.6 \times 1.43 \times 0.0245 = 2.09 \text{ ft. per second}$$
$$V_2 = 57.3 \times 1.345 \times 0.01895 = 1.455 \text{ ft. per second}$$
$$V_3 = 61.0 \times 1.496 \times 0.0175 = 1.595 \text{ ft. per second}$$
$$V_4 = 64.6 \times 1.64 \times 0.0203 = 2.15 \text{ ft. per second}$$

Using for values of A the mean areas computed above, the values of $Q = AV$, for the several sections are:

$$Q_1 = 267.25 \times 2.09 = 558.5 \text{ cu. ft. per second}$$
$$Q_2 = 364.5 \times 1.455 = 530.4 \text{ cu. ft. per second}$$
$$Q_3 = 344.5 \times 1.595 = 549.5 \text{ cu. ft. per second}$$
$$Q_4 = 254.8 \times 2.15 = 547.8 \text{ cu. ft. per second.}$$
$$\text{Average} \quad Q = 546.5 \text{ cu. ft. per second}$$

It is unlikely that the actual measurements made on a survey will give results which check as closely as the foregoing values assumed at random. The example, however, illustrates the method and also shows how great the accuracy of the level survey must be. By the same process, the maximum stream flow is computed from flood marks as has been previously set forth in this chapter.

The slope of the surface of the water in a stream corresponds approximately to the slope of the stream bed, but where the slope of the stream bed changes, that of the water surface undergoes a change before the point at which the stream bed slope change is reached. Thus, in Fig. 5, beginning at the left-hand end, the slope of the water in the first section corresponds approximately with the slope of the stream bed. The next adjacent section, toward the right, has less slope than the first section and the surface of the water begins to assume this decrease of slope before the point h_1 is reached. Also, the slope of the next succeeding section is greater than that of the second-named section, and the water surface correspondingly assumes this slope, but begins to take this more rapid fall some distance in advance of reaching the point h_2, at which the stream bed slope changes.

Backwater Curve.—The surface of a lake, formed by building a dam in a flowing stream, is not a level, horizontal plane, as is usually assumed. From the laws of flow in streams that have been given in this chapter, it is clear that the elevation of the water surface must continually rise from any point to any other point upstream, or, in other words, a difference in head must exist between two points, taken along the length of the stream, in order to cause flow from the higher to the lower elevation.

The determination of the slope of the water surface, or the "backwater curve," is of but little importance in practice, except in rare instances. In hydraulic-power developments the only computation necessary is to fix the approximate water-surface levels for maximum flood conditions. The calculated elevations depend on (1) the constants used, which are a matter of personal selection; (2) on several cross-sections of the stream bed, which are determined by soundings and, therefore, are not exact, the section being, moreover, subject to frequent changes due to erosion or deposits, and (3) on the assumed maximum value of flood discharge which may differ from the actual discharge by 20 per cent. or more. Hence, any attempt at accuracy in fixing the backwater curve can result in mathematically exact figures only, that will not locate the curve except approximately. The only thing to be sure of is that the actual backwater will never rise above the limiting elevations as computed. The backwater curve, thus becomes a limiting line of elevation—not a prediction of the exact contour of the water surface. Its object is, of course, to locate all lands, houses and other property that will be inun-

dated in time of flood, after the dam is built, so that proper provision can be made for such overflow.

Several formulæ of the exact variety have been devised for computing the backwater curve. None of these is really accurate except for canals, flumes or streams having vertical sides. Also, these formulæ are complex and require either difficult integrations or reference to voluminous tables.

Like many other physical problems, this one is most easily solved by trial and error methods. The following is the method of computation used by the author for practical power work.

Refer to Fig. 6. This shows a stream bed in which a dam has been erected, raising the level of the water at the site to a height H, which is equal to the height of the dam, plus the thickness of water over the crest.

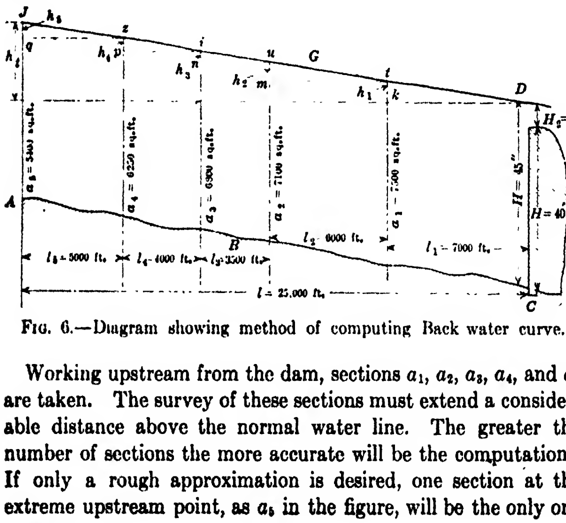


FIG. 6.—Diagram showing method of computing Back water curve.

Working upstream from the dam, sections a_1, a_2, a_3, a_4, and a_5 are taken. The survey of these sections must extend a considerable distance above the normal water line. The greater the number of sections the more accurate will be the computations. If only a rough approximation is desired, one section at the extreme upstream point, as a_5 in the figure, will be the only one needed.

The quantities which must be known are: l_1, l_2, l_3, etc., or the distance between sections—measured along the stream and following its meanderings.

a_1, a_2, a_3, etc., which are the areas of the sections up to the horizontal lines, k, m, n, etc. The area a_1 is first taken up to the horizontal line having an elevation the same as that of the water

surface at the crest of the dam. After computing h_1 and adding it to H, this fixes the elevation of the horizontal line above the surface of the water at a_1, and the area a_2 is taken up to this second horizontal line, *tm* in the figure. After finding the elevation of the water surface at a_1 the elevation of the horizontal line for determining a_2 is located, shown as *un* in the figure. This process is continued, step-by-step, until the last section is reached.

Q, the maximum flood discharge must be known.

C, the constant for velocity of flow in streams must be computed from the Kutter or Bazin formula.

p, the perimeter of wetted stream bed must be known. This is scaled from the cross-sections.

The approximate formula for the rise of water level, h_1, h_2, etc., above the elevation of the immediately preceding section is

$$h = \sqrt{FV^2 + (G - EV^2)^2} - (G - EV^2). \qquad (18)$$

F, G and E are constants for any stream and adopted values of l and resulting cross-sections. Their values are:

$$F = \frac{pl}{wC^2}, \quad G = \frac{a}{2w}, \quad E = \frac{1.25l}{wC^2}$$

a = area of section
V = velocity in ft. per sec.
w = width of section.

a, p and w are taken up to the elevation of the horizontal reference lines.

In using this formula, a value of V must be assumed for the first computation, and the calculations re-made after finding how far the first assumed velocity departs from that shown by the computations.

An example will illustrate the simplicity of this method.

Take the conditions shown in Fig. 6.

For section a_1,

a_1 = 7500 sq. ft.
l_1 = 7000 ft.
w_1 = 650 ft.
p_1 = 710 ft.
Q = maximum flood = 60,000 cu. ft. sec.

The coefficient C may best be computed from the Bazin formula

$$r = \frac{a}{p} = \frac{7500}{710} = 10.6 \text{ ft.}$$

Take $m = 1.30$.

Then,

$$C = \frac{87}{0.552 + \dfrac{1.30}{\sqrt{10.6}}} = 91$$

The constants can all now be computed

$$F = \frac{pl}{wC^2} = \frac{710 \times 7000}{650 \times (91)^2} = 0.9233$$

$$G = \frac{a}{2w} = \frac{7500}{2 \times 650} = 5.76$$

$$E = \frac{1.25l}{wC^2} = \frac{1.25 \times 7000}{650 \times (91)^2} = 0.00167$$

The velocity must now be assumed. If Q were passed through area a_1, the velocity would be $\frac{60,000}{7500} = 8$ ft. per second.

Since the area will be increased by the rise, h in the level of the water, the actual velocity will be less than $\frac{Q}{a_1}$.

Assume a velocity of 7 ft. per second.

$$h = \sqrt{0.9233 \times (7)^2 + (5.76 - 0.00167 \times (7)^2)^2} - (5.76 - 0.00167 \times (7)^2) = 3.12 \text{ ft.}$$

To compare this value of h with the conditions of assumed velocity, proceed as follows:

The addition of h ft. to the depth, increases the area of the cross-section hw sq. ft., approximately. For this case, $hw = 3.12 \times 650 = 2028$ sq. ft.

Total area at section $= 7500 + 2028 = 9528$ sq. ft.

Velocity through section $= \frac{60,000}{9528} = 6.3$ ft. per second.

Therefore the assumed velocity of 7 ft. per second is too great by 0.70 ft. per second or about 11 per cent.

In practice it is found that when the assumed value of V is too high, the correct value is nearly equal to the lower value found in the first computation, plus one-fourth the difference between the assumed and computed value. If the assumed value of V is too low, then deduct one-fourth the difference from the higher value computed. Taking the above case.

$$6.3 + \frac{0.70}{4} = 6.475 = V, \text{ and } V^2 = 41.925$$

Then $h = \sqrt{0.9233 \times 41.925 + (5.76 - 0.00167 \times 41.925)^2} - (5.76 - 0.00167 \times 41.925) = 2.79$ ft.

Computing velocity through section as before,

$hw = 2.79 \times 650 = 1813$ sq. ft. increase in sectional area.

Total area of section $= 7500 + 1813 = 9313$ sq. ft.

Velocity through section $= \dfrac{60{,}000}{9313} = 6.44$ ft. per second.

This differs from the second assumed value of V by $6.475 - 6.44 = 0.035$ ft. per second, or less than ½ per cent.

In the same way, the value of h_2 for section a_2 is found, h_3 for section a_3, and the process continued as far back upstream as may be found desirable. From the values of h_1, h_2, h_3, etc., the backwater curve may be plotted. The total rise from the dam back to the last section is, obviously, the sum of h_1, h_2, h_3, etc., as found.

The correctness of the head, velocity and area as found in the preceding example are shown by checking by formula $V = C\sqrt{rs}$.

The perimeter has added to it $1.25 \times 2 \times h$ (approx.) $= 7$ ft., so that total perimeter $= 717$ ft.

$$\text{New } r = \frac{9313}{717} = 12.98.$$

$$\text{New } C = \frac{87}{0.552 + \dfrac{1.30}{\sqrt{12.98}}} = 95.3.$$

$$\text{Slope} = \frac{2.79}{7000} = 0.0004$$

$$V = 95.3\sqrt{12.98 \times 0.0004} = 6.86 \text{ ft. per second}$$

which is within $\dfrac{6.86 - 6.44}{6.44} = 3.1$ per cent. of the value of V computed by the foregoing method.

MEASUREMENT OF STREAM FLOW

There are five practical methods for measuring the flow in streams which are:

1. Velocity of flow measurement with surface floats.
2. Velocity of flow measurement with current meter.
3. Velocity of flow measurement with Pitot tube.
4. Quantity of flow measurement with chemical dosage.
5. Quantity of flow measurement with weirs.

Of these, methods 1, 2, and 3 are simply velocity measurements, and to compute the quantity of flow, surveys of the cross-section of the stream are necessary, just as in the case of computation of flow, and as more fully described hereafter.

Method 4 requires neither the determination of velocity nor of stream cross-section.

Method 5 is, practically, independent of the stream cross-section, and depends only on the cross-section of the weir itself.

Accuracy of Determinations.—While it is not here suggested that measurements of stream flow should be carelessly or hurriedly made, nor that a mere superficial character of engineering is sufficient for hydraulic work, it should be understood at the outset, that attempts at extreme accuracy in flow measurements for water-power developments, result in a waste of time and money. Scientific exactness would be justified, if the régime of a river were constant, but with the flow varying from day to day, and from season to season, subject to no fixed laws, never repeating in any year the conditions of any preceding year, except partially and approximately, and subject to great differences of extreme low water, or heaviest floods; the futility of expecting to settle any question by an accurate measurement of flow at any time, should be obvious.

A river may have, say, 300 cu. ft. per second, as its lowest recorded stage, within a long series of years. Whether a water-power project is financially justified is, certainly, not based on the assumption of, exactly, 300 cu. ft. per second as the criterion. A 10 per cent. variation above or below this would not affect the question. Certainly no experienced engineer would report favorably on a proposed development based on a minimum of 300 cu. ft. per sec. if a low-water stage of 270 cu. ft. per second would mean an inability to meet the demand for primary power and result in a failure of dividends.

The previous history of a river is only an indication of what may be expected, and is, in no wise, a definite prediction of the future stream flow for any season. Hence, a measurement, that is within 2 or 3 per cent. of the exact value, is close enough for every practical purpose.

The one exception to this statement lies in the measurement of the water flowing through water wheels when efficiency tests are conducted. Accuracy within ½ per cent. is necessary this particular case

Measurement by Floats.—To find the velocity of a stream by floats, select two points along the stream about 300 ft. apart. These should be located somewhere along the stream where it runs straight, without curves, bends, falls, or eddy whirls, and the current is down the middle of the stream—not near either bank. Make a cross-section survey of the stream at both points, and determine the area of each section in square feet. Take the average of these two sections. This gives the mean section. Take the velocity of the stream by means of a float which rides on the surface of the water. The float is put into the current of the stream about 200 ft. above the upper reference point, so that by the time it has been carried down to this point it has attained the velocity of the stream. Observe, accurately, the time required for the float to travel from the upper point to the lower one. Knowing the time in seconds and the number of feet the two points are apart, the velocity of the stream flow at the time these observations are taken, may be computed. Several runs should be made at different points the stream surface. Not less than ten runs should be made and the average of these taken as the surface velocity of the water. The float may be a round billet of wood, 4 to 6 in. in diameter, and from 3 to 8 in long. Weights must be fastened to one end of the piece so that it will float vertically, with one end submerged and the other projecting only an inch or two above the surface of the water.

In order to observe, from the bank, the position of the float, it is usual to fasten a small piece of red cloth by a nail or piece of wire, to the upper end of the float.

Parker has found that spherical floats are less affected by wind, and states that oranges make excellent floats, easily detectable in the stream by their color.

The distance apart of the two points selected to observe the float velocity should be accurately measured and stakes driven in the ground near the water's edge to fix these reference points.

The velocity of the float is that of the surface of the water and is not to be taken as the mean velocity of the stream.

In reality, the velocities throughout the cross-section of a stream are widely variable. The velocity at the bottom and at the banks is lowest and it increases gradually, being greatest at some point near the center of gravity of the cross-section—usually slightly above it.

Figure 7 shows the variation in velocity at various parts of the cross-section of a stream. The irregular curved lines are boundaries of stream sections or filaments having the same velocity. The inner section, A, has the highest velocity and there is a continuous decrease in velocity from this section out to the portions in contact with the stream bed. In shallow streams the comparative velocities and section of highest velocity will vary appreciably with the direction and velocity of the wind when blowing strongly.

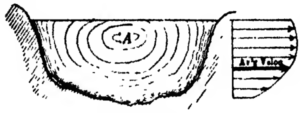


Fig. 7.—Comparative velocities in different sections of a stream.

The ratio of the float velocity to the mean velocity of the stream is widely variable, and depends on many factors, such as form of stream cross-section, depth of submerged portion of the float, direction and strength of the wind, and others.

Probably the most satisfactory coefficients for determining the mean from the surface velocities are those of Grunsky (*Trans.* Am. Soc. Civ. Eng., vol. 66, page 123) which are given in the following table.

Table 7.—Grunsky's Coefficients for Surface Floats

$\frac{W}{d}$	S
5	1.01
10	0 97
15	0.94
20	0.92
30	0 89
40	0.87
50	0.85
100 and above	0.82

W = width of stream in feet
d = average depth in feet
S = coefficient.
Vm = average velocity of stream flow
Vf = float velocity
$Vm = SVf$

These constants are for rivers with sandy beds and will apply only to streams having moderately smooth beds at the points of observation. For rough, stony bottoms, the values will be between 90 and 95 per cent. of the factors given in the table.

Current Meters.—One of the most accurate methods of measuring the velocity of a stream is by means of a current meter. There are several types of this instrument, the principal ones being the Price, Haskell, Fteley and Warren.

All current meters are forms of bucket-wheel, vane or screw

flow motors rotated by the current at a speed which is proportional to the stream velocity and provided with means for recording the number of revolutions in a given time.

According to Parker, the Fteley and Warren are the most accurate meters, but require careful handling and more expert knowledge on the part of the observers than do the others. The Price meter is the one best known and most used in America. It is sufficiently accurate for every practical purpose and is strong

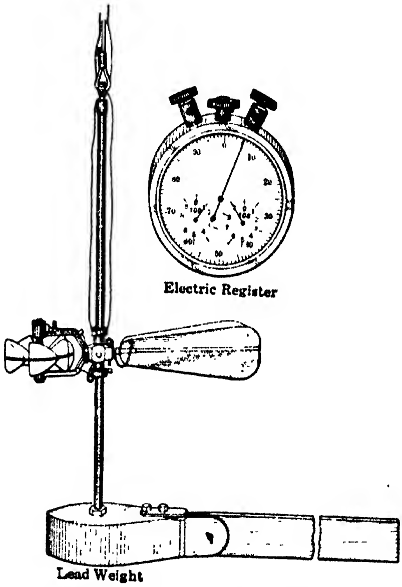


FIG. 8.—Price current meter.

and rugged, so that it maintains its condition as calibrated and is not easily injured.

This machine consists of a series of buckets forming the periphery of a wheel arranged to rotate in a horizontal plane as indicated in Fig. 8 herewith. The buckets are made in the form of hollow cones, as shown. The yoke in which the wheel rotates is pivoted to revolve about a vertical shaft from which latter, the instrument is suspended. A tail vane, having a cross-shaped section, is fastened to the supporting yoke on the other

side of the supporting rod as shown. This vane serves to balance the wheel and yoke at the supporting rod and to maintain the axis of the machine parallel to the stream lines when immersed in a body of moving liquid.

In order to prevent the force of the stream from swinging the supporting rod appreciably out of plumb, and thereby shifting the wheel from its normal horizontal position, a heavy weight is attached to the lower end of the supporting rod, as shown. This weight is shaped to present a sharp edge to the water flowing against it, and a wooden tail piece is fastened to the opposite end which keeps the weight steadily in its proper position, with the sharp edge turned upstream.

The speed of the bucket wheel is recorded by an electrical make-and-break contact on the wheel shaft which actuates a revolution counter that is connected, electrically, by means of twin wires, to the contact on the meter. The meter mechanism may be submerged at any depth and moved about at will, while the revolution counter remains in a boat or is otherwise conveniently located and under continuous observation. With this meter the velocities at various points in the stream cross-section may be taken by simply moving the meter from point to point and a fairly accurate value of the average velocity obtained.

The counting mechanism is frequently made acoustic, instead of visual. In this case, a small hammer strikes against a diaphragm—usually one blow for each ten revolutions. The sound is conveyed from the diaphragm on the instrument to the ear of the observer, through a flexible tube terminating in an ear piece. The acoustic meter is less complicated, has no batteries, counters nor electrical devices, and is, therefore the more dependable form of instrument. It can not be used if the observations are to be made from a bridge, high above the water surface, and is only suitable for measurements when the observer can hold the instrument by its suspension rod.

It should be observed that current meters will give erroneous readings if there are eddies or swirls in the water—the reading being too high or too low, depending on the direction of the swirls.

Hence, the point at which velocity observations are made should be carefully selected, so that the stream filaments flow in parallel paths, and there are no swirls or eddies.

Velocities should never be measured where rocks, boulders or other obstructions are near, nor at bends in the stream.

The current meter is not suitable for measurements where the velocity is less than ½ ft. per second.

There are a number of methods of obtaining the mean velocity of a stream by the use of a current meter. Of these the two principal ones are the *two-point* and the *summation* methods.

For either method the width of the stream is divided into an equal number of parts, just as in making soundings. The number of parts should be not less than ten. For wide streams the width of any section should be not over 20 ft., which will make the number of divisions $\frac{W}{20}$, where W = width of the stream. For two-point observations, two runs are made at each division, one being at a depth of $0.2d$, while the other is made at a depth $= 0.8d$, in which d is the depth of the stream at the division point. Then, one-half the sum of the two readings is the mean velocity of the section. The mean velocity of the stream is the average of the mean velocities of all the sections.

Owing to the fluctuations in velocity at any point in a stream, each reading should be prolonged over at least 2 min. and preferably 3 min.

The summation method consists in lowering the meter as far down toward the bottom of the stream as it can go and have the parts work freely. After remaining in its lowermost position about ¼ min., it is then slowly raised, at a uniform rate, not exceeding 6 ft. per minute, and then held near the surface for the same length of time it remained at the bottom. The average velocity of the meter during the time is the average velocity of the stream at the point selected. This observation is, of course, made at each of the division points, and the average of all the velocities thus obtained is the average velocity of the stream flow.

The summation method has the disadvantage that the raising of the meter at a uniform rate requires skill and patience. The two-point method is as accurate and much the easier of the two.

Current meters must be calibrated from time to time to insure their accuracy. This is usually done by drawing the meter through a body of still water, at a known velocity, and checking the readings against the actual speed of the meter through the water. Du Buat has, however, shown that the effect of drawing

any object through water at a given velocity is not the same as that of water moving at the same velocity, and striking against the object. The difference between the two effects is not very great, and current meters which are rated in this convenient manner will register as nearly the true stream velocity as one rated in a moving stream where the observations of velocity of stream flow are, necessarily, less accurate than the measurement of the speed of the meter drawn through still water.

The following notes on the use and care of the current meter, from a paper by Hoyt, are of value:

1. Be sure that the set screws are all tightened before putting the meter in the water; otherwise one of the parts may be lost.

2. Loosen the sleeve nut and see that the meter runs freely before beginning a measurement; and spin the meter cups occasionally during a measurement to see that they are running freely.

3. See that the weights play freely on the stem, so as to take the direction of the current and thus avoid a drag on the line.

4. If any apparent inconsistency in the results of an observation throws doubt on its accuracy, investigate the cause at once. Grass may be wound around the cup shaft; the cups may be tilted by tension of the contact wire; the channel may be obstructed immediately above the meter; the meter may be in a hole; or the cups may be bent so as to come into contact with the yoke.

5. After a measurement, clean and oil the bearings (in order to prevent rust) and inspect the cone point.

6. In packing the meter, turn the sleeve nut to lift the cups from the cone point.

7. Always see that the lock nut on the cone point is firmly screwed against the cone plug.

8. If the cone point is dulled, it can be sharpened with an oil stone.

9. In measuring low velocities, be sure that the meter is in a horizontal position. If it has a tendency to tip, it can be held in place by using a plug in the slot for the stem.

10. Avoid taking measurements in velocities of less than ½ ft. per second, as the accuracy of the meter diminishes as zero velocity is approached.

11. For velocities of less than 1 ft. per second, the bearing point should be sharp and smooth, as at the time of rating. As the velocity increases, the condition of the point is less important, for then the friction becomes a small factor.

12. In taking measurements at high velocities, sufficient weight, or a stay line, should be used to hold the hanger so that the meter will remain horizontal.

13. In very shallow streams the meter should be suspended from the lower hole on the stem, and the weight should be placed above.

14. If the cups of a small Price meter are bent, they may easily be put in place by using a wood or metal bar with a round, smooth end.

15. The telephone receiver is very sensitive to electric currents, and can be used to locate any break in the circuit. First try the telephone and battery together in a circuit having a make-and-break point. This may be done by using a knife blade or a screw-driver, making connection where the wires enter the plug. If there is no click in the telephone, then the battery or the telephone does not make a circuit. If there is a click, insert the meter in the line and test for a contact in the meter head by revolving the meter wheel. If the meter is all right, put the meter cord in the circuit and test both sides by making double connection and touching alternate sides of the line.

16. When the meter is not in use, disconnect the meter line from the battery, so that it will not become exhausted.

17. Do not strike the telephone receiver, as a heavy jar will, to a greater or less extent demagnetize the pole pieces, and to that extent will injure the receiver.

18. Care must be taken not to short-circuit the dry battery when the meter is not in use, as in that way the cell becomes exhausted in a short time. the energy being used in heating the cell. To avoid this, the poles are wound with adhesive tape.

19. If a dry cell which has been long in stock fails to work well, punch two nail holes in the wax on top of the cell and put it in water over night, when it will absorb enough moisture to renew it. The holes should then be coated over by heating the wax with a match and pressing it into place, or by pouring in melted paraffin. A cell which has been exhausted by use gives but little current after this treatment and may not give any appreciable amount.

Pitot Tube.—An excellent form of meter for measuring velocities is the *Pitot* tube. In its original form it was simply a glass tube having a curved bend in it, the two branches being at right angles to each other. Fig. 9 shows the principle of the device.

One end of the tube is immersed in the water, pointing upstream in the direction of the current, while the other end projects vertically upward from the water surface. The height to which the water rises in the vertical branch above the surface of the water gives the indication of the velocity of flow. Theoretically, this height h, in feet, is equal to $\frac{V^2}{2g}$, in which V = velocity in feet per second.

In the first practical form, the gauge consisted of two tubes, one straight, called the pressure tube, and one bent, called the impact tube, fastened together, as indicated in Fig. 10, each having a small opening at the lower end.

The two tubes are immersed in the water, then taken out and held in a position to be observed conveniently and obtain a

careful measurement of h, which is obviously, the difference in height between the level of the water in the vertical length of the bent tube, and that in the straight tube.

Stopcocks, s, s, are convenient to keep the water from passing through the small openings in the lower ends of the tubes when they are lifted from the water, although the fingers pressed over the upper ends of the tubes will retain the water at the respective original levels until h is read.

The size of the openings immersed in the water should be very small, varying from $\frac{1}{64}$ to $\frac{1}{8}$ in. in diameter. In the case of very high velocity measurements, the nozzle of a stylographic pen has been found to give a good size of opening.

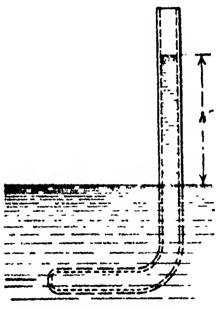


FIG. 9.—Principle of Pitot tube.

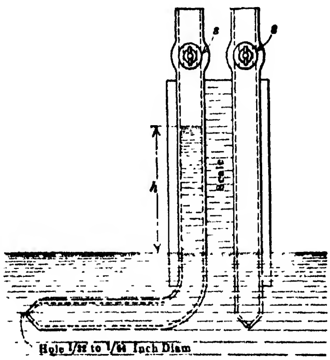


FIG. 10.—Pitot tube.

The simplicity of this device, its low cost, and the ease with which readings can be made in streams, or inside of pipes, would cause it to be universally used if it were not for the general belief that there is considerable difficulty in calibrating it. Ordinary forms of Pitot tubes must be calibrated, as the value of the difference in the water levels in the two tubes is seldom exactly equal to $\frac{V^2}{2g}$. This calibration, for accuracy, must be made under a water velocity nearly the same as that of the water which it will be used to measure. When it is considered that a velocity of 1 ft. per second will produce a rise in the impact tube equal to $\frac{1}{64.4}$ ft., or less than $\frac{1}{5}$ in., it is obvious

that the readings and calibration must be extremely accurate to record low velocities with any degree of exactness. The Pitot tube is not useful for practical measurements below 2 ft. per second.

The principal experimenters with these devices seem to have concluded that the necessity for calibration and the lack of accuracy, are due to the pressure tube and not to the impact nozzle.[1] White suggests a pressure tube of the form shown in Fig. 11. His experiments show, conclusively, that if the tube be smooth, inside and out, and the orifice be made without burrs, swells, or without changing the walls of the tube in any way, the value of h will be equal to the theoretical value, and no calibration is required, no matter what the form and size of the opening in the impact tube may be. In the light of these experiments and the important result, it is difficult to understand why this form of pressure tube has not been more universally used.

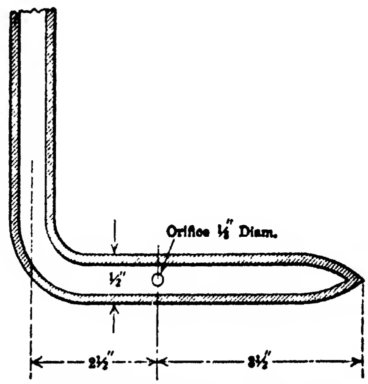


FIG. 11.—White's pressure-tube nozzle.

In measuring the velocity of streams having rough beds and in which the velocity is nowhere constant, the level of the water in the two tubes will oscillate and a definite reading is difficult. This is overcome by placing "damping" drums in the tubes as indicated in Fig. 12.

These are merely short sections of the tubes where the diameter is greatly increased, so that a mass of water is stored in them

[1] WHITE: *Journal of Assoc. of Engineering Societies*, August, 1901.

which has sufficient inertia to prevent the fluctuations in velocity from producing oscillations in the water columns in the tubes. These masses of water do not, however, influence the height of the water columns in the tubes, which heights are the averages of the changing velocities at the mouth of the impact tube.

Figure 12 also shows a construction frequently adopted, namely, that of joining the two tubes together at the top so that by placing the finger over the end of the single tube, at the top of the U bend, the whole apparatus may be lifted out of the water without any change in level of the water columns, and the reading more conveniently taken.

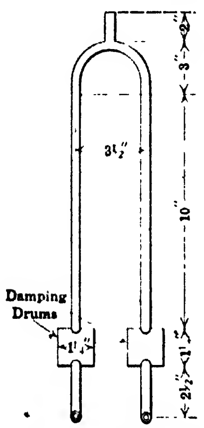


FIG. 12.—Pitot tube with common junction and damping drums.

Differential forms of Pitot tubes have been devised in which a column of some light liquid, such as kerosene, is lifted by the impact of the flowing water, the object being to magnify h, for a given value of V. These do not seem to be suitable for general hydraulic investigations and have, apparently never been used except in rare instances.

Chemical Methods of Stream Measurement.—In the foregoing methods of determining the flow in streams by various kinds of measurements, means for finding the velocities only, have been given. The quantity of water is, of course, computed by multiplying the cross-section of the stream by the average velocity of the water through the known cross-section.

The method of "dosing" the water with some cheap chemical and measuring the dilution, is a method for determining the stream flow which is independent of any knowledge of either stream velocity or cross-section. Hence the cost of cross-section surveys is avoided, when this chemical means is employed.

In his excellent work, "Control of Water," Mr. Philip A. Morley Parker gives some practical formulæ for application of this method.

If a weight of w lb. of a chemical be added each second to Q cu. ft. of water flowing each second, and, after thorough mixture, the water is found to contain 1 lb. of chemical for each n lb. of water, then $\frac{1}{n} = \frac{w}{62.5Q}$, whence, $Q = \frac{nw}{62.5}$ (19)

In order to get satisfactory results, it is essential that the chemical be dissolved in water and the solution added at some definite, uniform rate; the mixing of the solution in the water of the stream must be thorough before the sample for analysis is taken, and the solution itself should be thoroughly mixed and of uniform density.

The best chemicals are common salt, or calcium chloride. The solution made by dissolving the adopted chemical in water should be nearly, but not quite, saturated. This solution is added to the stream from a containing vessel, emerging through a small nozzle, the size depending on the size of the stream to be gauged. The nozzle must work under a constant head, and should be adjusted to give a rate of discharge of the solution of from 0.01 to 0.04 cu. ft. per second per 100 cu. ft. per second of stream flow.

Any form of constant head vessel will serve. Parker describes one with a weir overflow to prevent the head from changing, the solution being fed to the vessel from a larger reservoir at a slightly faster rate than it is discharged from the nozzle. There is, of course, a tank used as a reservoir from which the solution is fed to the constant head vessel through a pipe or hose. A valve in the feed pipe enables the flow from the tank to be adjusted to discharge the solution at any desired rate to the constant-head vessel. The tank and constant-head vessel may be made of any convenient material, such as wood, tin, or sheet steel.

Let V_m = mean velocity of stream in feet per second.
b = breadth of stream in feet.

Then for streams having an average depth between $\frac{b}{10}$ and $\frac{b}{3}$, complete mixture does not occur until a distance of at least $6b$ has been traversed, and the discharge of solution into the water has continued for a period of $\frac{24b}{V_m}$ sec. Samples must be taken while the chemical is still being added, though until a time $= \frac{6b}{V_m}$ sec. has elapsed after ceasing to add the chemical, the water downstream at a distance $= 6b$, still contains the normal amount of solution.

Before attempting a gauging by this method, the water should

be analyzed to discover its actual weight and whether it normally carries any salts in solution. If any salts are found, then the amount of chemical added to the water is found by subtracting from the total amount shown in the analysis, the quantity which the stream normally carries.

This method of measurement is, obviously, particularly adapted to measurement of quantities of water in making turbine tests, where the admixture is thorough, and the "dosing," into a flume or through a hole in the penstock, easy and definite.

It is, however, improbable that thorough mixture will take place in a wide large river unless dosing nozzles are placed at intervals of, say, 100 ft. Also, the quantity of chemical required to gauge a large stream would be considerable. Therefore, for streams discharging over 1000 cu. ft. per second, some other method of gauging will, in general, be preferable.

For common salt, the amount which should be used is 0.30 lb. per second, per 100 cu. ft. sec. of water in the stream, which for a 10-min. run would require 180 lb. of salt per 100 cu. ft. or 1800 lb. for a 1000 cu. ft. sec. measurement.

Parker indicates that a solution containing 22.5 per cent. of salt is a satisfactory one.

The determination of the chemical content of the water should be made by a skilled chemist. The chemistry of this process is beyond the scope of this treatise, and, likewise, beyond the experience of most hydraulic engineers; therefore, it should not be attempted by them. The samples should be carefully corked in bottles—any size from a quart up—and several samples taken from different parts of the stream at each observation.

CHAPTER III

WEIRS AND ORIFICES

Weirs.—A weir may be broadly defined as an artificial obstruction in a water-way, having a definite configuration and a sheer

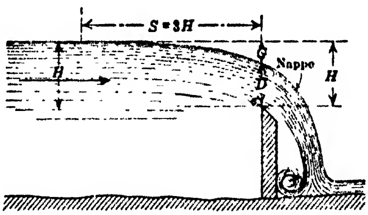


FIG. 13.—Sharp-edged weir.

fall on the downstream side over which the water falls in a sheet. Thus, the well-known measuring notch, made in a board set vertically in the stream, is one type of weir, while the spillway of a dam is another type.

In general, weirs are further subdivided as "broad-crested," and "thin-crested."

A thin-crested weir is one in which the edge, over which the water pours, is sharp, as indicated in Fig. 13. The bevelled surface is always turned downstream, the sharp edge being turned upstream as indicated.

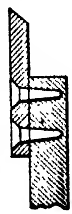


FIG. 14. Steel weir edge.

For accurate measurements, such as are required in testing hydraulic machinery, the edges of the notch should be made of flat steel, filed or machined fairly sharp, as indicated in Fig. 14. In the case of weirs having a depth of 1 ft., or more, over the crest, the edges may be $\frac{3}{16}$ in. in thickness.

A broad-crested weir is, obviously, one not sharpened, such as the spillway of a dam, or a flat board, placed as shown in Fig. 13, but without the edge sharpened.

Weirs are further classified according to whether they form merely a fallway, extending clear from one side of the stream to the other, or whether they also constrict the width of the stream. Thus, in Fig. 15, *a* is a weir without end contractions, *b* is a weir with one end contraction and *c* is a weir with two end contractions. The last one is the type used, almost exclusively in test measurements.

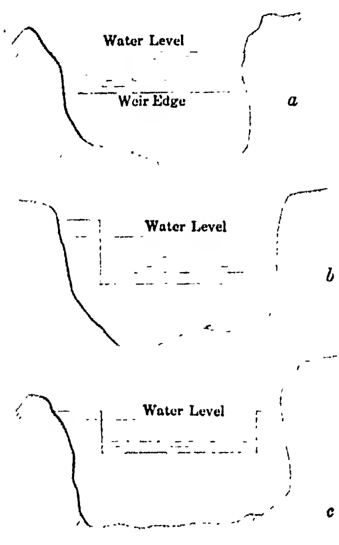


FIG. 15.—Forms of weirs.

Weirs may be further classified as rectangular, trapezoidal, (also called Cippoletti) and triangular. The weirs shown in Fig. 15 are rectangular weirs. The trapezoidal and triangular weirs will be discussed later in this Chapter.

Many investigations and experiments have been made on various forms and sizes of weirs, and under different conditions of stream flow, but the original formulæ, deduced by J. B. Francis from his Lowell experiments, in 1842, are still regarded and accepted as the most universally applicable, easily used, and accurate, when applied to rectangular weirs.

The experiments of Bazin,[1] Smith,[2] Rafter,[3] Fteley & Stearns,[4] Lyman[5] and others, all tend to confirm the real, practical value of the Francis formulæ.

In Fig. 13 is shown a sharp-edged weir, with the water spilling over it. The actual thickness of the water at the weir and passing over its crest is *D*, but the head acting on the weir is *H* as

[1] *Annales des Ponts et Chaussées*, October, 1888.
[2] "*Hydraulics*," HAMILTON SMITH, SR.
[3] *Trans.* Am. Soc. Civ. Eng., vol. XLIV.
[4] *Trans.* Am. Soc. Civ. Eng., vol. XII.
[5] *Trans.* Am. Soc. Civ. Eng., September, 1913.

indicated, H being the difference between the elevation of the weir crest and that of the water surface at a point some distance back from the weir crest. As shown, the water surface gradually drops as the weir is approached, and takes the form of a smooth, parabolic curve. This drop is due to the increasing velocity of the water, and the diminution of water level, at any point, is equal to $\frac{V_2^2 - V_1^2}{2g}$ ft., in which

V_1 = velocity at some distance back from the weir (feet per second).

V_2 = velocity at point where drop is to be computed (feet per second).

g = acceleration due to gravity = 32.2.

The distance S, from the weir crest back upstream to the point where the curve ceases and the water surface becomes level, varies with the depth of water over the crest. It is not a very definite function, and seems to vary from $3H$ to $3.5H$.

When measurements of H are made, the elevation of the water surface should be taken at a distance at least $5H$, upstream from the weir, but not further away than $10H$. Of course, H has to be first approximated to fix these distances, but as it is simply a question of getting a measurement upstream, a distance not less than $5H$, this presents no difficulty.

The drop, G, in water level, at the crest, is widely variable, lying between $\frac{H}{10}$ and $\frac{H}{4}$. Its value depends, largely, on the form of crest. It is much greater for broad-topped than for thin-crested weirs.

For approximate measurements of H, a stake may be driven into the bottom of the stream bed $5H$ ft. upstream, and cut off so that its upper end is at exactly the same height as the weir crest. The depth of water is then measured with an ordinary carpenter's rule, by placing it vertical, with one end resting on the stake, and reading the distance to the water surface.

If, however, the flow of water over the weir is to be determined with any degree of accuracy, a hook gauge and gauge well or "still box" are essential to measure the elevation of the water surface.

Hook Gauge.—The hook gauge is so called from its form. Fig. 16 shows one of a kind frequently made in the field. The sharp, upturned point, p, is raised until it is just level with the surface

of the water. The top of the rod, of which the hook and point form the lower portion, has its elevation measured against a scale *s*, the elevation of the scale being accurately fixed by a Y-level, or other equally exact means. The scale itself should be a standard, finely divided steel scale fastened to a wooden

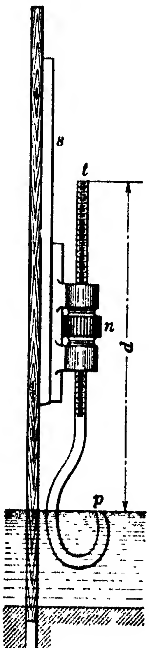


FIG. 16.—Hook gauge.

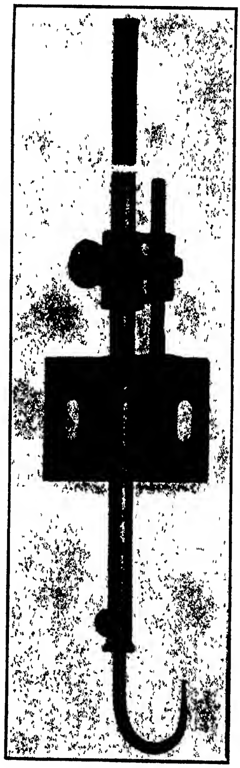


FIG. 17.—Hook gauge with vernier.

support. The gauge is made with a fixed distance, *d*, between the point and the top of the rod. Knowing the scale elevation, and *d*, the exact water level is easily calculated.

Movement of the hook rod is effected by the milled nut, *n*, encircling the rod and working between two rod guides, the rod

being threaded. In this way, extremely small changes in the elevation of the point can be effected.

These devices, while simple, must be made with extreme accuracy, as very small errors in determination of the head over a weir, will produce a considerable variation from the true value of water discharged.

A much better and more accurate hook gauge is that shown in Fig. 17 which is provided with a vernier for more exact readings of the head. It is recommended that this form be used whenever obtainable.

It takes but little practice to make accurate observations with a hook gauge. The lifting up of the water surface, without the point actually breaking through the surface film, is well-defined and easily observable.

Still Box.—Measurements of the elevation of the water level require that the surface be smooth and still. Hence, the hook gauge must be placed in a "still box." This device may take many forms. Essentially, it consists of a vessel having its bottom closed but the top open. Small holes are bored through the sides and bottom of the vessel, which is fastened to a support in the water, so that its bottom is at least 1 ft. below the surface of the water to be measured, and the top projects high enough above the surface to prevent water from spilling over it. The hook gauge is set inside the still box, and measures the elevation of the tranquil water inside, the level of which is the same as the mean level of the surrounding water into which the still box is immersed. Obviously, the still box cannot be set in the path of the water, but must be placed near one bank of the stream, and as far out of the current as practicable. Usually, a square box is made, measuring 8 to 10 in. inside, and from 2 to 5 ft. long, depending on the variation in the elevation of the water that it is intended to measure.

Figure 18 shows a satisfactory form of still box. The dimensions and construction are clear from the figure. As shown, a long plank forms one side of the box, but projects beyond the box at both ends. The upper projection serves to support the scale of the hook gauge and the gauge itself, while the lower end is sharpened and driven firmly into the stream bed, so that it forms the support for the entire device. Of course, many convenient structures will suggest themselves. The principal conditions to be maintained are, that (1) the size of the box be

amply large to operate and read the hook gauge; (2) the size be not too large (not above 18 in. square) so that small ripples inside the box may be avoided; (3) the height of the box above the water line should not exceed 8 in. as it becomes somewhat difficult to determine, exactly, just when the hook gauge point is adjusted to the water level; (4) the thickness of the material of which the box is made should be preferably ½ in. or more so that pulsations of the outside mass of water are not transmitted through the holes, but are "damped" out.

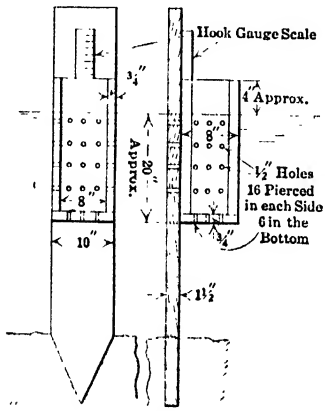


FIG. 18.—Still box.

Formulæ for Weir without End Contractions.—

If Q = quantity of water in cubic feet per second.
H = head on the weir.
Q_1 = cu. ft. sec. per foot length of weir (20)
L = length of weir in feet.
$Q_1 = 3.33H^{3/2}$
$Q = 3.33LH^{3/2}$ (21)

These formulæ assume that the velocity of the water flowing to the weir, or "velocity of approach," is low and its influence negligible.

If the velocity of approach is such that its equivalent head is 5 per cent., or more, of the head over the weir, the computation must be corrected to include it.

Since $h = \frac{V^2}{2g}$, the total head on the weir to give velocity to the water flowing over the crest is $H + h = H + \frac{V^2}{2g}$, in which
h = equivalent head due to velocity of flow.
V = velocity at point distant $3H$, or more, above weir crest—all in foot, second units. While the head to cause velocity is $H + h$, the height of water over the weir crest, which fixes the area of the overfall, is not changed by the velocity of approach. Since Q is dependent on both the velocity and the cross-section, the quantity discharged over the weir is *not* $3.33L\ (H + h)^{3/2}$ as is erroneously stated by some writers, but:

$$Q_1 = 3.33[(H + h)^{3/2} - h^{3/2}] \text{ per foot length} \tag{22}$$

or

$$Q = 3.33L[(H + h^{3/2}) - h^{3/2}] \tag{23}$$

all in foot, second units.

Formulæ for Weirs with End Contractions.—Where the width of the stream is constricted by the weir, as in *b* and *c*, Fig. 15, the formulæ previously given do not give the discharge accurately without a correction factor.

Francis formula for a constricted rectangular weir with very low velocity of approach is

$$Q = 3.33\ (L - 0.1nH)H^{3/2} \tag{24}$$

in which n = number of end contractions.

The weir almost universally employed for accurate measurements, is one which constricts the stream on both sides, like that shown in *c*, Fig. 15. For this case, the number of end contractions, n, is 2. Hence the formula (neglecting velocity of approach) becomes

$$Q = 3.33(L - 0.2H)H^{3/2} \tag{25}$$

Where the velocity of approach is considered, the value of H inside the parenthesis is unchanged, but $H^{3/2}$ becomes $(H + h)^{3/2} - h^{3/2}$ so that the complete general formula is,

$$Q = 3.33(L - 0.1nH)[(H + h)^{3/2} - h^{3/2})] \tag{26}$$

Following is a table giving the discharge over weirs of various lengths and depth of water over the crest. Velocity of approach neglected.

TABLE 8.—DISCHARGE OVER RECTANGULAR WEIRS, WITH AND WITHOUT CONTRACTION

Formula, $D = 3\frac{1}{3}(L - 0.2H)H^{3/2}$

Depth, H, of water on crest measured in still water. See page 43		Discharge in cubic feet per second						
		With two complete contractions No velocity of approach						Correction to be added to each of the preceding to give discharge with no contraction
In inches	In feet	1 ft. long	1½ ft. long	2 ft. long	3 ft. long	5 ft. long	10 ft. long	
0.3	0.025	0.0133	0.0200	0.0267	0.0400	0.0677	0.1330	0.0000
0.6	0.050	0.0369	0.0556	0.0743	0.1116	0.1863	0.3716	0.0004
0.9	0.075	0.0674	0.1015	0.1350	0.2040	0.3410	0.6830	0.0010
1.2	0.1	0.1033	0.1560	0.2087	0.3132	0.5240	1.0519	0.0021
1.5	0.125	0.1438	0.2175	0.2912	0.4385	0.7332	1.4695	0.0037
1.8	0.15	0.1879	0.2847	0.3816	0.5753	0.9627	1.9312	0.0058
2.1	0.175	0.2355	0.3575	0.4795	0.7235	1.2115	2.4315	0.0085
2.4	0.2	0.2861	0.4352	0.5843	0.8824	1.4787	2.9690	0.0119
2.7	0.225	0.3399	0.5177	0.6956	1.0513	1.7627	3.5412	0.0160
3.0	0.25	0.3959	0.6042	0.8126	1.2293	2.0227	4.1462	0.0208
3.3	0.275	0.4543	0.6946	0.9350	1.4157	2.3771	4.7803	0.0264
3.6	0.3	0.5149	0.7888	1.0627	1.6104	2.7059	5.4442	0.0328
3.9	0.325	0.5775	0.8863	1.1952	1.8129	3.0482	6.1363	0.0401
4.2	0.35	0.6420	0.9871	1.3321	2.0223	3.4032	6.8537	0.0483
4.5	0.375	0.7079	1.0909	1.4732	2.2335	3.7691	7.5976	0.0574
4.8	0.4	.....	1.1974	1.6189	2.4623	4.1485	8.3645	0.0675
5.1	0.425	.....	1.3070	1.7680	2.6926	4.5400	9.1565	0.0785
5.4	0.45	.....	1.4189	1.9221	2.9280	4.9404	9.9775	0.0906
5.7	0.475	.....	1.5333	2.0790	3.1708	5.3523	10.8094	0.1037
6.0	0.5	.....	1.6500	2.2392	3.4177	5.7748	11.6672	0.1178
6.3	0.525	.....	1.7689	2.4029	3.6709	6.2069	12.5469	0.1331
6.6	0.55	.....	1.8890	2.5698	3.9295	6.6489	13.4474	0.1496
6.9	0.575	.....	2.0129	2.7395	4.1928	7.0995	14.3668	0.1672
7.2	0.6	.....	2.1378	2.9123	4.4614	7.5596	15.3052	0.1859
7.5	0.625	.....	2.2646	3.0881	4.7351	8.0291	16.2641	0.2059
7.8	0.65	.....	2.3929	3.2665	5.0133	8.5069	17.2409	0.2271
8.1	0.675	.....	2.5234	3.3478	5.2960	8.9930	18.2354	0.2495
8.4	0.7	.....	.....	3.6313	5.5836	9.4832	19.2497	0.2733
8.7	0.725	.....	.....	3.8170	5.8747	9.9906	20.2796	0.2984
9.0	0.75	.....	.....	4.0052	6.1702	10.5007	21.3262	0.3248
9.3	0.775	.....	.....	4.1961	6.4704	11.0190	22.3895	0.3524
9.6	0.8	.....	.....	4.3888	6.7734	11.5444	23.4704	0.3816
9.9	0.825	.....	.....	4.5833	7.0810	12.0769	24.5659	0.4121
10.2	0.85	.....	.....	4.7806	7.3929	12.6169	25.6779	0.4440
10.5	0.875	.....	.....	4.9792	7.7075	13.1641	26.8056	0.4775
10.8	0.9	.....	.....	.....	8.0257	13.7177	27.9477	0.5123
11.1	0.925	.....	.....	.....	8.3509	14.2839	29.1164	0.5486
11.4	0.95	.....	.....	.....	8.6731	14.8461	30.2786	0.5864
11.7	0.975	.....	.....	.....	9.0012	15.4192	31.4652	0.6258
12.0	1.0	.....	.....	.....	9.3333	16.0000	32.6667	0.6667
12.3	1.025	.....	.....	.....	9.6685	16.5869	33.8829	0.7091
12.6	1.05	.....	.....	.....	10.0058	17.1789	35.1109	0.7531
12.9	1.075	.....	.....	.....	10.3471	17.7777	36.3552	0.7988

TABLE 8.—DISCHARGE OVER RECTANGULAR WEIRS, WITH AND WITHOUT CONTRACTION.—*Continued*

Depth, *H*, of water on crest measured in still water. See page 43		Discharge in cubic feet per second						
		With two complete contractions No velocity of approach						Correction to be added to each of the preceding to give discharge with no contraction
In inches	In feet	1 ft. long	1½ ft. long	2 ft. long	3 ft. long	5 ft. long	10 ft. long	
13.2	1.1	...	...	...	10.6907	18.3825	37.6100	0.8490
13.5	1.125	...	...	...	11.0376	18.9926	38.8801	0.8949
13.8	1.150	...	...	...	11.3866	19.6080	40.1625	0.9455
14.1	1.175	...	...	...	11.7388	20.2298	41.4573	0.9977
14.4	1.2	...	...	...	12.0935	20.8569	42.7664	1.0516
14.7	1.225	...	...	...	12.4507	21.4893	44.0866	1.1073
15.0	1.25	...	...	...	12.8100	22.1279	45.4204	1.1646
15.3	1.275	...	...	...	13.1733	22.7713	46.7653	1.2237
15.6	1.3	...	...	...	13.5375	23.4189	48.1224	1.2846
15.9	1.325	...	...	...	13.9067	24.0727	49.4927	1.3473
16.2	1.35	...	...	...	14.2740	24.7308	50.8733	1.4117
16.5	1.375	...	...	...	14.6450	25.3946	52.2671	1.4780
16.8	1.4	...	...	...	...	26.0625	53.6710	1.5460
17.1	1.425	...	...	...	...	26.7355	55.0870	1.6160
17.4	1.45	...	...	...	...	27.4127	56.5132	1.6878
17.7	1.475	...	...	...	...	28.0950	57.9515	1.7615
18.0	1.5	...	...	...	...	28.7814	59.3999	1.8371
18.3	1.525	...	...	...	...	29.4729	60.8604	1.9146
18.6	1.55	...	...	...	...	30.1680	62.3300	1.9940
18.9	1.575	...	...	...	...	30.8681	63.8116	2.0754
19.2	1.6	...	...	...	...	31.5717	65.3022	2.1588
19.5	1.625	...	...	...	...	32.2809	66.8049	2.2441
19.8	1.650	...	...	...	...	32.9935	68.3175	2.3314
20.1	1.675	...	...	...	...	33.7093	69.8393	2.4207
20.4	1.7	...	...	...	...	34.4299	71.3719	2.5121
20.7	1.725	...	...	...	...	35.1546	72.9146	2.6054
21.0	1.750	...	...	...	...	35.8827	74.4672	2.7009
21.3	1.775	...	...	...	...	36.6151	76.0286	2.7984
21.6	1.8	...	...	...	...	37.2510	77.6002	2.8979
21.9	1.825	...	...	...	...	38.0909	79.1814	2.9996
22.2	1.85	...	...	...	...	38.8346	80.7726	3.1034
22.5	1.875	...	...	...	...	39.5812	82.3717	3.2093
22.8	1.9	...	...	...	...	40.3321	83.9816	3.3173
23.1	1.925	...	...	...	...	41.0860	85.6005	3.4276
23.4	1.95	...	...	...	...	41.8436	87.2271	3.5399
23.7	1.975	...	...	...	...	42.6045	88.8635	3.6545
24.0	2.0	...	...	...	...	43.3695	90.5061	3.771
27.0	2.25	...	...	...	...	...	107.44	5.06
30.0	2.50	...	...	...	...	...	125.17	6.59
36.0	3.00	...	...	...	...	...	162.81	10.39

Broad-topped Weirs.—Although the discharge over broad-topped weirs is not accurately given by the Francis formula, the flow as computed from it, is so nearly the same as that de-

rived from later and more exact formulæ, that it may be employed for practical use in water-power engineering.

Usually, the sole computation that is made for discharge over broad-topped weirs, is for the purpose of fixing the length of the spillway of a dam, and the maximum thickness of flood waters over it. Since this is, at best, an approximation only, as has been previously indicated herein, there is no real advantage in following through the elaborate experiments of the several authorities who have produced other formulæ and graphical solutions of this problem. Any engineer who desires to follow these investigations as a matter of scientific interest, is referred to the several papers and treatises on the subject before mentioned.

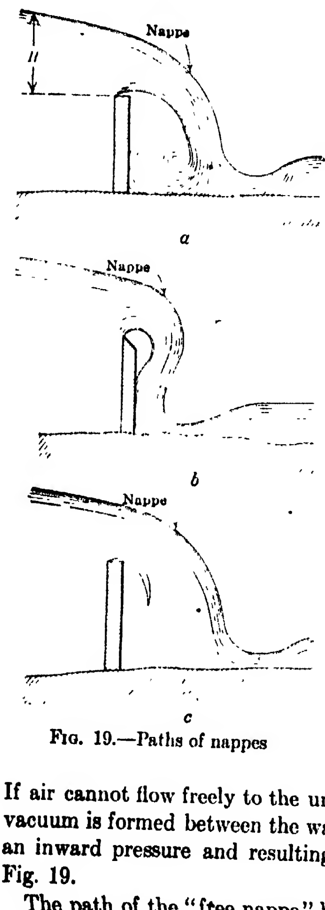


FIG. 19.—Paths of nappes

Path of Nappe over Weirs.—The sheet of water falling over the weir crest—called the "nappe"—may take any one of several paths, depending on the conditions.

The usual path is that of the "free nappe," shown in previous figures and also in *a*, Fig. 19. This form assumes that air can flow freely to the under-surface of the nappe, which is always the case with weirs having end contractions. If air cannot flow freely to the underside of the nappe, a partial vacuum is formed between the water sheet and the weir, causing an inward pressure and resulting in the path indicated in *b* of Fig. 19.

The path of the "free nappe" has been investigated by Bazin. Parker gives a table[1] which shows the ordinates of the curves

[1] PHILIP A. MORLEY PARKER: "Control of Water."

forming upper and lower boundaries of the nappe from the points where they begin, to a distance downstream equal to $0.7H$, H being the head on the crest of the weir.

Referring to Fig. 13, it is seen that the curve of the upper boundary begins at a distance about $3H$ upstream from the weir crest. Also, the under-surface of the water does not begin to fall immediately after it passes the crest but it actually rises slightly as indicated at e.

Taking the origin of the curve at the weir crest, the curve of the upper boundary curve can be plotted as follows:

TABLE 9.—COÖRDINATES OF UPPER CURVE OF FREE NAPPE

For $\frac{x}{H}$ =	−3	−1	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7
$\frac{y}{H}$ =	0.997	0.963	0.851	0.826	0.795	0.762	0.724	0.680	0.627	0.596

The coördinates for the lower boundary curve are as follows:

TABLE 10.—COÖRDINATES OF LOWER CURVE OF FREE NAPPE

For $\frac{x}{H}$ =	0.0	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70
$\frac{y}{H}$ =	0.0	0.085	0.109	0.112	0.111	0.097	0.071	0.035	0.009

The continuation of the lower boundary curve is plotted from the formula of Müller

$$x^2 = 2.3Hy \tag{27}$$

This latter is only an approximation, if H is less than 3 ft.

These tables and formulæ for the lower boundary are useful in laying out the curve of the upper section of the spillway of a dam. It has been customary to approximate this curve as a parabola having the equation,

$$x^2 = 1.78hy \tag{28}$$

The curve thus computed is probably as close to the actual possible contour of a masonry dam as workmen will make it, but to guard against the overflowing waters springing clear of the spillway and falling to the tail water, causing scour, instead of being guided over the curves of the natural water path and

easily down to tail water, the more accurate tables and formula should be used.

If the air is completely excluded from the under-surface of the nappe, the mass of water from the weir crest down to the stream bed becomes practically solid, as indicated in *c*, Fig. 19. A vacuum is formed as shown by the dotted lines in the figure.

The movement of the water, in cases of partial or complete vacuum, produces a downstream pull against the weir, which, in the case of large dams, may be great enough to form a considerable stress which must be guarded against by arranging for admission of air underneath the nappe.

"Drowned" Weirs.—When the water level on the downstream side of a river is higher than the weir crest, as indicated in *a* and

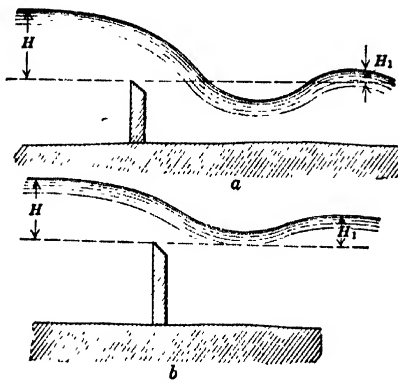


Fig. 20.—Submerged weirs.

b, Fig. 20, the weir is called a "drowned," or submerged weir. The Fteley & Stearns formula for discharge over a submerged weir is as follows:

Calling Z the net head over the weir, $Z = H - H_1$, where H = elevation of water upstream, above weir, and H_1 is elevation of water downstream, above weir.

Without end contractions,

$$Q = CL\sqrt{Z}\left(H + \frac{H_1}{2}\right) \tag{29}$$

C is a constant depending on the ratio $\frac{H_1}{H}$. Values of these constants are given in Table 11 herewith.

TABLE 11.—VALUES OF C FOR FTELEY & STEARNS WEIR FORMULA

$\frac{H_1}{H}$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
C	3.37	3.29	3.21	3.16	3.11	3.09	3.09	3.12	3.19	3.36

The correction for velocity of approach is not so definite as in the case of unsubmerged weirs, but is approximately given by adding $h_v = \frac{V_A^2}{2g}$, to H, V_A being the velocity of approach. The complete formula then becomes;

$$Q = CL\sqrt{Z}\left(H + h_v + \frac{H_1}{2}\right) \tag{30}$$

If there are end contractions, change L to $(L - 0.2H)$.

As an example, consider a submerged weir 10 ft. long, the water level upstream being 5 ft. above the weir crest, while the water level downstream is 3 ft. above the crest. Velocity of approach by measurement (float or meter) = 2 ft. per second.

$$h_v = \frac{4}{64.4} = 0.0622 \text{ ft.}$$

Hence, $H + h_v = 5.0622$. $Z = 5 - 3 = 2$.

$\frac{H_1}{H} = \frac{3}{5} = 0.6$. Value of C corresponding to 0.6 is 3.09.

$$Q = 3.09 \times 10\sqrt{2}\left(5.0622 + \frac{3}{2}\right) = 286 \text{ cu. ft. sec.}$$

Ratio of Length to Depth.—For measuring weirs, to which the Francis formula is to be applied, the results will be more accurate if the ratio of length to depth is between 5 to 1 and 10 to 1. Hence, for a Francis weir, with two end contractions, L should be between $1.625Q^{2/5}$ and $2.44Q^{2/5}$, the corresponding depths, H, being, $0.325Q^{2/5}$ and $0.244Q^{2/5}$.

As an example, to fix the length of weir for a discharge of 400 cu. ft. per second.

$$Q = 400$$
$$\log Q = \log 400 = 2.6021.$$
$$\log Q^{2/5} = \frac{2}{5}\log Q = \frac{2.6021 \times 2}{5} = 1.04084$$

Number, whose log is 1.04084, is 10.98

Hence,

$$Q^{2/5} = 10.98$$

and L must lie between

$$10.98 \times 1.625 = 17.84 \text{ ft., and}$$
$$10.98 \times 2.44 = 26.8 \text{ ft.}$$

or practically, between 18 and 27 feet.

If there are varying quantities of flow to be measured, and 400 cu. ft. per second is the largest, then the shortest length of weir should be adopted, because as the quantity of water is reduced, the ratio of length to depth is increased, and may, in certain cases, exceed the preferred limit of 10 to 1.

Bazin's Formula.—For most accurate determinations, with sharp-crested weirs, they should be made without end contractions, by lining the sides of the channel with wood sheeting or concrete, for a distance back from the weir equal to about $10H$, then building the weir across from side to side.

In this case *Bazin's* formula should be used which is

$$Q = mLH\sqrt{2gH} \text{ cu. ft. sec.} \tag{31}$$

in which

$$m = \left(0.405 + \frac{0.00984}{H}\right)\left(1 + 0.55\left(\frac{H}{p+H}\right)\right) \tag{32}$$

p = height from bottom of channel to crest of weir.

$Q_1 = 8.025\, H^{3/2} m$ cu. ft. sec. per foot length of weir.

Bazin's correction of H for velocity of approach is

$$H_1 = H + 1.68h \tag{33}$$

in which

$h = \frac{V^2}{2g}$, V being the velocity of approach.

The value of H_1 should be substituted for H in the preceding formula.

Weir Contractions.—In order that the equations for discharge over weirs may apply with reasonable accuracy, the contractions must have certain minimum values and these must be maintained. The vertical distance from the bottom of the stream bed to the crest of the weir should, under no conditions, be less than 1 ft. The best theoretical height is $3H$, though the readings will be well within the limits of good practice if this height is not less than H. Also, the distance from the side of the notch to the

adjacent bank of the stream, should be at least as great as H, and $2H$ is a better value. These relationships are all indicated in Fig. 21. In very large weirs, these clearances are not usually practicable, nor are they so necessary as in small weirs.

The side of the stream should not be constricted very close to the weir on the downstream side, as the nappe should be allowed room to expand until it falls down to the level of the tail water. Fig. 21 will make this requirement clear.

In silt-bearing, or rapidly moving streams, silt, sand or gravel will pile up in the stream bed and in this way suppress the contractions, either partially or completely, and thereby introduce errors in the determination of the stream flow. This should be prevented by keeping the bed cleared out on the upstream side of the weir, removing the deposits whenever they reach a point less than H ft. distant from the weir crest.

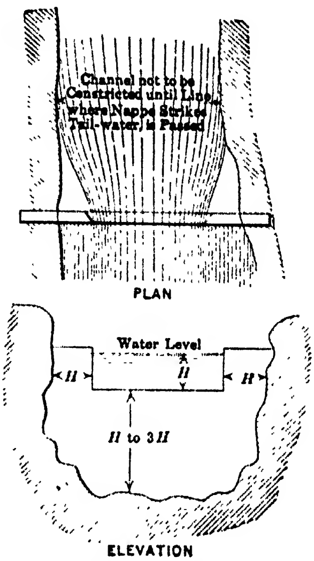


Fig. 21.—Plan and elevation of weir showing proper proportions.

Practical Weir Construction.—Where large weirs are to be built, say to measure above 300 cu. ft. per second, it is usually better to make them of concrete, using a thin steel inset to give sharp edges around the notch. Fig. 22 shows a design that has been used with satisfactory results. Such a weir is self-supporting, just as is a concrete dam.

Small weirs may be constructed of planks and timber. Fig. 23 shows in a general way, one of the approved methods of making a wooden weir. The weir must have a floor on the downstream side to prevent erosion of the stream bed from the overfall and consequent settling of the whole structure. This is important, as the crest must be maintained exactly level, and the whole weir must be exactly vertical. In order to prevent settling

of one side or the other, heavy stones should be sunk under the mud sills, as shown. The upstream mudsills and outer edges of the planking must be thoroughly puddled and all the joints must

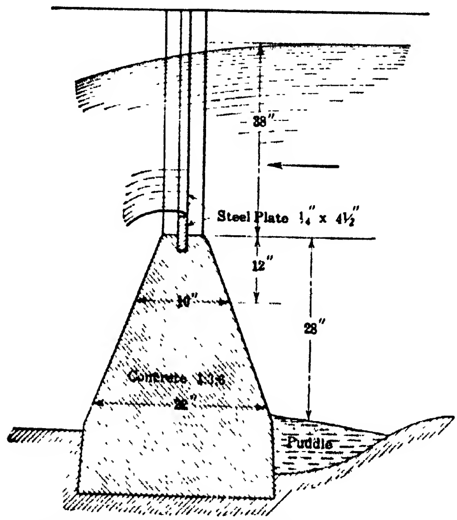


FIG. 22.—Concrete measuring weir with steel edges.

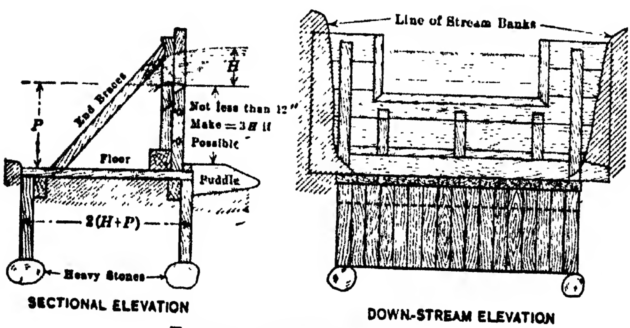


FIG. 23.—Timber measuring weir.

be covered with canvas and tarred. Obviously, there must be absolutely no leakage.

The water pressure tending to slide the whole structure down-

stream, and the moment to overturn the weir, can easily be computed by the formulæ given in the chapter on "Dams." The bracing, thickness of planking, and all dimensions and disposition of the timbers, must be fixed to resist these forces, and further, there must not be any appreciable deflection, either in the downstream direction or in the plane of the floor.

A sharp-edged weir is an accurate, water-measuring instrument, and it must be constructed with great care and in accordance with the requirements in making any other measuring apparatus.

Trapezoidal (Cippoletti) Weirs.—This form of weir is shown in Fig. 24, which also gives the relationship of the slope of the sides to depth of water. It is used principally for measuring small quantities of water, say up to 50 cu. ft. per second, but

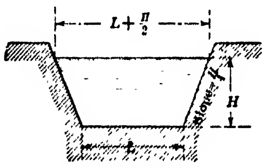


FIG. 24.—Trapezoidal weir.

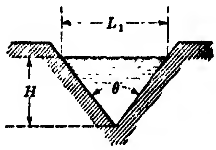


FIG. 25.—Triangular weir.

is seldom used for the large quantities of water generally encountered in water-power engineering.

The general formula for discharge through any trapezoidal weir is seldom used as it requires the determination of several constants. The specific form, shown in Fig. 24 and which was devised by the Italian engineer, Cippoletti, has the inclination of its sides such that the increase in the length with increase in H, just compensates for the correction factor for contraction in the Francis formula. The slope of the sides is one in four, as indicated. Hence, with a Cippoletti weir there is no correction for contraction.

Cippoletti's experiments showed that

$$Q = 3.367LH^{3/2} \text{ cu. ft. per second} \tag{34}$$

L = width at weir crest, in feet.

Triangular Weirs.—These are used almost exclusively for measuring small quantities of water in laboratories. Fig. 25 shows a weir of this kind. The formula for discharge over them is

$$Q = 1.27LH^{3/2} \text{ cu. ft. per second} \qquad (35)$$
$$L = \text{length, at water level, in feet.}$$

If the angle, ϕ is made 90°, then $L = 2H$, and the formula for this specific condition becomes,

$$Q = 2.54H^{5/2} \text{ cu. ft. per second} \qquad (36)$$

Both the foregoing formulæ neglect velocity of approach.

Orifices.—The complete theory of flow of water through orifices is not pertinent to the subject of development of water powers. The only three cases that are of importance here are:

(1) Efflux through sluice gates.
(2) Flow through penstock intakes.
(3) Efflux through nozzles or guide vanes of water wheels.

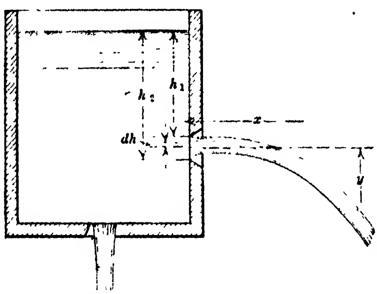


FIG. 26.—Sharp edged orifices and path of jet.

In general, the efflux through an orifice having a given area and under a constant head will differ with different forms of orifices. Theoretically, the velocity through any orifice will be $V = \sqrt{2gh}$, but this theoretical value of V is seldom attained.

One reason why the actual velocity of the jet is less than the theoretical, is that a small portion of the head is used up in the internal friction of the mass of water due to the converging of the stream lines from various parts of the mass to the opening, which loss of head does not appear as velocity, but is totally dissipated. Other reasons are, the friction of flow through the orifice, and the contraction of the cross-section of the jet.

If the edges of an orifice be made sharp, as in Fig. 26, the velocity is 0.97 of the theoretical.

As the jet acquires the velocity due to the head forcing it

through the orifice, it contracts in cross-section, so that the area of the jet becomes only 0.63 of the area of the orifice.

The discharge through the orifice being the product of velocity by area, the quantity of flow becomes

$$Q = 0.97\sqrt{2gh} \times 0.63A = 0.61A\sqrt{2gh} \qquad (37)$$

in which A = area of the orifice in square feet.

In other words, the actual discharge through a thin-edged orifice is 61 per cent. of the theoretical discharge.

The general form of equation is, $Q = CA\sqrt{2gh}$, in which C is a constant, and in the foregoing case, is equal to 0.61.

For other forms of openings the coefficient, C_v, C_c, and their product, $C_vC_c = C$, are given in Fig. 27.

Formula 37 is for small orifices in a horizontal plane in which the pressure, due to the head, is the same at every point in the periphery of the orifice.

Where the hole is in a vertical plane, and rectangular in form, the quantity discharged per second is,

$$Q = \tfrac{2}{3}\,Cl \times 8.02(h_2^{3/2} - h_1^{3/2})$$
$$= 5.347Cl(h_2^{3/2} - h_1^{3/2}) \qquad (38)$$

l being the length of the opening.

h_1 being the head to the top of the opening.

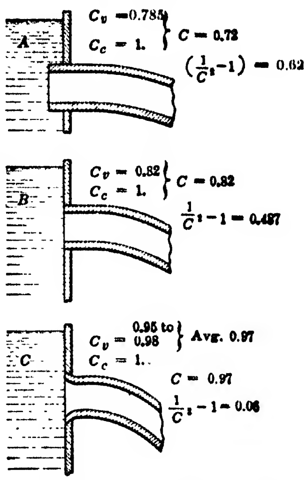


FIG. 27.—Tube ends of various forms with corresponding discharge coefficients.

h_2 being the head to the bottom of the opening.

This formula is derived as follows:

In Fig. 26, h_1 = head on upper edge of orifice while h_2 is the head on the lower edge, and l = width of hole. Call h the head on any horizontal element of the orifice such as dh. Then the velocity of efflux through the elementary area ldh will be $V_s = \sqrt{2gh}$. The discharge being the product of velocity of flow by area of orifice, the differential flow dQ through differential area dh

is $\sqrt{2gx}l\,dh$. The greatest value of h is h_2, and the smallest is h_1. Hence the limits of integration are h_1 and h_2.

$$\int dQ = \int_{h_1}^{h_2} \sqrt{2gx}\,l\,dh = l\sqrt{2g}\int_{h_1}^{h_2} x^{1/2}dh =$$

$$l\sqrt{2g}\left[\frac{2h^{3/2}}{3}\right]_{h_1}^{h_2} = l\sqrt{2g} \times \tfrac{2}{3}(h_2^{3/2} - h_1^{3/2}) = 5.347l\,(h_2^{3/2} - h_1^{3/2})$$

This is, however, the theoretical efflux through the orifice, and, as has been shown, the theoretical value must be multiplied by a correction factor, C, so that the equation becomes

$$Q = 5.347Cl(h_1^{3/2} - h_2^{3/2}) \qquad (38)$$

C being as given in Fig. 27.

If the orifice in the side of the vessel be circular instead of rectangular, an approximate formula for discharge is $Q = 6.298d^2\sqrt{h}$ cu. ft. per second,
in which,

d = diameter of hole, in feet.
h = head to center of hole, in feet.

For sluice gates, the coefficient is usually considered as from 0.61 to 0.8, depending on various conditions. The experiments of Benton (Punjab Irrigation Branch Papers No. 8) indicate the following values of the coefficient C.

TABLE 12.—COEFFICIENTS C FOR VARIOUS WIDTHS OF SLUICE GATES

Width of gate, ft.	4	6	8	10
C =	0.748	0.763	0.779	0.795

From these values, it would appear that the larger the orifice the higher will the value of C become.

This view is confirmed by Herschel's experiments at the Tremont mills, Lowell, Mass., in which he found that for a gate 9 ft. wide by 4 ft. high, submerged on both sides, the coefficients were

TABLE 13

For a 3-ft. net head	0.954
For a 4-ft. net head	0.943
For a 5-ft. net head	0.915

The formula for discharge through sluice gates is the same as that for vertical orifices, namely, formula (38).

When a vertical orifice, or sluice gate, has both sides of the

opening completely submerged as in Fig. 28, the effective head is simply the difference in head between the two bodies of water, and indicated by $''Hd$ in the drawing. In this case the formula becomes

$$Q = CA\sqrt{2g''H_d} \tag{39}$$

in which C = constant given in Tables 12 and 13.

When sluice gates, unsubmerged on the downstream side, are set inclined to the vertical as indicated in Fig. 29, the formula for discharge is the same as that for vertical gates except that H_1 and H_2 are measured vertically to the upper and lower edges of the gate, as shown.

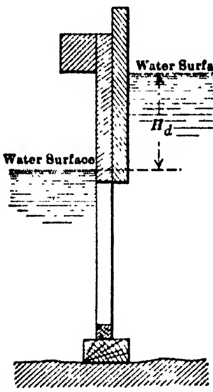


FIG. 28.—Submerged sluice gate.

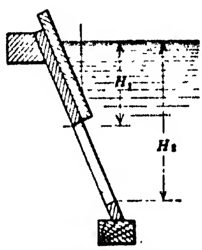


FIG. 29.—Inclined sluice gate.

Path of a Horizontal Jet.—Since a mass of water having an initial horizontal velocity, moves downward also, by the action of gravity, the combined motion of the two sets of forces cause the water to follow a curved path as in Fig. 26.

The equation of this curve is,

$$y = \frac{X^2}{4h} \tag{40}$$

in which

h = head acting on the water at the center of the orifice.

This equation is derived as follows:

The horizontal velocity, v of the water = $\sqrt{2gh}$. In the time t sec., the water will travel horizontally vt ft. = X.

During the time t, the water will be falling vertically with a uniformly accelerated velocity, $V = \sqrt{2gy}$.

The average vertical velocity is the average of the initial and the final velocity. The initial vertical velocity v being zero, the average velocity is $\frac{V}{2}$.

Hence,

$$y = \frac{Vt}{2}$$

$$V^2 = 2gy = 2\frac{gVt}{2}$$

whence,

$$V = gt$$

$$y = \frac{t}{2}gt = \frac{1}{2}gt^2.$$

Also from equation $X = vt$

$$t = \frac{X}{v}.$$

From equation $y = \frac{1}{2}gt^2$

$$t = \sqrt{\frac{2y}{g}}.$$

Equating these values of t,

$$\frac{X}{v} = \sqrt{\frac{2y}{g}} \quad \text{and} \quad \frac{X^2}{v^2} = \frac{2y}{g}.$$

Substituting for v^2 its value $2gh$

$$\frac{X^2}{2gh} = \frac{2y}{g} \quad \text{and} \quad y = \frac{X^2}{4h} \tag{40}$$

which is the equation of a parabola with its vertex at the orifice, and having a vertical axis.

Pipe Intakes.—In the case of penstock intakes, the conditions are as shown in Fig. 30. The water rushing into the end of the tube contracts in cross-section, beginning at the end of the tube and extending a distance equal to approximately the diameter of the tube. This contracted section is called the "*vena contracta.*" The unfilled space in the tube at this point is a partial vacuum. If a small tube be tapped into the penstock, so that it projects vertically down to a vessel of water, and with its lower end submerged, the water will be sucked up by the partial vacuum at the

vena contracta, as indicated in the figure, the rise of the water, above the surface of the liquid in the vessel being, approximately, $0.75H$, where H is the head acting on the center of the penstock orifice. Of course, this relation can hold only up to $H = 44$ ft.

The coefficients of discharge through the ends of the pipe depend on the shape and arrangement of the ends with reference to the reservoir walls. Fig. 27 shows the coefficients usually employed in making computations. For want of better data they may be used, but the author is of the opinion that the coefficients

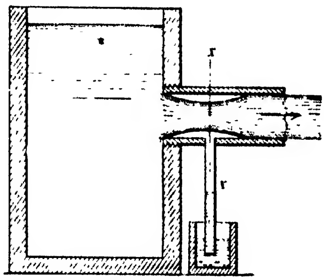


FIG. 30.—*Vena contracta.*

for straight-ended tubes should be higher than those in use. Apparently, all experiments to determine orifice coefficients were made on small pipes or openings, and it seems obvious that the coefficients should increase with the size of the orifice. (See, "Entry Head," Chap. VI).

Efflux from Nozzles.—A nozzle, having a smooth, tapered interior cross-section, will have a coefficient of discharge equal to nearly unity, and may be taken as = 1 without introducing an error greater than 2 per cent. The average coefficient is 0.99.

CHAPTER IV

POWER VARIATION AND STORAGE

Hydrographs are simply graphical representations of stream flow drawn to horizontal and vertical coördinates, the vertical height, or ordinate, at any point, representing the cubic feet per second flow, while the horizontal position of any chosen point, or abscissa, corresponds with the date on which the stream flow as given by the ordinate was recorded. They are customarily, though not necessarily, drawn to include one calender year each.

Figure 31 is a hydrograph of the Colorado River at Austin, Tex., for the year 1908. For any date given on the lower horizontal line, there is a certain vertical distance to the line of the hydrograph, which, to the vertical scale, represents the flow on that date. Thus, on Aug. 31, the flow was 760 cu. ft. per second. On Sept. 20 the flow was 5950 cu. ft. per second. The minimum flow occurred on Apr. 10 and was 180 cu. ft. per second. The maximum flow was 72,600 cu. ft. per second, on Apr. 21.

If the records of flow and its variation are made at some point on the stream distant from the power site, the necessity for computations to determine the rate of flow at the power house is obviated by the use of a double scale hydrograph, as shown in Fig. 32.

Plot the hydrograph with ordinates giving rates of flow as recorded at the gauging station and re-number the vertical distances, thereby changing the values of the ordinates, so that the ratio between the original numbering and the changed values is the same as the ratio of the drainage area at the proposed power site to that at the gauging station. With these changed values of the ordinates, the curve becomes a hydrograph for the power site. Thus, Fig. 32, with the values of the ordinates on the left-hand side of the figure, is a hydrograph plotted from the records of flow of the Juniata River at the U. S. Government

Gauging Station at Newport, Pa. The drainage area to this point is 3476 sq. miles.

At the power site at Warriors Ridge, Pa., the drainage area is

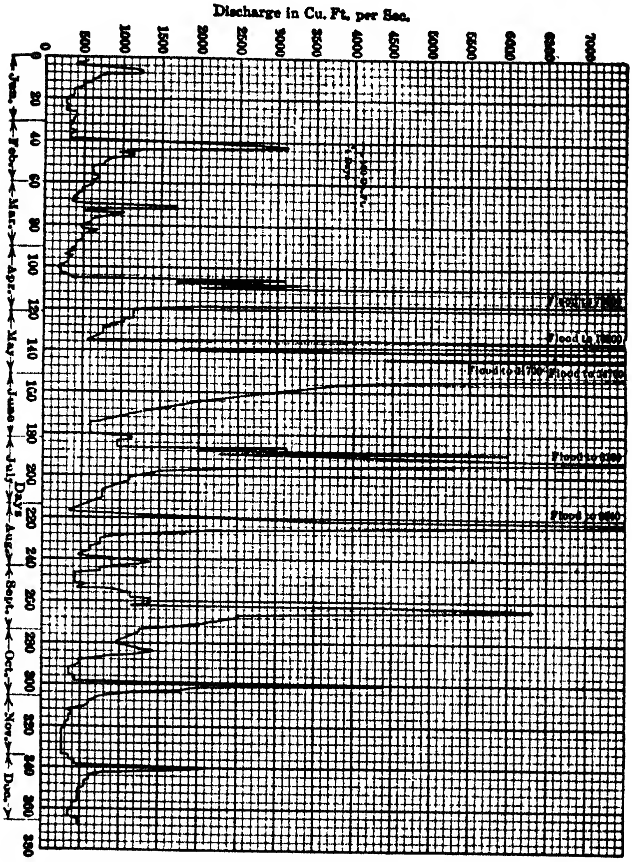


FIG. 31.—Hydrograph of Colorado River at Austin Texas.

759 sq. miles. Ratio of area at gauging station to that at Warriors Ridge is 0.219. Multiplying the values of the vertical distances by this ratio, the values placed on the right-hand side of the figure are obtained, and using the right-hand vertical and

bottom horizontal values, the figure becomes a hydrograph for Warriors Ridge. Thus, on Aug. 12, the stream flow at the

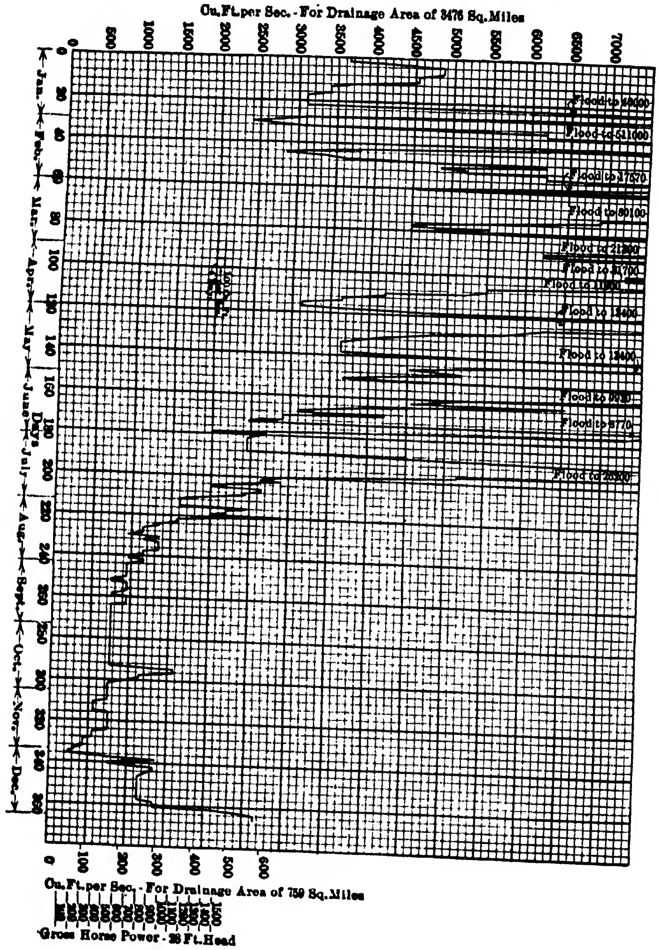


FIG. 32.—Hydrograph of Juniata River.

gauging station was 1150 cu. ft. while at Warriors Ridge it was 252 cu. ft., per second. The maximum flow at the power site was 17,600 and the minimum was 54.8 cu. ft., per second.

This method of computation is not accurate, as local rains above one of the selected points, but below the other one would change the ratio of the respective stream discharges. It is, however, the nearest guide to the determination of the actual flow at the power site and is the only basis available to settle the question of minimum power.

Another series of figures is shown on the hydrograph, Fig. 32, and marked "HP Gross." The head, H, at the power site is 28 ft. Then for each cubic foot flow per second the gross horsepower $= \frac{28}{8.8} = 3.182$. Multiplying the vertical figures by 3.182, the horsepower for any rate of flow of the stream is given and the hydrograph becomes a horsepower curve. Thus, on Aug. 12 the gross power of the stream at the power site was 802 hp. The minimum power was 174 hp. gross, and occurred on Dec. 1.

From this discussion, the importance of the hydrograph in studying water powers becomes evident.

Usually, there is a storage basin formed by the dam and, if this basin is of considerable area, it is evident that a greater amount of water may be used during the periods of minimum flow than is actually supplied by the stream during these periods. The excess is abstracted from the stored water in the storage basin, which diminished storage is replenished when the rate of stream flow exceeds the rate at which the water is used for power. When this condition exists, the hydrograph is mainly useful in furnishing data from which to compute a mass curve (also called Rippl curve) which is later fully explained. Before entering into the discussion of storage and the mass curve, it is necessary to define the term "load factor" and explain its relation to storage and the rate of usage of water for power supply.

Load Factor.—The consumption and production of power differ, essentially, from the consumption and production of any other commodity in that these two functions are simultaneous, and the quantity of power used, in any instant, is subject to the will and control of the various users of it and is not, in any way, under the control of the producer. At any instant, the amount of power produced must be exactly equal to the power consumption at that instant. This means that the output of any plant is subject to wide fluctuations, and, as is well known, the amount of power delivered throughout the 24 hr. of the day varies greatly from time to time; also, the amount of power for any given period

in the 24 hr. varies with the time of the year and with the day of the week. Therefore, in any power development, the only limiting characteristic of the plant is the maximum possible output, beyond which the plant is not capable of delivering energy. Throughout each hour of the day and the seasons of the year, the output will fluctuate, assuming various values that lie somewhere below this maximum possible output for which the equipment has been installed.

The load factor is defined as the ratio of the average to the maximum load. The *daily* load factor is the ratio of the average load to the maximum load for one day, while the *yearly* load fac-

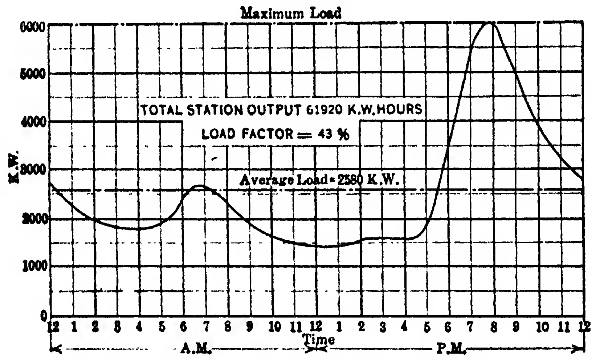


FIG. 33.—Load curve of power plant.

tor is the ratio of the average load for the year to the maximum load that occurred during the year.

It is usual to express the load factor as a percentage of the maximum load.

Obviously, the investment in generating apparatus, power house, transformers, transmission line and accessories is fixed by the *maximum* output. This equipment, however, does not run continuously at its full capacity, as has been explained. If a plant should be operated continuously throughout every hour in the year at a full-load output, the power station would have a 100 per cent. load factor. Fig. 33 is a curve which shows the variation in load in a power plant for 24 hr. The ordinates represent, to scale, kilowatts output, while the abscissæ represent the various hours in the day. The height of any point of

the curve, at any time, shows the load on the plant at that particular time. It will be observed that about one o'clock p.m. the load reached its minimum, being 1480 kw., while at 7:50 p.m. the load reached its maximum of 6000 kw. The average load throughout the day was 2580 kw. The ratio of the average to the maximum is therefore as 2580 is to 6000 and the average load is 43 per cent. of the maximum; hence, the load factor for this particular day was 43 per cent.

The production of a horsepower for 1 hr. is called a horsepower-hour. A horsepower delivered for 10 hr. is 10 hp.-hr.; 10 hp. delivered for 1 hr. is also equal to 10 hp.-hr., that is, the production of horsepower by the time in hours through which the power is delivered, is the number of horsepower-hours that are supplied during this period. All energy is measured and sold on the basis of horsepower- or kilowatt-hours, a kilowatt being $1\frac{1}{3}$ hp.

The usual way of determining the average load from a load curve, is to find the area included between the curve and the base line or X axis, and divide this area by the length of the measured diagram on X, the quotient being the desired average value; just as in the case of indicator cards or other figures having variable ordinates.

The usual load factors of plants are:

TABLE 14

LOAD FACTORS

	Per cent.
Lighting plants	25 to 40
Mixed service lighting and power	30 to 45
Large power distribution plants	40 to 55

Diversity Factor.—There is another factor which enters into the subject of station output and size of equipment, and this is called the diversity factor. If a power plant has a possible maximum output of 10,000 kw., it may easily sell power to users to such an extent that there will be 20,000 kw. connected to, and dependent on, the power station. Obviously, the plant could not supply all these users simultaneously, but it is a matter of experience that the various consumers of power never use their respective maxima at the same time. Owing to this diversity of use, a plant cannot obtain full benefit of its entire equipment at the time of the peak, or maximum load, unless the motors, lamps

and other devices, which are connected to the transmission lines, have a total rated capacity in excess of the maximum station capacity. This excess ranges from 50 to 100 per cent. of the maximum station capacity.

The ratio of the maximum connected load to the maximum station output is called the "diversity factor" and it ranges from 1.5 to 2.

Storage.—From the discussions of "Load Factor," and "Diversity Factor," it becomes clear that the amount of water required for the production of power in any hydro-electric power station, varies greatly throughout each day, and also, with the seasons of the year. From this, the necessity of water storage is obvious. A stream which supplies a sufficient amount of water to produce the average load throughout the day might be totally inadequate for the production of the maximum load. With a storage basin of sufficient capacity, the water furnished by the stream which is in excess of the amount required to deliver the necessary power, during those hours of the day when the load is small, can be stored, and the stored water, together with the normal stream flow, will be sufficient to produce the power required during the hours of heavy load.

If the power output be based on that produced by the minimum flow of the river, the area of the storage basin to equalize the 24-hr. load need be only great enough to hold all the water that the stream supplies during those hours of the day when the load is less than the average, minus the quantity of water drawn through the water wheels during this period of light load; that is, the amount of water stored during the hours of light load is equal to the difference between the quantity furnished by the stream and that passed through the water wheels.

Thus, in Fig. 33, the line drawn horizontally across the curve of station output and showing the average continuous load is at an elevation which represents the amount of power that the minimum continuous stream flow will produce. Where the load curve is below this value, the incoming water exceeds that necessary to supply the required energy to the water wheels and is stored in the basin. Where the load curve is above the line representing the power produced by the continuous stream flow, the amount of water delivered to the water wheels exceeds the stream flow, and the additional water is supplied from the amount previously stored in the basin when the load was small.

Obviously, the total 24-hr. energy output cannot exceed the total 24-hr. energy input of the stream.

The term "*area*" has been used in defining the size of the storage basin, although the production of power depends on the volume of water used. However, the entire amount of water in a storage lake cannot be drawn through the water wheels. The head, H, acting on the wheels is fixed by the level of the water in the lake. As the storage water is drawn off the lake level falls, thus diminishing the head. It is not feasible to lower the lake level to a point below which the reduction in head is 25 per cent. of the normal head. Hence, the upper layer of water in the lake, having a thickness equal to 25 per cent. of the normal head, is that useful for power production. The effective *volume* of water is the mean value of the areas of the lake at normal level and minimum level, multiplied by one-fourth the normal head or

$$U = \frac{A' + A''}{2} \times \frac{H}{4} \qquad (41)$$

in which

U = volume of water in storage lake available for power in cubic feet.

A' = area of lake at normal elevation.

A'' = area of lake at elevation = 75 per cent. of normal head.

H = head in feet with water at normal elevation.

This equation assumes a straight line slope of the banks of the storage basin, which is never strictly true. It is, however, sufficiently accurate for all practical purposes.

In computing the amount of power which may be delivered from a storage basin, the following formula is useful:

$$\text{Horsepower-hours, gross,} = \frac{UH_{av}}{3600 \times 8.8} = \frac{UH_{av}}{31{,}680} \qquad (42)$$

in which U is available cubic feet of stored water and H_{av} the average head in feet.

Referring again to Fig. 33, the average energy input due to stream flow is 2580 kw. and the daily energy input is 24 × 2580 = 61,920 kw.-hr. daily.

If the normal head is 28 ft., each cubic foot of water, per second, gives $\frac{28}{15.68}$ = 1.7857 kw. delivered by the generators, (equation (8)).

To produce 2580 kw., the required rate of flow is $\frac{2580}{1.7857}$ = 1444 cu. ft., per second, at 28-ft. head. The head, however, is diminished as the level of the lake sinks. If the lowest lake level is $\frac{H}{4} = \frac{28}{4} = 7$ ft. lower than the normal level, the *average* head will be, approximately, $\frac{28 + 21}{2} = 24.5$ ft. for the period during which the draft on the reservoir exceeds the rate at which water flows into the lake. The kilowatts per cubic foot, per second, at this head $= \frac{24.5}{15.68} = 1.562$ kw. The quantity of water required at this head to produce 2580 kw. is $\frac{2580}{1.562} = 1651$ cu. ft. per second which is the required average flow.

From 5.30 p.m. until 12.15 a.m. of the next day—a period of 6.75 hr.—the total station output is 29,520 kw.-hr., corresponding to an average of $\frac{29,520}{6.75} = 4373$ kw. This output corresponds to $\frac{4373}{1.562} = 2800$ cu. ft. per second, at 24.5-ft. head.

Total water to be stored daily = $(2800 - 1651) \times 6.75 \times 3600 = 27,920,600$ cu. ft.

The depth of water available from storage is $\frac{28}{4} = 7$ ft. From formula (41), $\frac{A' + A''}{2} = \frac{27,920,600}{7} = 3,988,700$ sq. ft.; that is, the average area of the lake, taken between its upper and lowest levels, must be about 92 acres. All this is simply an approximation, as the average head throughout the day may differ from that herein assumed as the mean of the upper and lower values. The more correct method is given hereinafter.

Also, this example illustrates the condition of a storage reservoir only large enough to equalize the daily or 24-hr. load. When the lake is large enough to equalize the 24-hr. load and also to furnish some additional water during periods of drouth, so that the power produced during such periods of a month, or more, may exceed that due to the stream flow during that time, the problem becomes more complicated and the use of mass curves is resorted to.

Mass Curves.—A mass curve is one in which the horizontal distances, or abscissæ, represent time and the vertical distances, or

ordinates, represent total quantities of water that the stream has delivered within the time represented by the horizontal distances. Unlike the hydrograph, it is not a curve that shows, directly, the rates of flow. It shows the rate of flow only indirectly. Its direct indication is the cumulative quantity of water that has passed down a stream within a given period of time.

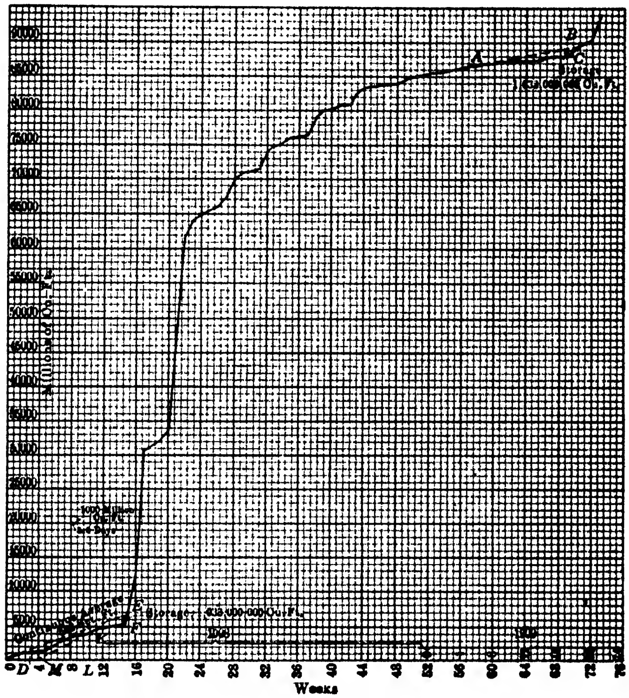


FIG. 34.—Mass curve of Colorado River at Austin, Texas.

Figure 34 shows a mass curve taken of the Colorado River for the year of 1908 and part of 1909. Beginning on Jan. 1, 1908, the amount of water that passed down the river at the instant of beginning the curve was zero. The rate of flow for Jan. 1, as shown by the hydrograph, was 543 cu. ft. per second. Since there are 86,400 sec. in a day, the total amount of water which flowed from Jan. 1 to Jan. 2 was 543 × 86,400 = 46,915,200

cu. ft. Hence, the ordinate of the point on the curve for Jan. 2 is 46,915,200. On Jan. 2, the rate of flow was 530 cu. ft. per second and the total flow for Jan. 2 = 530 × 86,400 = 45,792,000 cu. ft. Adding this to the amount that flowed in Jan. 1, the sum, 92,707,200, is the total quantity that passed down the stream in the 2 days, and the ordinate on the point for Jan. 3 is this total of 92,707,200. Proceeding in this way, the total mass curve is plotted.

Since mass curves are needed only for investigations where a large storage lake is contemplated, it is unnecessary to go through the tedious process of computations for the flow each day. The average of the flow for a week is quite sufficient for each point on the curve and, in the case of very large storage lakes, the use of the average of the flow for a month to determine points on the curve gives results that are accurate enough for any power determinations.

The greater the rate of flow, the higher will be the corresponding points of the curve and therefore, the steeper the curve. Hence, the slope of the curve at any point gives the rate of flow. This slope is identical with that of the tangent to the curve at that point and is equal, numerically, to the quantity in cubic feet shown by the vertical scale at the point taken, divided by the time in seconds, as shown on the horizontal scale, and included between a vertical line dropped from the point on the curve to the horizontal scale and a line tangent to the curve at that point, prolonged to intersect the horizontal scale. Thus, in Fig. 34, at the point K, the total quantity of water is 4,200 million $= 42 \times 10^8$ cu. ft. Dropping a vertical line KL to the horizontal scale and drawing the line K tangent to the curve at K, and prolonged to intersect the horizontal scale at M, the distance included between L and M represents 46.9 days, which is equal to 4,052,160 sec. Dividing the total cubic feet of water by this time in seconds, the result is 1037, which is the rate of flow, in cubic feet per second, at which water was being supplied by the stream on the date shown on the horizontal scale at point L, namely, Mar. 13, 1908.

If, at any time, the rate at which water is used is equal to the rate of stream flow, the slope of the mass curve will be the same as the slope of the line representing the rate of draft for that particular period.

If the rate at which the water is used is greater or less than the

rate of stream flow, the slope of the line representing the use of water will be greater or less than that of the curve.

Since the use of water is limited in rate and quantity by the minimum stream flow, those portions of the mass curve with the least slope are the points which are used in the determination of available water.

Without any storage greater than that sufficient to equalize the 24-hr. load, the average rate at which water may be drawn through the water wheels is equal to the minimum stream flow, and the slope of the line of draft simply coincides with the slope of that part of the curve where its slope is least. With larger storage, however, the average rate of draft through the water wheels exceeds the rate of stream flow by an amount which is computed from the curve. If the normal head of water is 60 ft., and the allowable reduction in head due to draft on the reservoir, is 25 per cent. or 15 ft., and the area of the storage reservoir is such that the upper 15-ft. layer of water has a volume of U cu. ft., this volume of water is represented by a certain length on the vertical scale of the mass curve.

If the point where the minimum slope of the curve begins be taken as one point, the point where the minimum slope ends be taken as another point, a vertical be erected on this second point and extending above it a distance equal to U cu. ft. to the same scale as that of the curve, and a straight line be drawn from the first point through the extreme upper end of the vertical, then the slope of this line will represent maximum average draft through the water wheels that the stream flow plus storage can supply.

The prolongation of the line until it intersects the mass curve in some other point shows the date of refilling the reservoir.

In Fig. 35 is plotted an enlarged portion of the mass curve shown in Fig. 34.

The periods of lowest flow of the river, namely, from Jan. 7, 1909, to Apr. 28, 1909, are included in this figure. The average area of the reservoir between the normal level and the lowest allowable level is 68,866,666 sq. ft. The depth of draft is 15 ft. Consequently, the available volume of water in the upper 15-ft. layer is 1,033,000,000 cu. ft.

The lowest rate of flow, as indicated by the slope of the curve, Fig. 35, began on Jan. 7, 1909, and ended Apr. 28, 1909, after which the rate of flow increased rapidly.

Erecting a vertical on the curve at the end of this portion of minimum rate of flow, and making the height of this perpendicular equal to 1,033,000,000 cu. ft. by the scale of the curve, the point G is located.

Drawing a straight line from point A through point G and prolonging it until this line cuts the curve at point B, the following facts are ascertained:

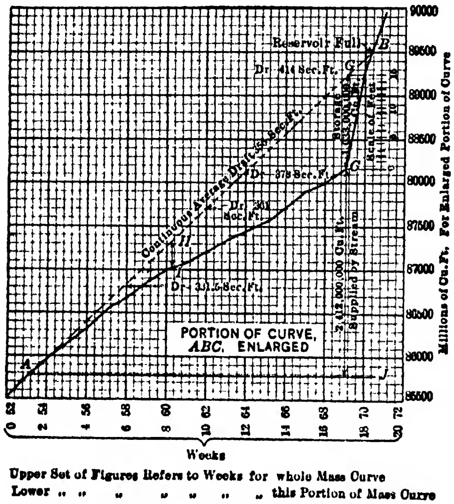


FIG. 35.

1. The rate at which water may be used, during the period of lowest water, is 3,445,000,000 cu. ft. in 112 days, which is an average of 356 cu. ft. per second.
2. The draft on the reservoir began on Jan. 1.
3. The maximum depth of draft was reached on Apr. 28.
4. After Apr. 28, the water flowing into the storage lake exceeded that used and the lake began to refill from that time on.
5. The reservoir was again full and the normal level of the lake restored on May 17, 1909.

The drop in the level of the water in the reservoir is indicated by the height of the line of draft above the curve at any point, and is equal to the normal level less the drop in level as found from the curve.

6. The vertical distance from the curve to the draft line represents a certain known quantity of water drawn from the reservoir, which draft causes a known drop in the elevation of the water level in the lake. If the length of the vertical line from the curve to the draft line, at the point where the maximum draft is indicated, be given a scalar value, the height of the line from the curve to the draft line at any other point will show the drop in the lake level to this same scale. For instance, in the example just given the maximum draft on the reservoir results in a drop in level of 15 ft. Making the length of the vertical line CG = 15 ft., it is found that the length of the line HI, to the same scale, is 5 ft.

7. The average head for the period during which the level of the lake was below the normal can be determined by taking a series of points, equidistant horizontally, and determining the net head at each of these points, as indicated in paragraph (6). The average of these is the average head. Where the line of draft and the elements of the curve form the boundaries of a triangle, this computation is unnecessary. The average head in that case is the average of the normal and the minimum heads. In the case where that portion of the mass curve below the draft line is made up of several broken lines or curves, and the area lying between the draft line and the curve is not a simple triangle, the average head must be computed either by the method indicated, or measurement of the area between curve and draft line. Thus in Fig. 35, there are eight equidistant points taken and the average of the ordinates drawn through these points is 7.59 ft., which is the average reduction in head below normal. Hence, the average head, during the period of draft on the stored water, is equal to the normal value of 60 ft. minus 7.59, or 52.41 ft.

If the area ABC be taken by planimeter, or by counting cross-section squares, or by any other method, and divided by the horizontal length AJ, the result will be the average vertical height of the figure and, therefore, the average reduction in head.

8. With the average rate of flow, as given in paragraph (1), and the average head, as given in paragraph (7), the average continuous power output of the power site can be computed. Horsepower available throughout the period = $H_{avg.} \times Q_{avg.} \times$ 0.1135, gross. The actual daily draft will not be the average

except on the one day when the actual head is equal to the average head. At the beginning of draft on the reservoir, the amount of water required for the production of the average power will be less than the average draft during the period when the level of the water in the reservoir is being lowered, and the rate of draft will gradually increase, being a maximum when the water in the reservoir is lowest.

The rate of draft for any decrease in elevation of the lake level is

$$Q_1 = \frac{Q_{avg} \times H_{avg}}{(H - d)} \qquad (43)$$

in which

Q_1 = rate of draft at any time, in cubic feet per second.
Q_{avg} = average rate of draft as found from mass curve.
H = normal head when lake is full, in feet.
d = drop in lake level below normal head H, in feet.
H_{avg} = average head as found from mass curve.

Obviously, the total daily kilowatt-hours can be distributed over any load factor, as the volume of stored water is many times greater than that necessary to equalize the 24-hr. load.

It is clear that the mass curve may be used in a manner converse to the preceding, that is, the required area of storage lake to deliver a given quantity of water during the period of low water, may be computed from it.

Beginning at any point on the curve where the slope decreases to less than the slope of the draft line, the maximum vertical ordinate, from the draft line to the curve will give, to scale, the total volume of water required to augment the stream flow. Dividing this volume by the depth of the allowable draft—usually 25 per cent. of the head—the result is the average area of the storage lake, taken between the normal and minimum levels.

Reduced Head Due to Floods.—During floods, the water below the dam—or "tail water"—rises until it is elevated considerably above its normal level. At the same time, the thickness of water over the crest of the dam increases, raising the lake level. The increase in the elevation of the tail-water level is considerably greater than the increase in elevation of the lake level, so that the net head diminishes with increase of water in the stream. For a complete investigation of the change in head, curves may be plotted, one showing the variation in lake level,

the other the change in tail-water level for various values of discharge up to that of the maximum flood. The two curves should be plotted on the same sheet, and the ordinate extending from the lower, or tail-water curve, to the upper, or lake-level curve, represents the net head, to scale. This is, however, somewhat academic. The only information needed in practice is the net head at the time of maximum flood. The generating equipment must be designed to meet this extreme condition, and if it does, it will certainly operate satisfactorily during water stages lower than the maximum. The question of efficiency does not enter into the problem. This is important only for water stages in which the flow equals, or is less than, the quantity required for power purposes, as is later discussed more fully.

Evaporation.—In computing available amounts of storage, it is customary to consider the evaporation from the surface of the storage lake. The *Bulletins* of the United States Geological Survey give data which show the average evaporation from water surfaces for different periods of the year and in different sections of the United States. As a matter of fact, the introduction of evaporation into storage computations of water-power developments is an unnecessary refinement because the cycles of high and low water and the quantity of water supplied by the stream during these periods never repeat themselves, and to attempt the introduction of accuracy into a problem which is fundamentally an approximation, does not produce results which are any nearer the truth than the approximations themselves.

For the use of engineers who desire to compute the effect of evaporation on the available storage, the following table is supplied, which is an average for those parts of the United States in which the rainfall is from 48 to 52 in. per annum.

TABLE 15.—MONTHLY EVAPORATION IN INCHES DEPTH FROM WATER SURFACES. AVERAGES FOR 50-IN. TOTAL PER ANNUM

	Inches		Inches
January	1.25	July	7.64
February	1.34	August	7.00
March	2.16	September	5.24
April	3.80	October	4.02
May	5.70	November	2.87
June	7.05	December	1.93

For special conditions of hot, dry sections of country, the reader is referred to the *Bulletins* of the United States Geological Survey.

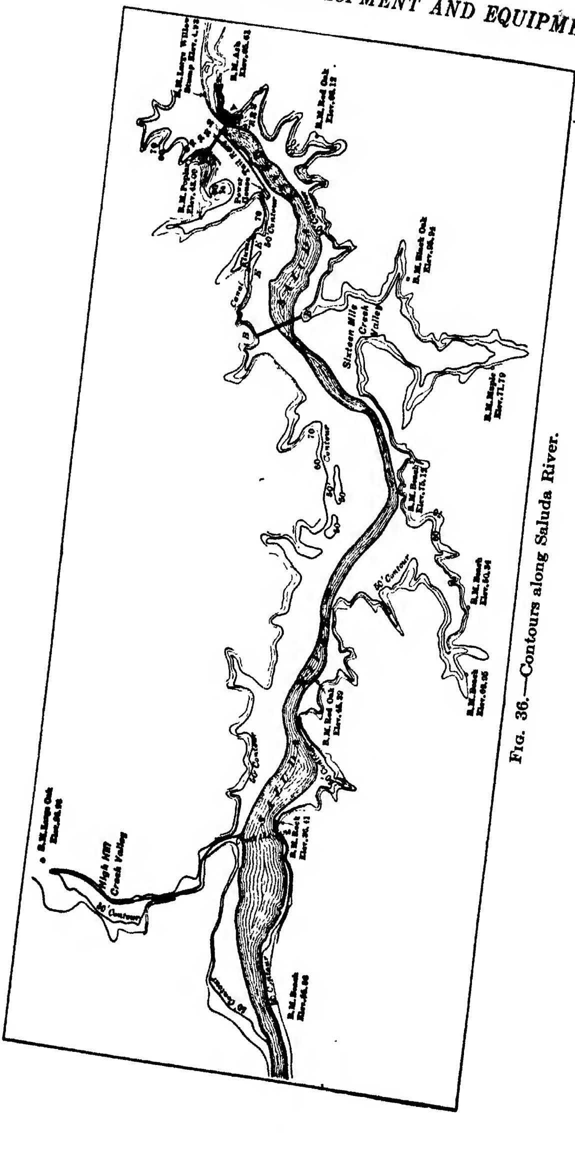


FIG. 36.—Contours along Saluda River.

Survey of Storage Basin.—After the fall and height of dam have been determined, a survey should be made to locate the contour which indicates the boundary of the lake that will be formed when the dam is built. This contour is simply the level line beginning at one end of the dam and at the elevation of its crest, following along up the stream until the contour strikes into and crosses the stream bed, then returning downstream until it finally ends at the other end of the dam. Several other contours should be run at different elevations. There should be one above this first-named contour and at an elevation above it equal to the height of the water surface with the maximum flood passing over the spillway crest. There should be another contour made below the first-named contour at an elevation about one-fourth the head below the lake level contour for the purpose of determining the storage volume. The area included in the first contour gives the value of A' and the area in the lower contour gives the value of A'' for formula (41).

Figure 36 shows a survey of a water-power site on the Saluda River near Columbia, S. C. It will be observed that a number of contours have been run in order to determine the most economical development to be made. The higher the dam, the greater will be its cost and the greater the area of the overflowed lands which must be purchased, but this increase in height and area of storage lake gives a corresponding increase in power. Therefore, for preliminary investigations and in order to study carefully the general conditions for making the development which will give the lowest cost per horsepower of delivered output, a number of contours are advisable.

In making the survey for contours, stadia measurements are quite accurate enough for determining distances, and with a good theodolite, having its telescope clamped level so that it can be used as a level and also turned about the vertical axis to indicate the angular measurements, contours can be rapidly and cheaply run.

CHAPTER V

ARTIFICIAL WATER-WAYS

Under the classification "artificial water-ways" are comprised canals, flumes and tunnels.

Canals.—The laws governing the stream flow for given slope and hydraulic radius, as set forth in Chap. II, apply equally to canals, except that in artificial water-ways the bed and sides are usually made smooth so that the coefficient, C, is smaller than in the case of natural water courses. Also, the hydraulic radius, r, is usually larger for canals, so that the required slope for a given velocity of water is less than for natural streams.

In excavating a canal there must be a slope to the sides, as indicated in Fig. 37.

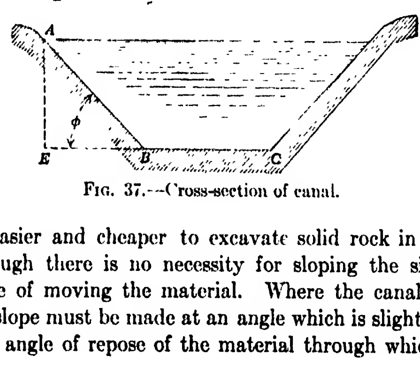


FIG. 37.—Cross-section of canal.

It is easier and cheaper to excavate solid rock in this way, even though there is no necessity for sloping the sides other than ease of moving the material. Where the canal is cut in earth, a slope must be made at an angle which is slightly greater than the angle of repose of the material through which the cut is made.

The usual limiting steepness of slope, *i.e.*, the limiting ratio which the side makes with a horizontal line is 1:1¼. This means that the tangent of the angle ABE, or ϕ, is $\frac{1}{1.25} = 0.8$, which corresponds to approximately 38°.

There is some economical section which, for fixed slope of material excavated and area of cross-section, gives the maximum hydraulic radius and, hence, the maximum flow of water with a given loss of head.

The formulæ for the relationship of the widths of top and bottom to the depth, for a given area of cross-section and maximum hydraulic radius are

$$d = \sqrt{\frac{a \sin \phi}{2 - \cos \phi}} \tag{44}$$

$$\text{Width at top} = \frac{a}{d} + \frac{d \cos \phi}{\sin \phi} \tag{45}$$

$$\text{Width at bottom} = \frac{a}{d} - \frac{d \cos \phi}{\sin \phi} \tag{46}$$

d = depth in feet.
a = area in square feet.
ϕ = exterior angle made by side with the horizontal.

As an example, consider a canal to carry 240 cu. ft. per second at a velocity of 3 ft. per second.

$$a = \frac{240}{3} = 80 \text{ sq. ft.}$$

$$\text{Take } \phi = 38°$$

$$\text{Sin } \phi = 0.61566$$

$$\text{Cos } \phi = 0.788$$

$$d = \sqrt{\frac{80 \times 0.61566}{2 - 0.788}} = 4.06 \text{ ft.}$$

$$\text{Average width} = \frac{a}{d} = \frac{80}{4.06} = 19.7 \text{ ft.}$$

which is the width at half depth.

$$\text{Width at top} = \frac{a}{d} + \frac{d \cos \phi}{\sin \phi} =$$

$$19.7 + \frac{4.06 \times 0.788}{0.6156} = 19.7 + 5.13 = 24.83 \text{ ft.}$$

$$\text{Width at bottom} = \frac{a}{d} - \frac{d \cos \phi}{\sin \phi}$$

$$19.7 - 5.13 = 14.57 \text{ ft.}$$

and the area of this trapezoid is 80 sq. ft.

Also, for a canal having a trapezoidal section, the wetted perimeter, $p = b + \frac{2d}{\sin \phi}$, and the hydraulic radius, $r = \frac{a}{p}$.

The velocity of flow in canals must be limited within certain fixed values for two reasons; one is to prevent too great a loss of head, which increases with the velocity, the other is to prevent wash of the canal sides if they are not protected by a masonry or concrete lining. The question of limiting velocity to prevent

excessive loss of head is an economic one, the conditions of which must be worked out for each case. Usually, the area of cross-section should be such that the velocity head loss will equal in value the interest on the cost of the canal at 8 to 10 per cent. This general rule, of course, assumes that there is a market for all the available power. The loss in head, due to movement of the water at some fixed velocity, is computed by Bazin's formula, as given in Chap. II.

Usually, however, the allowable velocity in the canal, based on financial economics, is greater than that which the earth will withstand.

The allowable velocities in different classes of material are given in the subjoined table.

TABLE 16.—LIMITING VELOCITIES OF WATER FOR VARIOUS MATERIALS

Very fine sand	0.3 ft. per second
Very fine clay	0.5 ft. per second
Ordinary sand	1.0 ft. per second
Ordinary clay	1.5 ft. per second
Gravel	2.0 ft. per second
Loose broken stone	5.0 ft. per second

In preliminary computations, the velocity is taken at 2 to 2.5 ft. per second.

Canals in earth, unlined, are subject to loss of water by leaks and seepage. It will usually be found that excessive losses from these sources take place over some short section of a canal and by lining this portion with concrete or masonry, or puddling it (plastering with a thick layer of clay), the defect can be remedied.

Also, the original cross-section is subject to decrease owing to the gradual deposition of silt in canals where the velocity of the water is low, and also, because of the growth of shrubs and brushwood. Therefore, it is probable that any canal will require occasional cleaning at periods from 1 to 3 years apart.

Effects of Ice.—In the case both of canals and open flumes, the formation of a sheet of ice over the surface decreases the cross-section and increases the hydraulic radius, thereby reducing the velocity of flow. If an open channel, of any kind, is subject to freezing, its cross-section must be made of sufficient area to carry the desired quantity of water, when ice-covered.

The computations for the area required and the loss in head are made in the same manner as has been outlined for flow in streams,

except that the length of wetted perimeter is equal to the length of the boundary of the stream-bed cross-section plus the width of the ice surface. This means that $\frac{A}{p}$ is much less than in the case of open streams.

The value of Bazin's m for this condition is

$$\frac{p_1 \times m_1 + p_2 \times 0.06}{p_1 + p_2} \qquad (48)$$

in which

p_1 = length of wetted perimeter of canal bed.
p_2 = perimeter of ice surface = width of stream.
$p_1 + p_2$ = total wetted perimeter.
m_1 = Bazin's coefficient for the surface of the canal bed, usually 1.30.
0.06 = coefficient for smooth surfaces.

As an example, take a canal having sloping sides: depth = 4 ft., width at top of water surface = 20 ft., width at bottom = 12 ft., length of each sloping side = 5.65 ft.

Then $p_1 = 12 + 2\ (5.65) = 23.3$ ft.

$$\begin{aligned} p_2 &= 20 \text{ ft.} \\ 23.3 \times 1.3 &= 40.29 \\ 20 \times 0.06 &= \underline{\ 1.20} \\ \text{Sum} &= 41.49 \end{aligned}$$

The wetted perimeter is 43.3 ft. and the area is $4\left\{\frac{12+20}{2}\right\}$ = 64 sq. ft. The hydraulic radius, therefore, is $\frac{64}{43.3} = 1.48$. When there is no ice coating, the hydraulic radius is $\frac{64}{23.3} = 2.74$. Assuming a slope of 0.00075, the values of C become:

For canal with ice coating, $C = \dfrac{87}{0.552 + \dfrac{0.957}{\sqrt{1.48}}} = 65.2$

For canal without ice coating, $C = \dfrac{87}{0.552 + \dfrac{1.30}{\sqrt{2.74}}} = 65.0$

Velocity in canal with ice coating $= 65.2 \times \sqrt{1.48} \times \sqrt{0.00075}$ = 2.38 ft. per second.

Velocity without ice coating $= 65 \times \sqrt{2.74} \times \sqrt{0.00075} = 2.94$ ft. per second,

so that, with the same loss of head, the canal discharges over 23 per cent. more water without the covering of ice.

Seepage losses in canals have been investigated on some of the irrigation projects of the United States Reclamation service. The average losses from leakage and seepage that may be expected in a canal during each 24 hr. are Jpl, in which

p = wetted perimeter in feet.
l = length of canal in feet.
pl = total wetted area of canal.
J = a constant which depends for its value on the character of material of the canal bed.

The loss of seepage in cubic feet per second is

$$\frac{Jpl}{86,400} \tag{49}$$

The values of J are given in Table 16 herewith. They actually represent the loss in depth (in feet) per 24 hr., as is obvious from the formula.

TABLE 17.—VALUES OF J FOR SEEPAGE LOSS

Material of canal bed	J
Cement gravel and hard pan with sandy loam	0.34
Clay and clay loam	0.41
Sandy loam	0.66
Volcanic ash	0.68
Volcanic ash with some sand	0.98
Sand with volcanic ash	1.20
Sandy soil with some rock	1.68
Sandy and gravelly soil	2.20

Just how valuable and authoritative these data are, the author is unable to say. The losses appear to be excessive, and certainly no power canal would be a satisfactory one with any such continuous loss as even the smallest figures in the table would indicate. The results of the tests are merely given here as an indication that some seepage does take place, and materials should be used—such as clay puddling or concrete lining—to prevent it, unless the water supply is greater than the demand for power.

Where the expense is justified, concrete makes one of the most satisfactory linings and is usually laid in thicknesses of from 4 to 6 in., depending on the depth of water to be maintained in the canal, the material on which the concrete is to be placed, etc. Thicknesses less than 4 in. are seldom used,

as the saving in concrete is more than offset by the cost of trimming the slopes and the cost of placing. In climates which have large variations of temperature, expansion joints should be provided at proper intervals. Sometimes, when the canal has large embankments, it is advisable to reinforce the lining with a steel wire mesh or light reinforcing rods. In canals whose side slopes are flat enough to permit the placing of concrete without forms, the cost of placing is greatly reduced, but in most power canals the side slopes are such that forms are required for lining the sides. Unless the canal embankments have been allowed to stand for some time before the lining is placed, the bank sometimes shrinks away from the lining at the top. These cracks between the lining and the bank should be carefully sought out and churned full of soft clay, as otherwise the lining may crack when the canal is filled with water. Special care must be taken in joining the lining to bulkheads and flume entry-ways in order that vertical cracks may not form at the joint and serious leaks occur. The thickness of the lining should be gradually increased up to such junctions and short lengths of reinforcing rods used as dowels.

On sidehills, or wherever the ground slopes toward the canal, some means, such as surface ditching, should be taken to prevent surface water from collecting behind the lining and upturning it. In describing a canal, several miles of which was lined, F. W. Hanna, of the United States Reclamation Service, says:

"In order to protect the lining from the collection of water behind it from surface drainage on the uphill side of the canal, a complete system of drainage ditches was dug to collect the water and bring it into the canal over the top of the lining. A sudden melting of snow before a portion of the ditches was completed demonstrated the necessity of this plan as several panels of the lining were ruptured by the hydrostatic pressure behind them."

When large quantities of storm waters are permitted to enter a canal, the surplus water is discharged over waste weirs or through waste gates, or over automatic gates.

Where power canals traverse an irrigated country, the subsurface water level is often above the bottom of the canal and when the canal is to be lined, care must be taken that the pressure produced by this water does not upturn the lining. During the canal construction, in such cases, the subsurface water must be drained away in order that the construction work may proceed

more advantageously. When possible, this may be effected by leading such drainage to some natural water course crossing the canal, by means of a small drainage ditch in the middle of the canal bottom.

The following tables give some of the approximate figures on slopes, velocities and costs of canals for a given discharge in cubic feet per second.

TABLE 18.—APPROXIMATE DATA ON CANALS IN ORDINARY EARTH

Second feet	Velocity in feet per second, V	Area of wet section, sq. ft	Water depth, ft.	Approximate slope in feet per mile	Approximate cost per running foot	
					Low	High
50	2	25	2.5	4	$0.375	$0.75
100	2	50	3.5	2	0.75	1.50
200	2	100	5.0	1½	1.50	3.00
300	2	150	6.0	1	2.25	4.50
400	2	200	7.0	¾	3.00	6.00
500	2	250	7.0	¾	3.75	7.50
1000	2	500	10.0	½	7.50	15.00
1500	2	750	12.0	½	11.25	22.50
2000	2	1000	12.0	⅓	15.00	30.00
3000	2	1500	15.0	¼	22.50	45.00

In rock, the velocity in a canal may be much higher, and if the canal be lined, 8 ft. per second is admissible. For preliminary calculations this velocity will give approximate results. The following approximate figures are based on this velocity:

TABLE 19.—APPROXIMATE DATA ON CANALS IN ROCK

Second feet	Velocity in feet per second, V	Area of wet section, sq. ft.	Water depth, ft.	Approximate slope in feet per mile	Approximate cost per running foot	
					Low	High
50	8	6.25	2.5	40	$0.32	$1.28
100	8	12.5	3.5	25	0.63	2.50
200	8	25.0	5.0	16	1.25	5.00
300	8	37.5	6.0	12	1.87	7.50
400	8	50.0	7.0	10	2.50	10.00
500	8	62.5	7.0	9	3.25	13.00
1000	8	125.0	10.0	6	6.00	24.00
1500	8	175.0	12.0	4½	8.75	35.00
2000	8	250.0	12.0	3½	12.50	50.00
3000	8	375.0	15.0	3	18.75	75.00

These values are for lined canals only, since a higher velocity will be allowable without producing too great a loss of head and the cost will probably be more favorable.

It must be considered that these figures are wholly approximate since for any desired section the amount of excavation, per lineal foot of canal, must vary with the character of the route—whether this be flat, rolling or sidehill.

In this connection, the following approximate costs, based on the annual reports of the United States Reclamation Service, are of interest.

TABLE 20.—APPROXIMATE COST OF EXCAVATION PER CUBIC YARD

	Low	High	Fair value
Earth—plowable with four horses	$0.10	$1.00	$0.18
Earth—plowable with six horses	0.12	1.50	0.30
Indurated material	0.29	2.00	0.60
Loose rock	0.35	2.50	0.75
Solid rock	0.60	5.00	2.00
Earth below plane of saturation	0.20	3.00	1.80
Rock below water		....	4.50

Open flumes are simply elevated aqueducts made of wood, steel or reinforced concrete. Their use is, principally, in connection with canals, and they are, generally, for the purpose of avoiding long detours which would have to be made if the canal followed along the path of the natural contour. They are also used for carrying water across depressions or ravines from the end of a canal on one side of the ravine to the other end of the canal on the opposite side.

The cross-sectional area of a flume can be made much less than that of an unlined canal to carry a given quantity of water, because the frictional coefficient is less and the velocity of the water is not limited by the physical characteristics of material.

Wherever a canal discharges into a flume, or *vice versa*, the cross-section of the flume being considerably less than that of the canal, curved approaches are made so that the water from the canal section is gradually, and not abruptly, contracted, until it finally takes the cross-section of the flume. Unless these gradual approaches are made, there will be a loss of head due to the abrupt increase in velocity, and decrease in cross-section. The values of these losses are given in the formula for loss of head due to

abrupt decrease in cross-section of pipes, in Chap. VI. Also, where the water emerges from the flume at a given velocity and passes into a canal having a greater cross-section and, therefore, at a lower velocity, if this change be made abruptly, there is a loss in head, as is set forth in the succeeding chapter. With the easy approach between the flume end and the connecting canal end, this latter loss is avoided and there is a definite pressure head added to the water due to the reduction of velocity, which reduces the kinetic energy in the water, and this kinetic energy goes into the form of pressure head. Under these conditions, water can be made to run uphill and still maintain the velocity required in the canal, provided the rise in elevation is made from the end of the flume, where the higher velocity begins to diminish, to the end

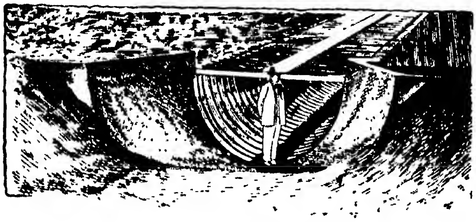


FIG. 38.—Tapered flume entry of concrete.

of the approach in the canal, at which point the lower velocity is finally attained.

The height to which water can be made to rise under these conditions, is

$$h = \frac{V_1^2 - V_2^2}{2g} \tag{50}$$

in which V_1 is the velocity of the water when just emerging from the flume, V_2 is the final velocity in the end of the approach to the canal, in feet per second, h is the rise of the water in feet, and g is 32.2.

As an example, consider a flume velocity of 9 ft. per second and velocity in canal of 2 ft. per second; then

$h = \frac{(9)^2 - (2)^2}{2g} = 1.2$ ft., which is the height to which the water may be elevated by its initial velocity V_1, and still leave the required velocity $V_2 = 2$ ft., per second, for flow through the canal.

The simplest and most durable method of making a proper,

flared, or tapered, entry-way, is to construct it of concrete, the end of the flume being set into the concrete and making a permanent, water-tight joint. This form of connection is suited to metallic, wood or concrete flumes, and it not only provides the proper form of entry, but largely removes the difficulty of making an impermeable joint between the bed of the canal and the flume end. Fig. 38 shows a concrete canal entry-way leading to a metallic flume. Tapered metallic entry-ways are made by The Hess Flume Co. Fig. 39 shows one of these.

The theoretical economical section of any open conduit is semicircular, but for wood and concrete the labor cost for making a semicircular form usually exceeds the value of the saving in material as compared with a rectangular section. Semicircular wood stave flumes are sometimes installed, as later described. Also, certain types of sheet-steel flumes have a semicircular section and are, therefore, subject to a slightly smaller loss of head than other forms.

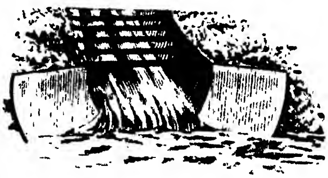


Fig. 39.—Flared flume entry of metal.

The most economical rectangular cross-section for a given area of water, and the one that will give the largest hydraulic radius, is that in which the width is twice the depth. This fact is shown as follows:

If a = depth and b = width of cross-section, the area $A = ab$, and for a given area, ab is constant.

The wetted perimeter p of an open rectangular trough is $2a + b$; and the hydraulic radius is, therefore

$$\frac{A}{p} = \frac{A}{2a + b}.$$

As A is constant, $\frac{A}{2a + b}$ will be maximum when $2a + b$ is minimum. Since $A = ab$, or $b = \frac{A}{a}$,

$$2a + b = 2a + \frac{A}{a}.$$

The minimum of $2a + \frac{A}{a}$ is obtained by taking its derivative and

equating it to zero, *i.e.*, from the equation

$$\frac{d}{da}\left(2a + \frac{A}{a}\right) = 2 - \frac{A}{a^2} = 0$$

which gives $A = 2a^2$ and therefore

$$b = \frac{A}{a} = \frac{2a^2}{a} = 2a;$$

that is, the width for minimum perimeter is twice the depth.

Sheet-steel Flumes.—Manufacturers are now supplying sheet-metal troughs bent to semicircular form, made up of sections which are easily and quickly jointed together, and which, supported on a trestlework, make an efficient, durable and economi-

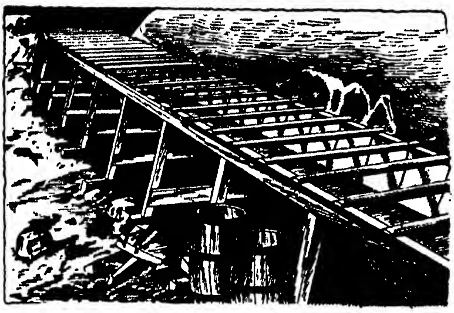


FIG. 40.—Steel flume.

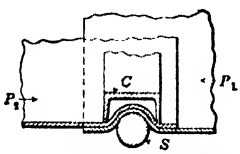
cal flume. The value of n in Kutter's formula for these flumes is 0.012 to 0.015 and the value of m for Bazin's formula is 0.16.

The "McGinnis" flume is one which is typical of this kind. The individual, bent sheets are placed end to end and are supported at each joint by a bent iron rod or bar which encircles the outside of the sheet. The ends project upward past the edges of the sheets and pass through a cross-member, which is usually of wood. This cross-member is parallel with the joint and, therefore, lies across the flume, spanning it. The cross-members are longer than the flume diameter and project out at either side. These projecting portions form the flume support, as they rest on stringers which run along the flume on either side.

This construction is shown in Fig. 40 which shows the completed flume following along a hillside.

The joint between adjacent sheets is made mechanically. The ends of each sheet are beaded, as shown in Fig. 41. The beaded end of one sheet mates with the beaded end of the next one, and the bent supporting rod lies in the groove formed by the bead. A light channel, also bent to semicircular shape, to conform to the curve of the sheets, is put inside the sheets, the flanges of the channel lying against the surface of the plates on either side of the bead. A detail of this joint is shown in Fig. 42.

At the upper ends, the channel bars are bent outwardly over the upper edges of the sheets and the ends of the supporting rods pass through these bent channel ends. The rod ends are threaded.



P_1 and P_2 – Adjacent Sheets
C = Channel
S = Supporting Rod

FIG. 41 —Section through "McGinnis" flume joint.

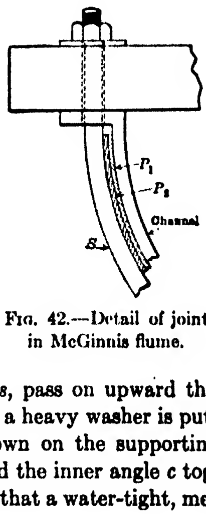


FIG. 42.—Detail of joint in McGinnis flume.

The ends of the supporting rods, s, pass on upward through the transverse supporting pieces, and a heavy washer is put over the ends. A nut screwed firmly down on the supporting rod ends, clamps the supporting rod s and the inner angle c together with the sheet ends between them, so that a water-tight, mechanical joint results.

The Hess flume is also made of curved sheets which are provided with a clamp joint that does not require any interior bracing strip, so that the interior of the flume is smooth. The manufacturers claim that tests by United States Reclamation Service engineers show the value of n for Kutter's formula to be 0.011, which is certainly a low value.

The following table gives areas, velocities, heads, slopes and carrying capacities of semicircular flumes, based on n for Kutter's formula = 0.015.

TABLE 21.—VELOCITY AND DISCHARGE IN SEMICIRCULAR STEEL FLUMES

Kutter's Formula, $n = 0.015$

D = Discharge—cubic feet per second F = Feet per mile—grade
V = Velocity—feet per second S = Slope per unit of length

Dia. ins.	Area sq. ft.	F	1.584	2.112	2.64	3.96	5.28	7.92	10.56
		S	0.0003	0.0004	0.0005	0.00075	0.0010	0.0015	0.0020
15	0.63	D	....	....	0.62	0.72	0.84	1.04	1.20
		V	....	....	0.98	1.15	1.34	1.65	1.90
19	0.99	D	....	....	1.07	1.33	1.54	1.90	2.19
		V	....	....	1.08	1.34	1.56	1.92	2.21
23	1.43	D	....	....	1.82	2.23	2.56	3.19	3.69
		V	....	....	1.27	1.56	1.79	2.23	2.58
30½	2.54	D	....	3.66	4.09	5.05	5.87	7.21	8.33
		V	....	1.44	1.61	1.99	2.31	2.84	3.28
38	3.97	D	5.68	6.55	7.30	9.01	10.48	12.86	14.85
		V	1.43	1.65	1.84	2.27	2.64	3.24	3.74
46	5.73	D	9.62	11.12	12.38	15.30	17.71	21.72	25.10
		V	1.68	1.94	2.16	2.67	3.09	3.79	4.38
53½	7.81	D	14.21	15.70	18.35	22.57	26.24	32.18	37.10
		V	1.82	2.01	2.35	2.89	3.36	4.12	4.75
61	10.17	D	20.44	23.39	26.14	32.44	38.03	46.38	53.60
		V	2.01	2.30	2.57	3.19	3.74	4.56	5.27
68¼	12.89	D	27.82	32.38	36.22	44.70	52.01	63.61	73.42
		V	2.15	2.50	2.81	3.47	4.03	4.93	5.69
76½	15.88	D	36.84	42.72	47.80	59.07	68.92	84.32	97.34
		V	2.32	2.69	3.01	3.72	4.34	5.31	6.13
84	19.24	D	47.90	55.41	61.95	76.19	88.50	108.32	125.06
		V	2.49	2.88	3.22	3.96	4.60	5.63	6.50
92	22.92	D	60.51	70.14	78.62	96.26	111.16	136.14	157.23
		V	2.64	3.06	3.43	4.20	4.85	5.94	6.86
107	31.17	D	91.02	105.67	118.13	144.63	167.07	204.79	236.27
		V	2.92	3.39	3.79	4.64	5.36	6.57	7.58
122	40.69	D	129.80	150.15	167.64	205.48	237.22	290.53	335.69
		V	3.19	3.69	4.12	5.05	5.83	7.14	8.25

Wooden Flumes.—Wooden flumes have a comparatively short life but their first cost is low and, in many instances, their yearly cost of interest and maintenance is less than the yearly cost of more expensive but more durable structures on which the annual interest charge is heavy.

They are generally of rectangular form, though, sometimes, they are made of wood staves and are semicircular in cross-section, as shown in Figs. 43 and 44.[1]

Fig. 43.—Semi-circular wood-stave flume under construction.

Figure 43 depicts a wood-stave flume in process of construction, while Fig. 44 shows a section of the completed flume. As

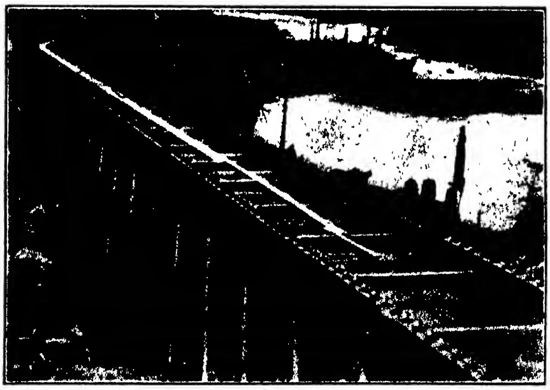


Fig. 44.—Semi-circular wood-stave flume complete.

indicated, the trough is carried in bent iron rods, the ends of which pass upward through supporting stringers, and are held

[1] Courtesy of Messrs. Owens & Bouillon.

in place by washers and nuts—these latter being plainly shown in Fig. 44. The general construction is very similar to that of semicircular metallic flumes, while the trough itself is made up

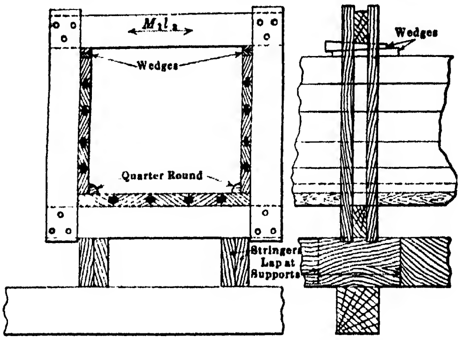


FIG. 45.—Section and elevation of wooden flume at support.

like wood-stave pipe, the end joints for the staves and other details being the same.

Figures 45 and 46 show designs of rectangular wooden flumes in general use. The construction is indicated in the drawings,

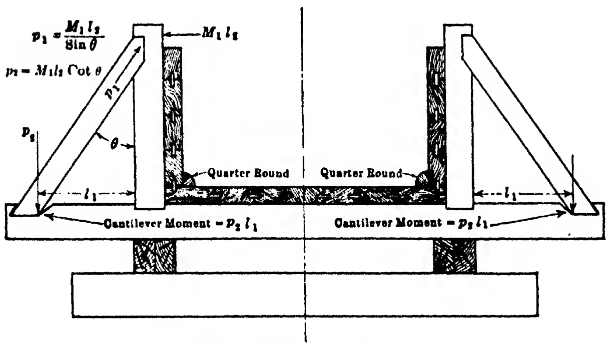


FIG. 46.—Section of wooden flume.

and from the equations, later given, for the forces acting on the troughs, any engineer can properly proportion the parts. From 12 to 20 ft. will usually be found the most economical spacing of the trestle-bents to support the trough. The longi-

tudinal spacing of the cross-beam supports carrying the side frames is from 4 to 6 ft. Of course, the farther apart the side supports, the thicker the planking of the trough must be to prevent side and bottom deflection.

Where flumes of rectangular cross-section are used, the following equations will give the values of the principal forces and moments set up by the water in the trough.

Refer to Fig. 47 and take all units in feet and pounds for 1 ft. length of flume.

Transverse Forces.—Force tending to push side walls outward is

$$P = \frac{d^2 \times 62.5}{2} \text{ lb. per foot length} \qquad (51)$$

This force is applied at a distance $= \frac{d}{3}$ ft. above bottom.

$$\text{Moment of } P \text{ about bottom} = M_1 = 10.4d^3 \text{ lb.-ft.} \qquad (52)$$

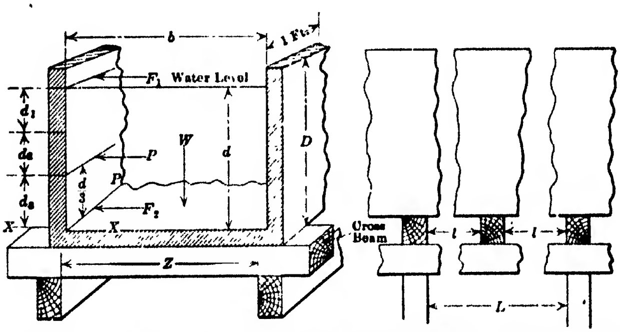


Fig. 47.—Diagram showing dimensions and forces used in flume formulæ.

This is the moment tending to overturn side about bottom joint at $X - X$.

If the depth d be divided into several parts, say d_1, d_2, d_3, beginning at the top, then the overturning moments at the several sections will be $m_1 = 10.4d_1^3$ for the uppermost section, $m_2 = 10.4\ (d_1 + d_2)^3$ for the second section, and $10.4\ (d_1 + d_2 + d_3)^3 = 10.4d^3$ for the total section.

F_1 = force acting at top of trough, and representing the

amount of a force which, if applied at the top, will resist the overturning moment of P.

$$F_1 = \frac{10.4d^3}{d} = 10.4d^2 \tag{35}$$

F_2 = force tending to burst side out from bottom if side is supported at top.

$$F_2 = 20.8d^2 \tag{54}$$

Total pressure on side at bottom if the side is unsupported at top is

$$F_1 + F_2 = 31.2d^2 \tag{55}$$

Moment of flexure in cross-beam is

$$M_b = \frac{3W_1Z}{2} \text{ lb.-in.} \tag{56}$$

$W_1 = \omega_1 + \omega_2$, per foot length.
ω_1 = weight of bottom of flume.
ω_2 = weight of water in flume = $62.5bd$.
Z = distance apart of supporting stringers.

If l_2 be the longitudinal distance apart of the cross-beams and side braces, the total forces and moments are those as given, multiplied by l_2. Thus, F_1l_2 is the total force acting at the top on each side brace.

Longitudinal Forces.—Moment of flexure between adjacent cross-beams, where the beams are l ft. apart, is

$$M_e = \frac{Wl^2}{12} \text{ lb.-ft.} = Wl^2 \text{ lb.-in.} \tag{57}$$

This moment must be resisted by the trough sides.

Moment of flexure of stringers is (approximately)

$$M_s = \frac{WL^2}{8} \text{ lb.-ft.} = \frac{3WL^2}{2} \text{ lb.-in.} \tag{58}$$

L = length between supports;
W = *wt* of flume and water, per ft.

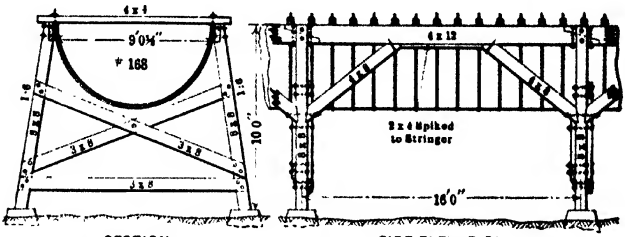
At the ends, and for the spans between joints, one-half the span is cantilevered, the other half simply supported, and for these spans the value of the flexure moment becomes

$$M'_s = \frac{WL^2}{10} \text{ lb.-ft.} = 1.2WL^2 \text{ lb.-in.} \tag{59}$$

Flume Trestlework.—The trestle bents to support a flume across depressions and ravines are to be proportioned in accord-

ance with the load to be carried and the height of the bents, just as for the carrying of any other trestle-supported load.

Figure 48 shows the design and dimensions of a trestlework to support a metallic flume. The design is also applicable to wooden



NOTE: Substructure designed for indicated height, ten feet. For greater heights increase dimensions of posts according to requirements. Side stringers and braces are standard for all heights with 16 ft. span. Sizes of timbers designed for Oregon pine.

FIG. 48.—Trestle-work for flume.

flume trestlework with proper modifications to accommodate the rectangular cross-section and the reduction in height for carrying the flume on top of the bents instead of between them.

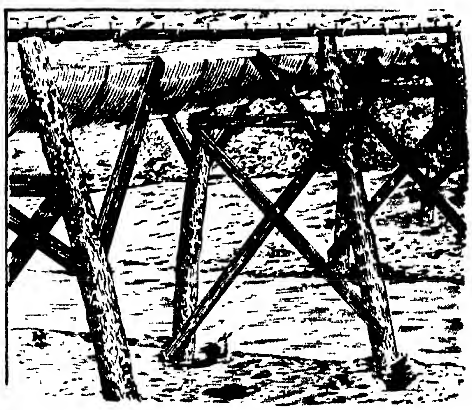


FIG. 49.—Steel flume on rough-timber supports.

These designs are also suitable for supporting semicircular, wood stave flumes, the small changes necessary to adapt them to this use being obvious.

Of course, the dimensions and sizes of timbers shown must be modified for different loadings and lengths of span, which must be made to conform to economical requirements in the use of materials. Also, the stresses in the timbers and joints must be within the limits of safe practice.

Flume-supporting timbers should be made larger than necessary to support the load safely, owing to the possibility of decay and consequent weakening of the supports. Rough timbers cut in the vicinity of the work may be used instead of dimension lumber, saving time and money. Such timbers fulfill every requirement and are, usually, more durable than sawed lumber. Fig. 49 shows a short section of flume on a trestlework made from rough materials.

Concrete Flumes.—Reinforced concrete has been used in flume construction and will probably be employed more in the future

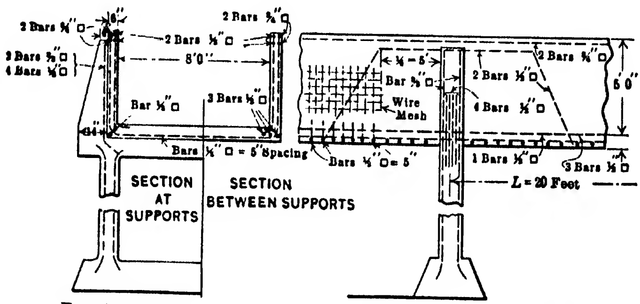


FIG. 50.—Section and elevation of reinforced concrete flumes.

since reinforced-concrete construction has become general for nearly every character of permanent engineering structure. These flumes are built of the same form as wooden flumes, the usual rules of concrete construction being followed:

The safe limiting stresses for this work are:

Concrete compression............	500 lb. per square inch
Concrete shear..................	50 lb. per square inch
Steel tension....................	18,000 lb. per square inch

Hydrated lime, or some equivalent waterproofing, should be used in the concrete forming the trough.

The designs shown in Figs. 50 and 51 are both used, though the second is usually the more economical.

In the first type, Fig. 50, the side walls of the flume are thin and only strong enough to act as a beam from one support to the next.

The stresses acting against bursting, or against the sides turning outward, must all be taken by the vertical braces surrounding the trough at each support.

There is a transverse flexure tending to break the bottom through, as indicated in Fig. 52. Hence, the bottom must be

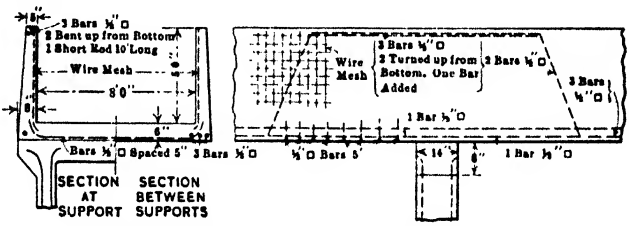


FIG. 51.—Section and elevation of reinforced concrete.

transversely reinforced. As is obvious, the total weight of flume and water is carried by the side walls acting as girders from support to support.

The side walls have the upper edges heavily reinforced to act as beams between supports, resisting the horizontal flexure due to side thrust while, at the bottom, where they join to the trough-bottom, they are anchored by short tension rods. In

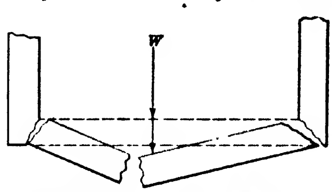


FIG. 52.—Diagram showing effect of transverse flexure in unreinforced concrete.

this way, the sides are supported against bursting outward at top and bottom. Hence, each vertical element of the side wall is a simple beam, supported at each end and having the equivalent of a concentrated load at a point $\frac{d}{3}$ from the bottom. This condition requires that vertical reinforcing rods be placed near the outside surfaces of the walls.

The design shown in Fig. 51 differs from the one just described, in that the side walls are held against both bursting and overturning by cantilever rods, the bottom of the side walls being thickened in order to give a greater lever arm to the steel. This design also avoids the steel reinforcement in the upper edges of the side walls.

The distance apart of supports is a question that must be decided for each locality. It depends on the cost of concrete materials, steel, labor and the height of the flume above ground. Seldom will a span exceeding 30 ft. be economical, and a 12 ft. span is as small as should ever be used.

The ordinary formulæ for design of reinforced-concrete structures are used in fixing the various dimensions, and the amount of steel and its location.

Provision for expansion must be made at intervals not exceeding 100 ft. apart, otherwise the trough will crack. The same forms of expansion joints, as described in the section on Reinforced-concrete Dams, are suitable for flume troughs. Each joint must be made at a support and each adjacent end of the trough should have bearing of at least 8 in. on the support.

This may require the supports underneath the joints to be wider than those which are at other points.

In the type of flume shown in Fig. 50 the forces to be resisted and the formulæ for quantities of steel reinforcement and location of bars are as follows:

The flume will act as a continuous beam, and the *bending moment between supports* (except the last section at each end and the sections adjacent to expansion joints) is

$$M_c = \frac{WL^2}{12} \text{ lb.-ft.} = WL^2 \text{ lb.-in.} \qquad (60)$$

L = distance between supports, in feet.
D = height of side wall, in inches.
d = depth of water in flume, in feet.
b = width of section, in feet.
W = weight per foot of flume, filled.
$= \omega_1 + \omega_2$.

ω_1 = weight of water, per foot length = $62.5\,db$.

ω_2 = weight of flume = $145X$ (approx.), in which X = volume, in cubic feet, of the concrete, per foot length of flume.

The constant 145, is the average weight of concrete, per cubic foot, with steel reinforcement.

Area of steel for longitudinal flexure between supports, is

$$A_c = \frac{8WL^2}{7S(D-2)} \text{ sq. in. (approx.)} \tag{61}$$

S being the stress, in pounds per square inch, in the steel.

Thickness of stressed concrete is given by formulæ in section on "Reinforced-concrete Dams," Chap. VII.

Where the flume does not act as a continuous, but as a simple, beam, namely, at the two ends, and where the spans are adjacent to expansion joints, the spacing can be only 90 per cent. of the intermediate spans, or the steel must be increased 20 per cent. above the amount shown by the formula, if the end spans be equal in length to the intermediate spans. These changes are due to the fact that a beam having one end cantilevered, as a portion of a continuous beam, the other end not cantilevered, but simply resting on the support, has a maximum bending moment = $\frac{WL^2}{10}$, instead of $\frac{WL^2}{12}$ lb.-ft.

Obviously, where the flume is continuous across supports, the longitudinal reinforcing steel at the bottom must be bent up to the top of the trough section, as indicated in the accompanying designs.

If the tops of the side walls are too thin, the compression in the concrete will be excessive. If this is found the case, the design should be changed by thickening the side walls.

Bending moment, per foot length of flume, of the transverse flexure tending to break the bottom through is

$$M_2 = \frac{3W_1b}{2} \text{ lb.-in.} \tag{62}$$

W_1 = weight per foot length of water and bottom of flume.
b = inside width of flume, in feet.

Hence, area of transverse steel, per foot length of flume, is

$$A_2 = \frac{8 \times 3W_1b}{7 \times 2 \times S(t_2 - 1.5)} = \frac{1.71\,W_1b}{S(t_2 - 1.5)} \text{ sq. in.} \tag{63}$$

t_2 = thickness of bottom, in inches.
S = stress in steel, pounds per square inch.

Spacing, J, of steel bars is

$$J = \frac{12 \times A_0}{A_2} \text{ in.,} \tag{64}$$

in which A_0 = area of one bar.

A_2 = total area of steel required.

Force acting outwardly at top of side wall, per foot length, is

$$F_1 = 10.4\, d^2 \text{ lb.} \tag{53}$$

$$\text{Moment of } F_1 = M_3 = F_1L^2 = 10.4\, d^2L^2 \text{ lb.-in.} \tag{65}$$

$$\text{Area of steel} = A_3 = \frac{10.4\, d^2L^2 \times 8}{7\, S(t_3 - 1.5)}$$

$$= \frac{11.9\, d^2L^2}{S(t_3 - 1.5)} \text{ sq. in.} \tag{66}$$

t_3 = thickness of side wall

Force, F_2, acting at bottom of side wall, per foot length, is

$$F_2 = 20.8\, d^2 \text{ lb.} \tag{54}$$

This stress is simply tension, as this force has no lever arm and hence, no moment.

Area of anchoring steel at bottom is

$$A_4 = \frac{20.8\, d^2}{S} \text{ sq. in.} \tag{67}$$

Flexure in side walls, between top and bottom supports, per foot length of wall is found as follows:

The force acting on the side walls to bulge them outward, is zero at the water surface, and 62.5 d pounds, per foot length, at the bottom, so that the stresses in a vertical element of the side walls are similar to those on a beam triangularly loaded. The maximum flexure stress for this loading is slightly greater than that caused by a uniform load having the same weight as the triangular load. In practice, the calculations will be sufficiently close to the actual conditions if the flexure stresses in the side walls be computed as if the vertical elements were uniformly loaded.

Total pressure, or load on side wall, per foot length, is $\frac{62.5\, d^2}{2}$, d, being in feet. This value corresponds to WL in the equation for the moment of flexure, while d corresponds to the span of the

beam and equals L in the equation. Hence,

$$M = \frac{WL^2}{8} = \frac{62.5\, d^3}{16} = 3.9d^3 \text{ lb.-ft.} = 46.8\, d^3 \text{ lb.-in.}$$

Area of steel required is

$$A_5 = \frac{53.5\, d^3}{S\,(t_5 - 1.5)} \text{ sq. in.} \tag{68}$$

Cantilever moment for vertical braces at each support.

Total force acting at top of trough, and equal to the outward pressure for one span length $= F_1L_1 = 10.4\ d^2L$ lb.

Cantilever moment $= M_6 = 12 \times 10.4\ d^2L \times d = 124.8\ d^3L$ lb.-in.

Dividing d into two parts, each equal to $\frac{d}{2}$, the cantilever moment, taken from the top of the side brace, halfway down is

$$M_{6t} = \frac{124.8\, d^3L}{8} = 15.6\, d^3L \text{ lb.-in.} \tag{70}$$

and area of steel required is

$$A_{6t} = \frac{17.8\, d^3L}{S(t_6 - 2)} \text{ sq. in.} \tag{71}$$

which area of steel extends from top to bottom of the sidewall.

The area of the cantilever steel for the bottom section is

$$A_{6b} = \frac{142.6\, d^3L}{S(t_6 - 2)} \text{ sq. in.} \tag{72}$$

which area of steel extends from the bottom halfway up the side.

For holding the sides of the trough by cantilever steel, as indicated in the design shown in Fig. 51, formulæ (70), (71) and (72) apply, except that the constant 2 is changed to 1.5, and L cancels out, unit length of the wall being taken, so that

$A_{7t} = \dfrac{17.8\, d^3}{S(t_5 - 1.5)}$ sq. in., for the top section, per foot length of trough. (73)

$A_{7b} = \dfrac{142.6\, d^3}{S(t_5 - 1.5)}$ sq. in., for the bottom section, per foot length. (74)

All of these equations assume that the reinforcing bars are placed with their centers 1½ in. from the surfaces of thin walls, which are in tension, and 2 in. from the tension surfaces in the thicker portions of the concrete.

As an example, consider, first, the flume shown in Fig. 50, in which

d = 4.5 ft. = maximum depth of water.

b = 8 ft.

D = 5 ft. = 60 in.

l = 20 ft.

S taken at 18,000 lb. per square inch.

$\omega_1 = 8 \times 4.5 \times 62.5 = 2{,}250$ lb.

To find ω_2, assume an average thickness of concrete = 6 in. = 0.5 ft.

$X = 0.5 \times [8 + (2 \times 5)] = 9$ cu. ft., per foot length.

$\omega_2 = 145 \times 9 = 1{,}305$ lb.

$W = \omega_1 + \omega_2 = 2{,}250 + 1{,}305 = 3{,}555$ lb. per foot length.

For longitudinal flexure, $M_1 = WL^2 = 3{,}555 \times (20)^2 = 1{,}422{,}000$ lb.-in. Total area of longitudinal steel required for flexure is, by equation (61),

$$A_1 = \frac{8 \times 1{,}422{,}000}{7(60 - 2)\,18{,}000} = 1.56 \text{ sq. in.}$$

The bars should be located near the edges of the flume, as the vertical walls will resist, practically, all the compression stresses which are opposite to the tension stresses of these longitudinal bars.

The area of a ½-in. square bar is 0.25 sq. in., and to give a total area of 1.56 sq. in. the number of ½-in. bars required is $\frac{1.56}{0.25} = 6.2$, or practically six bars. The location of these bars should be three on each side, as is shown in the figure. In order to make the trough a continuous girder, the rods must be bent up to the top and continue near the upper edge, over the supports. To prevent the trough from cracking, only two rods on each side should be bent upward, an additional short rod being placed at the top to give the proper area of metal. In the special case of the design reinforced at the top, all the way between sections, these rods will take up the cantilever stresses, and the additional short rods are not required.

The spans at the ends and at sections adjacent to expansion joints, with this amount of reinforcement, should be 90 per cent. of 20 ft. or 18 ft. If the end and joint spans be 20 ft. in length, the steel will have to be increased 20 per cent.

and, therefore, will be 1.87 sq. in., requiring eight bars, or four on each side.

The area of the transverse steel to resist the transverse stress at the bottom, per foot length of flume, is, from equation (63),

$$A_2 = \frac{1.71 \times 2{,}975 \times 8}{18{,}000\,(6 - 1.5)} = 0.502 \text{ sq. in.}$$

For ¾-in. square rod the spacing is, from equation (64),

$$\frac{0.5625 \times 12}{0.502} = 13.44 \text{ in, or practically, } 13 \text{ in.}$$

Or, if ½-in. square rods are used instead, the spacing will be

$$\frac{0.25 \times 12}{0.502} = 6 \text{ in.}$$

This is a better construction, and the same quantity of steel is used.

The area of the reinforcing steel to be placed in the top edges of each of the side walls is, from equation (66),

$$A_3 = \frac{11.9 \times (4.5)^2 \times (20)^2}{18{,}000\,(6 - 1.5)} = 1.19 \text{ sq. in.}$$

This reinforcement can best be made of two ¾-in. bars having an area of

$$2 \times 0.5625 = 1.125 \text{ sq. in.}$$

Area of anchoring steel, at bottom of side walls, per foot length is, from equation (67),

$$A_4 = \frac{20.8 \times (4.5)^2}{18{,}000} = 0.0233 \text{ sq. in.}$$

Since the bottom transverse rods will turn up into the sides, and the cross-section of these is greatly in excess of the anchoring steel required for the side walls, no provision is to be made for the anchor rods.

The reinforcement necessary in the side walls to prevent flexure or "bulging" of sides is, from equation (68),

$$A_5 = \frac{53.5\,(4.5)^3}{18{,}000\,(6 - 1.5)} = 0.06 \text{ sq. in. per foot length of flume.}$$

This amount of steel is too small to be conveniently placed in bars. A heavy wire mesh will be the best form of reinforcement here, and will keep the concrete from cracking.

The vertical wall braces at each support are made tapering toward the top as shown. These braces are 14 in. thick at the bottom and 4 in. thick at the top. Halfway up the brace the thickness is 9 in.

Hence, area of steel running all the way up the height of the wall braces is, from equation (71),

$$A_{61} = \frac{17.8 \times (4.5)^3 \times 20}{18{,}000\,(9 - 2)} = 0.258 \text{ sq. in.}$$

Use two $\frac{3}{8}$-in. square bars here. These have a total area of 0.282 sq. in.

Area of cantilever steel running from bottom of wall brace halfway up to the top is, from equation (72),

$$A_{6b} = \frac{142.6 \times (4.5)^3 \times 20}{18{,}000\,(14 - 2)} = 1.203 \text{ sq. in.}$$

Deducting the area of the steel which runs all the way up the brace (= 0.282), the area of the additional steel running from the bottom halfway up the brace is 1.203 − 282 = 0.921. Use four $\frac{1}{2}$-in. square bars for this reinforcement.

Referring to the drawing, it will be seen that the design is in accordance with these computations.

For the second type of flume, shown in Fig. 51, the size is to be the same as that of the flume used in the preceding example. The thickness of the side wall is made 5 in. at the top and 8 in. at the bottom, so that the thickness halfway up the height of the wall is $6\frac{1}{2}$ in.

The longitudinal reinforcement in the bottom of the walls to carry the flume across supports, and the transverse rods in the bottom, to resist the transverse flexure stresses, will be identical with the areas and spacings previously computed for the other form of flume.

The side walls are, however, held up by cantilever steel, and the area of this is computed as follows:

For steel running from top to bottom of the walls, by equation (73).

$$A_{7t} = \frac{17.8 \times (4.5)^3}{18{,}000\,(6.5 - 1.5)} = 0.0225 \text{ sq. in. per foot length.}$$

Use heavy wire mesh for this.

For steel running halfway up the walls from the bottom, the area is, by equation (74),

$$A_n = \frac{142.6 \times (4.5)^2}{18{,}000 \times (8 - 1.5)} = 0.115 \text{ sq. in. per foot length.}$$

The wire mesh used for the reinforcement of the upper portion of the walls will be ample for this area also.

From these data the design shown in Fig. 51 was prepared.

It is not to be assumed that these designs are the most economical for any condition or locality. As is obvious from the formulæ and calculations, the ratio of thickness of concrete and area of steel may be varied as the particular conditions may require. Where steel is costly and concrete cheap, thick walls will be the more economical. Where the reverse condition prevails, use thin walls and greater quantities of steel.

Do not, however, under any conditions, make the proportions of steel and concrete such that the area of the steel is greater than 0.4 per cent. of the area of the section of concrete, perpendicular to the axis of the reinforcing bars, otherwise too great a compression stress may be set up in the concrete.

Tunnels.—The territory over which the water has to be carried sometimes assumes such a configuration that tunnels are necessary, because the altitude to which pipes would have to be carried in order to pass the intervening elevations, would rise far above the hydraulic gradient and be too great for them to act as siphons. Where absolutely necessary and unavoidable, tunnels should, of course, be driven, but wherever they can be avoided, their construction is an unnecessary expense, and, as compared with well-protected steel or reinforced-concrete pipe, a tunnel offers no advantage. Where the tunnels are driven, they should be made as small as possible, and in order to limit their cross-section, should be subsequently lined with concrete, so as to present a smooth surface and allow a high velocity of water through them without too great a loss in head.

The cost of driving tunnels varies greatly with the conditions and locality, but ranges from $7 to $12 per cubic yard of material removed.

Following is a table giving approximate preliminary data on tunnels. The costs are based on the unit figure of $10 per cubic yard, including every expense of excavation, timbering, engineering, and equipment costs.

TABLE 22.—APPROXIMATE DATA ON TUNNELS

Carrying capacity in second feet	Velocity in feet per second	Net sectional area, square feet	Dimensions, width by height	Approximate slope feet per 1,000 ft.	Approximate cost per linear foot
100	3.6	28	4 by 7	0.65	$10
500	10.0	50	7 by 7¼	3.2	19
1,000	10.0	100	10 by 10	2.4	37
1,500	10.0	150	12 by 12½	1.8	58
2,000	10.0	200	14 by 14¼	1 5	77
5,000	10.0	500	20 by 25	0.91	187
10,000	10.0	1,000	30 by 33	0 62	334

The slope per 1,000 ft. is computed by Bazin's formula. The value of m is taken as 0.16, it being assumed that the tunnel walls are made smooth with concrete.

CHAPTER VI

PIPE LINES AND PENSTOCKS

Pipes, or tubes, for conducting water to water wheels are called pipe lines when of considerable length, while the short sections leading to the water wheels are called penstocks.

When canals or open flumes are used to conduct water, they must be nearly level from end to end, the entrance end being higher than the discharge by an amount equal to the head, h, necessary for producing the desired velocity of flow. With tubes, any contour may be followed, and the elevation of the pipe may be varied within any practicable limits, provided it be not raised above the hydraulic gradient, as will later be explained.

Also, no matter what path the pipe may follow, if it is kept below the hydraulic gradient, the net head acting at the discharge end of the pipe will be equal to the difference between the elevation of the receiving and the discharge ends, minus the loss in head due to the resistance of the water to flow through the pipe.

Pipes may be made of steel plate, cast iron, wood staves, or reinforced concrete.

Cast iron is seldom used for water-power conduits, owing to the fact that the diameters required are, usually, greatly in excess of the practicable diameters of cast-iron pipe. Also, the weight of cast iron, its cost and the difficulty of handling and joining the lengths together greatly exceed the corresponding characteristics in pipes of other materials.

Energy in Flowing Water.—A mass of water at a given elevation has a certain potential energy with respect to some lower elevation to which it may be allowed to flow. This energy goes into various forms in the course of the progress from the upper to the lower level. In general, the energy is divided into:

1. Energy required to give the water its initial velocity.

2. Energy required to move the water into the mouth of the orifice through which it passes in going from the reservoir to the conduit, or pipe, that carries it to the lower level.

3. Energy lost in friction due to passage of water through the conduit, or pipe.

4. Energy loss due to abrupt changes in cross-section of the conduit, or pipe.

5. Energy abstracted by water wheels, or other energy using devices in the path of the descending mass of water.

6. Energy remaining in the water, in the form of kinetic energy, when the water emerges from the end of the conduit, or pipe, due to its velocity of efflux.

The sum of all these must always be exactly equal to the potential energy which the mass of water originally possessed, due to its elevation above the lower level to which it passes. From this arises a condition which appears at first glance to be a curious one; namely, that if a pipe has its cross-section gradually increased, the head or pressure of the water in it may be greater in the larger section, though this may be some distance from the entrance end of the pipe, than in the smaller section at some other point nearer the entrance end, despite the greater loss of energy in pipe friction from the entrance end to the larger section.

This is more fully explained later in the example showing how the hydraulic gradient is determined.

Bernouilli's Theorem.—The foregoing statement that the total energy in a moving stream of liquid, at any point in its path, must be equal to that which the liquid originally possessed, less the energy consumed in reaching the chosen point, is another way of stating Bernouilli's theorem, which, algebraically, is

$$p_1 = p + \gamma\left(h + \frac{V^2}{2g} - \frac{V_1^2}{2g}\right) \qquad (75)$$

in which

p_1 = unit pressure at any point in the stream flow.
p = unit pressure at some other point.
γ = weight per unit volume of the liquid (= 62.5 for water).
h = difference in static head between points at which p_1 and p are taken.
V and V_1 = velocities at points p and p_1, respectively.

This is shown as follows: Let B, C, E, F, in Fig. 53, represent a mass of liquid moving in the direction of the arrow u. The boundaries of this mass, BC and FE, are elements of any cross-section, the section being of any shape.

The boundaries may be actual physical walls which confine the liquid, or they may be simply stream lines, or veins of flow in a greater mass of liquid which is moving, but in different stream paths at different points in the cross-section of the mass.

Consider the flow for a very brief period. The mass of liquid at the top will travel through a distance, d, as indicated in the figure by the dotted lines. The portion of the mass at k, near the bottom, will move a distance d', such that $dA = d'A'$, in which A is the area of the top section, and A' that at the bottom. In other words, the volumetric displacement at the top, in a given time, must equal the volumetric displacement at the bottom, or anywhere else in the vein. Otherwise, the liquid would either be compressed, or discontinuous.

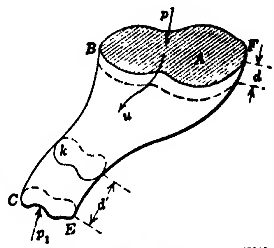


FIG. 53.—Illustrating Bernouilli's theorem.

If a pressure p, per unit area, be acting on the upper surface, A, then the work done in moving through the distance d, is pAd.

Also, if a pressure p_1 per unit area, be acting against the bottom surface, the work done by the movement of the liquid through the distance d' is $p_1A'd'$.

Since the pressure against the bottom of the liquid is opposite to that against the top surface, the signs of the two must be likewise opposed, so that the work done at the lower end is $-p_1A'd'$.

If the volume of liquid Ad, produced by the displacement of the surface A through the distance d, weighs γ lb. per unit volume, the weight of liquid displaced is γAd.

If h is the difference in head between the upper and lower sections, the work done by gravity for a movement of the volume Ad, having a weight γAd, through the distance h, is $h\gamma Ad$.

If the flow is steady, the velocities at A and A' are constant. Call these V and V_1 respectively. Then kinetic energy of weight of water γAd, at velocity V, is $\frac{\gamma AdV^2}{2g}$, while the kinetic energy at velocity V_1, is $\frac{\gamma AdV_1^2}{2g}$.

The difference between these two values represents the

change in the kinetic energy in passing from the upper to the lower positions. This value is $\frac{\gamma A d}{2g}(V_1^2 - V^2)$ (76)

Equating the total work done to the change in the energy of the liquid,

$$pAd - p_1A'd' + h\gamma Ad = \frac{\gamma Ad}{2g}(V_1^2 - V^2) \tag{77}$$

Since $Ad = A'd'$, and this factor cancels on both sides of the equation, this reduces to

$$p - p_1 + h\gamma = \frac{\gamma}{2g}(V_1^2 - V^2)$$

or

$$h\gamma + p + \frac{\gamma V^2}{2g} = p_1 + \frac{\gamma V_1^2}{2g} \tag{78}$$

and

$$p_1 = p + \gamma h + \frac{\gamma}{2g}(V^2 - V_1^2) \tag{79}$$

which shows that if, at any point, the velocity of the liquid increases, thereby increasing its kinetic energy, the pressure must undergo a corresponding reduction, and *vice versa.*

This is a most important theorem and by its use all problems of pressure and flow are easily solved.

If, instead of velocities V and V_1, their equivalent static heads H and H_1 are used, the formula is somewhat simplified.

Since $V^2 = 2gH$, this value can be substituted for values of V^2 and V_1^2 in formula (79) which becomes

$$p_1 = \gamma h + p + \gamma H - \gamma H_1$$

or

$$p_1 = p + \gamma(h + H - H_1) \tag{80}$$

In addition to the hydraulic, or mechanically applied, pressures that may act on a mass of water, the pressure of the atmosphere must be included in all computations when air is excluded from any pipe, or vessel, which may form a portion of the path travelled by the water. Usually, air pressures are neglected because they apply, equally, to all parts of the mass of water and, therefore, cancel out of the calculation.

The laws of flow of water in pipes are similar to those of flow in open channels. There is a resistance to the passage of the water through the pipe and a pressure, or head, must be present to overcome this resistance and cause flow. The resistance de-

pends for its value on the character of the surface of the pipe, the length, the diameter, and some power of the velocity of the water. Obviously, the head required to cause flow is lost and is deducted from the gross head to obtain the net head.

There are several formulæ for computing the head required for producing flow in pipes. Kutter's, Bazin's, and Hazen & Williams', as given in Chap. II, are all applicable.

For many years it was assumed by various investigators that the head required to force water through a pipe, or conduit, varies as the square of the velocity. It has long been known that at extremely low velocities the required head varies, approximately, as the first power of the velocity. Such velocities are very much lower than those used in practical hydraulics and need not be considered here.

At practicable velocities, the head required to move water through a conduit varies more rapidly than the first power, but not so greatly as the square of the velocity.

The more recent formulæ for the flow of water in pipes are those of Tutton, and Hazen & Williams, both of which are exponential, and of the general form, $V = Cr^nS^m$.

Tutton's formula is, perhaps, as accurate as that of Hazen & Williams, but this accuracy is obtained by varying both the coefficient and the exponents for varying conditions. Hence, it is not a general formula at all, but an algebraic expression of empirical results.

The formula of Hazen & Williams is, apparently, the most reliable of them all, particularly for power-plant work, as the experiments to which these investigators had access included large pipes, while the tests on which most other formulæ are based were made on pipes having diameters of less than 3 ft. For these reasons, the Hazen & Williams formula has been adopted in this work.

The use of an exponential formula, in which the powers are fractional, may appear, at first sight, to offer difficulties of computation greater than the old, and less exact formulæ, based on the variation in head with the square of the velocity, and in which the powers are whole numbers. This is, however, not the case. Nearly all exponential formulæ are most easily and accurately solved by the use of logarithms, and, moreover, the possibility of arithmetical errors is greatly reduced, even if the powers are simply squares and cubes.

The adoption of fractional exponents merely compels usir the simpler method. Also, the objection, which has some times been offered to equations in which fractional exponent occur, that they are not adapted for solution on the slide rule, does not hold. The ordinary slide rule has a logarithmic graduation on the under side of the slide, and it is as easy to read logarithms on the rule as to read the figures on the upper side of the slide. Any engineer who will make a few computations with this formula, will conclude that it is just as simple and easily used as any of the older and less accurate ones.

The tests investigated by Hazen & Williams, and also by Tutton, show, conclusively, that the resistance to flow of water in pipes varies less rapidly than the square of the velocity. The Tutton formula shows a variation proportional to the 1.73 to the 1.96 power of the velocity. The Hazen & Williams formula shows the variation to be proportional to the 1.853 power of the velocity.

The Hazen & Williams formula for flow is

$$V = 1.32\, Cr^{0.63} s^{0.54}. \tag{81}$$

or placing $1.32\ C = k$,

$$V = kr^{0.63} S^{0.54}. \tag{82}$$

V = velocity in feet per second.

r = hydraulic radius $= \dfrac{\pi D^2}{4} \div \pi D = \dfrac{D}{4}$ for a circular pipe, D being the diameter.

s = slope $= \dfrac{h}{l}$, in which h = difference in head between the two ends of the pipe, and l = length of pipe, both in feet.

C is a constant, which has different values for different kinds of pipes.

The average values of the constants C and k are as follows:

TABLE 23.—VALUES OF C, k AND LOG k FOR HAZEN & WILLIAMS FORMULA
(Values for Open Channels given in Chap. II)

	C	$k = 1.32\,C$	Log k
Smooth wood-stave pipes......	125	165	2.2175
Unplaned wood flumes........ ..	112	148	2.1703
Smooth concrete...............	130	172	2.2355
Cast-iron pipes, new...........	130	172	2.2355
Rivetted steel pipes, new.........	110	145	2.1614
Rivetted steel pipes, old........	100	132	2.1206
Rough, tuberculated iron pipes..	40 to 90	53 to 119	1.7243 to 2.0755

From formula (82), and the foregoing table of constants, the velocity for any given slope and size of pipe can be computed.

The head required to maintain water flow at a given velocity, or the "friction-head loss" in pipes, is computed from the following modifications of the Hazen & Williams formula.

$$h_1 = \frac{FV^{1.853}}{D^{1.166}} \text{ ft.} \tag{83}$$

$$h = \frac{lFV^{1.853}}{D^{1.166}} \text{ ft.} \tag{84}$$

h_1 = head loss per foot length of pipe.
h = head lost in length, l, of pipe.
V = velocity in feet, per second.
D = diameter of pipe, in feet.
F = a constant.

Following is a table giving values of constant, F, for various kinds of pipes.

TABLE 24.—VALUES OF F AND LOG F FOR HAZEN & WILLIAMS FORMULA

	F	log F
Smooth wood-stave pipes.......	0.000392	$\bar{4}.59302$
Unplaned wooden flumes.......	0.00048	4.68048
Smooth concrete, Cast-iron pipes—new	0.000363	$\bar{4}.55966$
Rivetted-steel pipes—new.......	0.0005	$\bar{4}.69481$
Rivetted-steel pipes—old........	0.0006	$\bar{4}.77257$
Rough, tuberculated iron pipes.	0.0032 to 0.00072	$\bar{3}.50692$ to $\bar{4}.85614$

As an example of the use of these formulæ, take the following conditions:

D = diameter of pipe = 6 ft.

Kind of pipe; rivetted steel, new.

l = length of pipe = 1500 ft.
h = difference in elevation between two ends of pipe = 6.3 ft.

$$\text{Hydraulic radius, } r = \frac{D}{4} = 1.5 \text{ ft.}$$

$$S = \text{slope} = \frac{h}{l} = \frac{6.3}{1{,}500} = 0.0042.$$

What is the velocity, in feet per second, through the pipe? Constant k, from table for rivetted-steel pipe, is 145.

$$V = 145 \times (1.5)^{0.63} \times (0.0042)^{0.54}$$

$$\log 1.5 = 0.1761$$

$$0.1761 \times 0.63 = 0.11094$$

Number corresponding to log 0.11094 is 1.291, so that

$$(1.5)^{0.63} = 1.291$$

$$\log 0.0042 = \bar{3}.62325$$

$$\log (0.0042)^{0.54} = \bar{3}.62325 \times 0.54 = \bar{2}.71653$$

Number corresponding to log $\bar{2}.71653$ = 0.0521
Hence $(0.0042)^{0.54} = 0.0521$
Hence $V = 145 \times 1.291 \times 0.0521 = 9.75$ ft. per second.

Of course, the easier and quicker way to perform this operation would be to add the logarithms of the several quantities, and take as the final result the number corresponding to the algebraic sum. Thus,

For k = 145, log k =	2.1614
log 1.5 × 0.63 =	0.11094
	2.27234
log 0.0042 × 0.54 =	−1.28347
Algebraic sum of logs =	0.98887

Number corresponding to log 0.98887 = V = 9.747 ft. per sec.

As an example of the use of the formula for computing loss of head, take the size, length and kind of pipe as given in the preceding example and, with a velocity of 9.75 ft. per second,

TABLE 25.—FRICTION LOSS IN NEW RIVETTED-STEEL PIPES PER 1000 FT. AND DISCHARGE

H = Loss in feet head. Q = Discharge in cubic feet per second

Pipe diameter		Velocities in feet per second											
Inches	Feet	1	2	3	4	5	6	7	8	9	10	11	12
12	1.0	Q = 0.7854 H = 0.50	1.5708 1.81	2.357 3.8	3.1415 6.5	3.927 9.86	4.714 13.5	5.498 18.00	6.28 23.6	7.068 29.4	7.854 35.6	8.66 42.6	9.425 50.0
18	1.5	Q = 1 766 H = 3.12	3.53 1.125	5.29 2.395	7.06 4.07	8.82 6.16	10.60 8.45	12.36 11.47	14.12 14.7	15.9 18.31	17.66 22.2	19.4 26.55	21.28 31.2
24	2.0	Q = 3.14 H = 0.223	6.28 0.80	9.41 1.71	12.56 2.91	15.7 4.40	18.85 6.2	22.0 8.2	25.14 10.5	28.25 13.1	31.41 15.8	34.6 18.9	37.7 22.30
30	2.5	Q = 4.91 H = 0.172	9.82 0.62	14.73 1.3	19.64 2.24	24.55 3.4	29.46 4.55	34.37 6.33	39.28 8.11	44.2 10.08	49.1 12.25	54.0 14.61	58.91 17.2
36	3.0	Q = 7.07 H = 0.139	14.14 0.502	21.21 1.062	28.28 1.81	35.35 2.74	42.42 3.84	49.49 5.11	56.56 6.55	63.63 8.15	70.7 9.9	77.77 11.81	84.84 13.9
42	3.5	Q = 9.62 H = 0.116	19.24 0.418	28.86 0.887	38.48 1.514	48.10 2.29	57.72 3.21	67.34 4.26	76.96 5.46	86.58 6.81	96.2 8.26	105.8 9.88	115.4 11.6
48	4.0	Q = 12.57 H = 0.100	25.14 0.361	37.71 0.765	50.28 1.30	62.85 1.97	75.42 2.70	68.0 3.68	100.56 4.71	113.13 5.86	125.7 7.13	138.27 8.51	150.84 10.0
54	4.5	Q = 15.9 H = 0.0867	31.8 0.313	47.7 0.665	63.6 1.132	79.5 1.71	95.4 2.34	111.3 3.20	127.2 4.09	143.1 5.08	159.0 6.19	174.9 7.40	190.8 8.67
60	5.0	Q = 19.63 H = 0.0765	39.26 0.276	58.89 0.586	78.52 1.00	98.15 1.512	117.78 2.07	137.41 2.70	157.04 3.61	176.67 4.49	196.3 5.46	216.0 6.52	235.5 7.65
66	5.5	Q = 23.76 H = 0.0685	47.52 0.248	71.28 0.525	95.04 0.893	118.80 1.354	142.56 1.87	166.32 2.52	190.08 3.22	213.84 4.02	237.6 4.88	261.36 5.84	285.12 6.85
72	6.0	Q = 28.27 H = 0.062	56.54 0.224	84.81 0.475	113.08 0.808	141.35 1.225	169.62 1.67	197.89 2.28	226.16 2.92	254.43 3.64	282.7 4.43	311.0 5.28	339.24 6.20
78	6.5	Q = 33.18 H = 0.0564	66.36 0.204	99.54 0.433	132.72 0.735	165.90 1.113	199.08 1.52	232.26 2.08	265.44 2.66	298.62 3.32	331.8 4.02	365.0 4.80	398.16 5.64
84	7.0	Q = 38.48 H = 0.0516	76.96 0.186	115.44 0.396	153.92 0.671	192.40 1.02	230.88 1.40	269.36 1.90	307.84 2.43	346.32 3.03	384.80 3.68	423.28 4.40	461.76 5.16
90	7.5	Q = 44.18 H = 0.0477	88.36 0.173	132.54 0.365	176.72 0.621	220.90 0.944	265.08 1.32	309.26 1.76	353.44 2.25	397.62 2.80	441.8 3.40	486.0 4.06	530.16 4.77
96	8.0	Q = 50.27 H = 0.0442	100.54 0.160	150.81 0.339	201.08 0.575	251.35 0.876	301.62 1.20	351.89 1.630	402.16 2.06	452.43 2.60	502.70 3.16	553.0 3.77	603.24 4.42
102	8.5	Q = 56.75 H = 0.0412	113.50 0.149	170.25 0.316	227.0 0.536	283.75 0.814	340.5 1.14	397.25 1.52	454.0 1.94	510.75 2.42	567.5 2.94	624.25 3.51	681.0 4.12
108	9.0	Q = 63.62 H = 0.0386	127.24 0.139	190.86 0.296	254.48 0.502	318.10 0.763	381.72 1.07	445.34 1.42	508.96 1.82	572.58 2.26	636.20 2.75	699.82 3.28	763.44 3.86
114	9.5	Q = 70.88 H = 0.0362	141.76 0.130	212.64 0.276	283.32 0.466	354.40 0.71	425.28 0.995	496.16 1.33	567.64 1.69	637.92 2.11	708.8 2.56	779.7 3.06	850.6 3.59
120	10.0	Q = 78.54 H = 0.0341	157.08 0.123	235.62 0.261	314.16 0.443	392.70 0.675	471.24 0.924	549.78 1.255	628.32 1.615	706.86 2.00	785.4 2.43	863.94 2.90	942.48 3.41
126	10.5	Q = 86.59 H = 0.0322	173.18 0.116	259.77 0.247	346.36 0.419	432.95 0.636	519.54 0.890	606.13 1.188	692.72 1.52	779.31 1.89	865.9 2.30	952.49 2.74	1039.1 3.22
132	11.0	Q = 95.03 H = 0.0305	190.0 0.110	285.1 0.234	380.1 0.396	475.15 0.605	570.2 0.815	665.2 1.124	760.24 1.44	855.3 1.79	950.3 2.18	1045.3 2.60	1140.4 3.05
138	11.5	Q = 103.9 H = 0.029	207.8 0.105	311.7 0.222	415.6 0.277	519.5 0.574	623.4 0.785	727.3 1.07	831.2 1.37	935.1 1.71	1039 2.07	1143 2.47	1247 2.90
144	12.0	Q = 113.1 H = 0.0275	226.2 0.0994	339.3 0.211	452.4 0.357	565.5 0.544	678.6 0.745	791.7 1.013	904.8 1.30	1018 1.62	1131 1.96	1244 2.34	1357 2.75

For loss in wood-stave pipe, multiply tabular figures by 0.80.
For loss in concrete pipe, multiply tabular figures by 0.725.
For loss in cast-iron pipe (new), multiply tabular figures by 0.725.
For loss in old rivetted pipe, multiply tabular figures by 1.20.

compute the head lost in forcing the water through it. The result should be a check on the first assumed head of 6.3 ft. within the limits of accuracy of the constants F.

The formula is

$$h = Fl\frac{V^{1.853}}{D^{1.166}} \text{ and, for this case,}$$

$$F = 0.0005$$
$$l = 1.500 \text{ ft.}$$
$$V = 9.75 \text{ ft. per second.}$$
$$D = 6.0 \text{ ft.}$$

$$\log V = \log 9.75 = 0.98887$$
$$\log V^{1.853} = 0.98887 \times 1.853 = \quad 1.83138$$
$$\log D = \log 6 = 0.7782$$
$$\log D^{1.166} = 0.7782 \times 1.166 = \quad 0.90738$$
$$\log V^{1.853} - \log D^{1.166} = \quad 0.92400$$

Number corresponding to log 0.92400 is 8.395.

Hence $h = 0.0005 \times 1500 \times 8.395 = 6.296$ or, practically, 6.3 ft.

Table 25 gives the friction head loss in new, rivetted, steel-plate pipes per 1000-ft. length of pipe, and the discharge in cubic feet per second for various sizes and velocities.

For convenience in calculating the loss of head for velocities other than those given in Table 25, Table 26 of values of functions for various pipe diameters has been computed.

Determination of Size of Pipe.—If the amount of power to be delivered at the water-wheel shaft, and the allowable loss of head be known, the proper diameter D, of pipe, is given by the formula

$$D = \left(\frac{33.5P}{k\epsilon H}\right)^{0.38} \times \left(\frac{l}{pH}\right)^{0.205} \tag{85}$$

k = factor for specific kind of pipe as given in Table 23.

P = number of horsepower to be delivered at the water-wheel shaft (assumed efficiency = 80 per cent.).

H = total head, in feet, *i.e.*, difference between elevation of the water surface at intake and of tail water at discharge.

ϵ = efficiency of pipe line expressed as a decimal fraction.

p = ratio of head lost in pipe line to total head, expressed as a decimal fraction of total head.

$\epsilon + p = 1.$

l = length of pipe in feet.

TABLE 26.—VALUES OF FUNCTIONS OF D FOR FORMULÆ FOR FRICTION LOSS IN PIPES

Diameter, Inches	Diameter, D. Feet	log D	log $D \times 1.166$	$D^{1.166}$	Area $= \frac{\pi D^2}{4}$ sq. ft.
12	1.0	0.00	0.00	1.000	0.7854
18	1.8	0.176091	0.205322	1.604	1.767
24	2.0	0.301030	0.351001	2.244	3.141
30	2.5	0.397940	0.463998	2.911	4.909
36	3.0	0.477121	0.556333	3.600	7.069
42	3.5	0.544068	0.634383	4.309	9.621
48	4.0	0.602060	0.702002	5.035	12.57
54	4.5	0 653213	0.761646	5.776	15.90
60	5.0	0.698970	0.814999	6.531	19.63
66	5.5	0.740363	0.863263	7.299	23.76
72	6.0	0.778151	0.907324	8.078	28.27
78	6.5	0.812913	0.947857	8.869	33.18
84	7.0	0 845098	0.985384	9.669	38.48
90	7.5	0.875061	1.020321	10.480	44.18
96	8 0	0.903090	1.053003	11.300	50.27
102	8.5	0.929419	1.083702	12.126	56.75
108	9 0	0.954243	1.112647	12.961	63.62
114	9.5	0.977724	1.140026	13.802	70.88
120	10.0	1.00000	1.166000	14.658	78.54
126	10.5	1.021189	1.190706	15.513	86.59
132	11.0	1.041393	1.214264	16.378	95.03
138	11.5	1.060698	1.236773	17.250	103.9
144	12.0	1.079181	1.258325	18.127	113.1

This formula is derived as follows.

The actual head to force the water through the pipe is pH, and the net head at the water wheel is $H-pH = \epsilon H$.

The velocity in the pipe is equal to $V = kr^{0.63}s^{0.54}$.

r = hydraulic radius $= \frac{D}{4}$; D = diameter of pipe, in feet.

The slope is,

$$s = \frac{pH}{l}.$$

A = area of pipe $= \frac{\pi D^2}{4}$.

Q = cubic feet of water per second through the pipe =

$$VA = k\left(\frac{D}{4}\right)^{0.63} \times \left(\frac{pH}{l}\right)^{0.54} \times \frac{\pi D^2}{4} \qquad (86)$$

P = horsepower = $\frac{QH_1}{11}$ for a water-wheel efficiency of 80 per cent. H_1 being the net head at the water wheel = ϵH.

Then

$$P = k\left(\frac{D}{4}\right)^{0.63} \times \left(\frac{pH}{l}\right)^{0.54} \times \frac{\pi D^2}{4} \times \frac{\epsilon H}{11} \qquad (87)$$

Whence,

$$D = \left(\frac{33.5P}{k\epsilon H}\right)^{0.38} \times \left(\frac{l}{pH}\right)^{0.205} \qquad (88)$$

It is obvious from the derivation, that no allowance is made for entry and velocity heads. These are usually negligible for pipe lines of any considerable length, and their inclusion in the formula would needlessly complicate it. In view of the fact that pipes are not constructed to exact, computed dimensions but in accordance with standard shop or field practice, which requires that the diameter of large pipes be some multiple of ⅓ or ½ ft., it is clear that absolute exactness in the computation of pipe sizes will not lead to any engineering construction in accordance with the mathematical result. If the computed diameter were 6.32 ft., the nearest standard size above 6.32 would be taken, which is 6.5 ft., and this slightly larger pipe would cost less than the smaller, odd size.

An example will illustrate the use of the formula.

Assume the following conditions:

P = horsepower to be delivered at the water-wheel shaft = 1500.

H = gross head = 80 ft.

l = length of pipe line = 12,500 ft. Pipe to be of rivetted steel plate. Hence, k (from Table 23) = 145.

Allow a 4 per cent. loss of the total head in the pipe, giving a pipe efficiency of 96 per cent. Then,

$$p = 0.04$$
$$\epsilon = 0.96$$
$$D = \left(\frac{33.5 \times 1500}{145 \times 0.96 \times 80}\right)^{0.38} \times \left(\frac{12{,}500}{0.04 \times 80}\right)^{0.205}$$
$$= (4.51)^{0.38} \times (3903)^{0.205}$$

$$\text{Log } 4.51 = 0.6542$$
$$\text{Log } 3903 = 3.5917$$
$$0.6542 \times 0.38 = 0.248596$$
$$3.5917 \times 0.205 = 0.736298$$
$$\text{Sum} = 0.984894$$

Number corresponding to log 0.984894 $= D = 9.658$ ft.

In practice, this pipe would be made 9.5 ft. in diameter.

Minimum Size of Pipe.—Occasionally, a limited amount of power only, is required and the stream flow, with the available head, is capable of delivering considerably more power than is demanded. In this case, if a long pipe is necessary to carry the water to the water wheels, the problem is to determine the smallest size of pipe, and, therefore, the lowest cost pipe which will deliver the required power to the water wheels. The question of efficiency of the pipe line does not enter into the problem because all unused water and the energy in it go to waste.

The diameter of the smallest size pipe to deliver a given amount of power is that in which the loss of head in the pipe is 0.351 of the total head. By putting $p = 0.351$ and $\epsilon = 0.649$ in formula (88), the resulting diameter will be the smallest size of pipe that will deliver the required power under the given head and length of pipe. This relation is derived as follows:

$P = \dfrac{Q(H-h)}{11}$ in which $h = pH$ is the head lost in the pipe.

By formula (86),

$$Q = \frac{k\pi D^{2.63} h^{0.54}}{4^{1.63} l^{0.54}}$$

Whence,

$$P = \left(\frac{k\pi D^{2.63} h^{0.54}}{11 \times 4^{1.63} l^{0.54}}\right)(H - h) \qquad (89)$$

To obtain the maximum power, the derivative of P with respect to h is taken, and equated to zero.

Casting out the factors which do not involve h, and differentiating,

$$\frac{d}{dh} h^{0.54}(H - h) = 0$$

or

$$0.54Hh^{-0.46} = 1.54h^{0.54}.$$

Multiplying both sides of the equation by $h^{0.46}$ it becomes

$$0.54\,H = 1.54h.$$

Whence,

$$h = 0.351H.$$

Obviously, this is also the condition for the maximum delivery of power through any size of pipe, because the quantity of water delivered through a pipe absorbing a friction head of $0.351H$, and this quantity, multiplied by $0.649H$, or the net useful head, gives the maximum possible product of these two factors. Under the condition of delivery of maximum power, the efficiency of the pipe line is only 64.9 per cent., which is much too low for any condition except the rare one which has been set forth. The delivery of the maximum power must not be confused with high efficiency, nor is it the maximum quantity of water the pipe can discharge under the given head. This latter condition is, of course, attained when the entire head is used to overcome friction in the pipe, and to produce the velocity head. Under this condition, the velocity head only, is available for power and ϵ is very small, while p is very great. Thus, the maximum efficiency of the pipe, as a water carrier only, is reached when its efficiency as a power carrier reaches its lowest value.

Velocity Head.—The foregoing discussion of friction-head loss in pipes assumes that a certain velocity is maintained throughout the length of the pipe. The water must be given this initial velocity, and this requires the further expenditure of head, which is $h_v = \frac{V^2}{2g}$.

This head may or may not be lost. It is lost if the pipe discharges into a pond or forebay, or an open flume. If the pipe discharges into a cylindrical wheel chamber, about 50 per cent. of this head is lost. If the pipe discharges into a scroll-case wheel chamber, only about 10 per cent. of the velocity head is lost, while if it discharges through a nozzle against the vanes of an impulse wheel, none of this head is lost.

Under usual, practical conditions, V will be between 4 and 8 ft. per second, so that h_v will be from 0.25 ft. to 1 ft.

Entrance Head.—In addition to the loss in head due to the friction of water conduits, there is the loss due to entrance head. Whenever water is drawn through a canal or penstock from some large body of water, the particles of water all move in toward the mouth of the conduit. This means either the starting of the particles from rest, to bring them to the conduit entrance, or, possibly, a deflection of them from their path in some other direction, which obviously requires the expenditure of energy.

The entry head is given by the formula

$$h_2 = \frac{V^2}{2g}\left(\frac{1}{C^2} - 1\right) \qquad (90)$$

in which C is an empirical constant, varying with the character of the entrance orifice. In Fig. 27 are shown various forms of entrance orifices and the corresponding values of C and $\frac{1}{C^2} - 1$ are given on the figure.

For instance, in Fig. 27*a* is shown a pipe projecting into the reservoir.

$$\left(\frac{1}{C^2} - 1\right) = 0.93$$

$$h_2 = \frac{V^2}{2g} \times 0.93$$

Figure 27*c* shows a bell-mouthed orifice, and the value of $\frac{1}{C^2} - 1$ for this form is 0.06. Hence, $h_2 = \frac{0.06V^2}{2g}$. These figures show the importance of making a flared entrance end for penstocks in low head developments. As shown by the foregoing computations, the entry head required for the inwardly projecting, straight tube, is over 15 times as great as for a flared end.

If the velocity were 10 ft. per second through the orifice, the loss of head for the inwardly projecting tube would be

$$\frac{10^2}{64.4} \times 0.93 = 1.44 \text{ ft.}$$

while for the bell-shaped end, the loss would be

$\frac{10^2}{64.4} 0.06 = 0.093$ ft.—a difference of 1.347 ft., which, for a 20-ft. head development, is 6.7 per cent. of the total head.

Losses Due to Changes in Section.—When the size of a conduit is suddenly changed, there is a certain loss of head due to impact and swirls, which goes into heat. If the area be very smoothly and gradually increased, there is no perceptible loss. Hence, the necessity for long tapers in pipes and flumes when the size of the section changes.

The loss of head due to sudden increase in size is

$$h_1 = \frac{(V_1 - V_2)^2}{2g} \qquad (91)$$

in which V_1 = velocity in small section.

V_2 = velocity in large section.

It is to be noted that the loss of head due to sudden increase in the size of pipe is an energy loss, but this does not mean a *net* reduction of the *pressure* inside the larger section equal to $\frac{(V_1 - V_2)^2}{2g}$. As has been explained previously, a reduction in velocity is accompanied by a corresponding increase in pressure head. Hence, the pressure head p_2 in the larger pipe will be

$$p_2 = \frac{V_1^2 - V_2^2}{2g} - \frac{(V_1 - V_2)^2}{2g} + p_1 = \frac{V_2(V_1 - V_2)}{g} + p_1$$

in which V_1 = velocity in smaller pipe, V_2 = velocity in larger pipe, and p_1 is the pressure head in the smaller pipe. The first term of this equation is the net increase in pressure head, due to the reduction of the velocity in the enlarged section. The drop in head, $\frac{(V_2 - V_1)^2}{2g}$, due to the eddy swirls, is a net loss and can not be recovered as either pressure or velocity head.

The loss of head, if the section is suddenly contracted, is

$$h_1 = \frac{nV_2^2}{2g} \qquad (92)$$

in which n is an empirical constant.

If A_1 = area of the smaller pipe, and

A_2 = area of the larger pipe.

Then, for different values of the ratio $\frac{A_1}{A_2}$, the following are values of n.[1]

TABLE 27.—VALUES OF R FOR LOSS OF HEAD DUE TO CONTRACTION IN PIPES

$\frac{A_1}{A_2}$	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0,100
n	0.362	0.338	0.308	0.267	0.221	0.164	0.105	0.053	0.015	0.00

In sudden contraction of the section, the net pressure head in the smaller pipe will be

$$p_1 = p_2 - h_1 - \left(\frac{V_1^2 - V_2^2}{2g}\right) \qquad (93)$$

h_1 = loss of head due to sudden contraction $= \frac{nV_2^2}{2g}$

n = constant from Table 27.

[1] HOSKIN'S "Hydraulics," p. 74.

Hence

$$p_1 = p_2 + \frac{V_2^2(1 - n) - V_1^2}{2g} \tag{94}$$

Loss of Head due to Bends.—The data from various experiments to determine the loss of head due to bends, are so conflicting and so limited in range—all experiments, apparently, having been made on sizes of pipe below 24 in. in diameter—that it may be said that at this time, (1916), no formula has yet been devised applicable to power pipe lines and penstocks.

This deficiency in the available engineering data of the present day is not so important as may, at first, appear. It is expensive, generally unnecessary, and certainly bad practice to make elbows or quick bends in large pipes. The loss in large bends having a radius 6 times or more the diameter of the pipe will not differ, appreciably, from the friction loss in the same length of straight pipe, the length, of course, being measured along the center line of the curved section of pipe.

Computation of Q for System of Varying Cross-section

If the head lost in any pipe, or system of piping and orifices, is h_1, with some known or assumed quantity of water Q_1 passing through the pipe or system, and h is the allowable loss of head, then the quantity of water Q that the head h will force through the pipes is

$$Q = Q_1\left(\frac{h}{h_1}\right)^{0.54} \tag{96}$$

And conversely, if Q_1 cu. ft. sec. of water are passed through a pipe or system, with loss of head h_1, then Q cu. ft. per second will require the expenditure of a head

$$h = h_1\left(\frac{Q}{Q_1}\right)^{1.853} \tag{97}$$

For example: If a head of 3.5 ft. is absorbed in causing a discharge of 400 cu. ft., per second, through a series of pipes and orifices, the discharge for a 6.5 ft. head will be $Q = 400 \times \left(\frac{6.5}{3.5}\right)^{0.54} = 558$ cu. ft. per second. Also, if 400 cu. ft. per second be passed through a system of piping with an expenditure of 3.5 ft. head, then the head required to force 558 cu. ft. per second through the same system will be

$$h = 3.5 \times \left(\frac{558}{400}\right)^{1.853} = 6.5 \text{ ft.}$$

In the case of a number of pipes and orifices, or several variations in cross-section, a formula for the direct solution of V becomes so long that the computation is tedious. The simpler method is to assume a velocity in one of the sections and find the corresponding velocity for each of the other sections. Then compute the loss in head for each section and the sum of these is the total head required to force water through the system at the assumed velocity. Then, by formula (96), the velocity for the given head can be determined.

Thus, for the system indicated in Fig. 54, the computations would be as follows:

Assume a canal N, discharging through an orifice M in a bulkhead to pipe A, which is 1000 ft. long, from pipe A to pipe B, through opening l. Pipe B is 1200 ft. long. From pipe B,

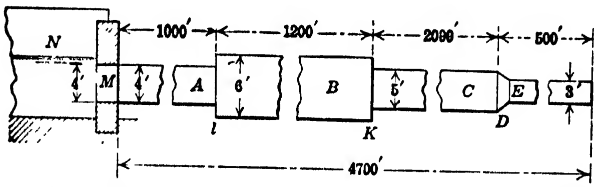


Fig. 54.—Example of piping system to compute friction losses.

through opening K into pipe C, which is 2000 ft. long. From pipe C, through throat D into pipe E, 500 ft. long, and finally discharging from the end of pipe E.

Assume that the diameters are: throat M, 4 ft., pipe A, 4 ft., pipe B, 6 ft., pipe C, 5 ft., E, 3 ft. All pipes of steel plate. Then the respective areas and velocity ratios, considering the largest opening, or B, as the unit, are as follows:

Area of M =	12.56 sq. ft.	Vel. ratio = 2.25
" " A =	12.56 sq. ft.	" " 2.25
" " B =	28.27 sq. ft.	" " 1.00
" " C =	19.63 sq. ft.	" " 1.44
" " E =	7.07 sq. ft.	" " 4.00

Assume a velocity of 1 ft. per second through pipe B. The velocities through the other sections of the system are equal to the assumed velocity for the largest section, multiplied by the velocity ratios, which velocities, in this case, are numerically equal to the velocity ratios as given.

The velocity head will change for every change in cross-section. So in computing the loss due to the velocity head, always take its value from the velocity through the discharge end of the system. Thus, for the present example, the velocity head to be taken is that to give the water the required velocity through pipe E which is

$$\frac{(4)^2}{2g} = 0.25 \text{ ft., which is lost at the discharge end.}$$

Loss in entry head is

$$h_2 = \frac{(2.25)^2}{2g} \times 0.5 = 0.0393 \text{ ft.}$$

$$\text{Loss through } A = \frac{1200 \times 0.0005 \times (2.25)^{1.853}}{(4)^{1.166}} = 0.446 \text{ ft.}$$

Loss at L in passing suddenly from pipe of smaller to one of greater section is

$$\frac{(2.25 - 1.00)^2}{2g} = 0.0243 \text{ ft.}$$

$$\text{Loss of head in } B = \frac{1200 \times 0.0005 \times 1^{1.853}}{6^{1.166}} = 0.0743 \text{ ft.}$$

Loss at K, passing suddenly from a large to a small pipe, is

$$h = n\frac{V_2^2}{2g}$$

$$\frac{A_1}{A_2} = \frac{19.63}{28.27} = 0.694. \quad n \text{ (from table)} = 0.105$$

$$V_2 = 1.44$$

$$\text{Loss of head} = \frac{(1.44)^2}{2g} \times 0.105 = 0.00338 \text{ ft.}$$

$$\text{Loss in section } C = \frac{2000 \times 0.0005 \times (1.44)^{1.853}}{5^{1.166}} = 0.31 \text{ ft.}$$

$$\text{Loss in section } E = \frac{500 \times 0.0005 \times (4)^{1.853}}{3^{1.166}} = 0.91 \text{ ft.}$$

Total loss of head = 2.05728 ft.

Discharge in cubic feet per second = any area × velocity through the area taken = 28.27 × 1 or 7.07 × 4 = 28.27 cu. ft. second.

For any other velocity, loss of head, or rate of discharge, the corresponding quantities can be computed from equations (96) and (97).

For instance, if the available head were 5 ft., the quantity of water which would pass through the system would be =

$$28.27 \times \left(\frac{5}{2.057}\right)^{0.54} = 45.7 \text{ cu. ft. per second.}$$

Of course, the results from formula (96) will only approximate the actual values of Q or h for values of h or Q other than those for the assumed velocity, if the losses of head due to orifices, entrances and changes in pipe section are appreciable as compared with the total loss, because they vary as the square and not as the 1.853 power of the velocity. The approximation will be close enough, however, in most cases, and a re-computation of the value of Q or h, using the approximate value as found from the formula, will show how far it departs from the actual, so that one or two trial and error calculations will fix the desired value with sufficient accuracy.

If the greater loss is in the factors that are functions of the square of the velocity, compute values of Q or h by formula

$$Q = Q_1 \sqrt{\frac{h}{h_1}} \tag{98}$$

instead of formula (96).

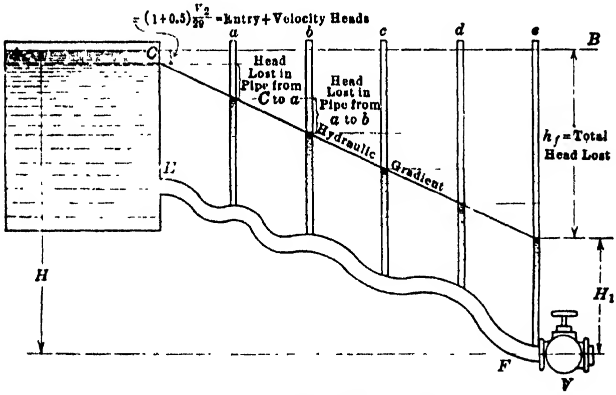


FIG. 55.—Hydraulic gradient.

Hydraulic Gradient.—From the conditions of flow, as indicated by the previously given formulæ, there is a continual absorption of head to maintain the flow, and this is proportional to the length of pipe. Hence, if vertical tubes were inserted in the pipe, these tubes being spaced along the length of the pipe, the height to which the water would rise in any tube would be less than in the tubes nearer to the entrance end of the pipe, and greater than in any tube farther from the entrance end of the pipe. This condition is illustrated in Fig. 55.

EF is a pipe, straight or curving, which takes water from a stream or reservoir having a water level *AC*, and the vertical tubes *a*, *b*, *c*, *d* and *e* are inserted in the main pipe *EF*. When the valve *V* is closed and no water flows through the pipe, the level of the water in each of the tubes will be the same as that in the reservoir, namely, up to line *CB*, and the gross head, *H*, is present at the end of the pipe at *V*. If, however, *V* be opened and water begins to flow, the level in the tubes will sink, the diminution in elevation increasing from tube to tube, taken from the reservoir outward. First, there is a drop at the reservoir equal to the entry head + velocity head $= (1 + 0.5)\frac{V^2}{2g}$. The decrease in elevation from tube to tube is equal to the head taken up in forcing the water through the length of pipe included between the vertical tubes. For instance, the drop in height between *a* and *b* is equal to the head required to force the water through the pipe from tube *a* to tube *b*. The total drop in head is the difference in elevation between *AB* and the height of water in tube *e*, which is h_f for the whole pipe. The net, working head is $H - h_f = H_1$ as shown.

If a straight line be drawn from a point at the edge of the reservoir where the pipe enters, which point is $(1 + 0.5)\frac{V^2}{2g}$ below the surface of the water in the reservoir or stream, to the point at the end of the pipe, which is at an elevation equal to the net head at that point, this line will be the *hydraulic gradient*. Also, it will, necessarily, pass through the several water levels in the vertical tubes. This assumes a pipe in which the cross-section, and character of surface are constant.

Where there are variations in diameter, or pipe surface, the hydraulic gradient will be a broken line and is found by taking the lengths of pipe in which the conditions are constant and calculating the drop in head for each section. This gives the decrease in elevation of the gradient line for each section. Beginning at the reservoir level and working out to the end of the pipe, and making due allowance for change in pressure head with changes in velocity, the gradient is easily found.

As an example, illustrating the changes in water pressures inside pipes with changes in velocity, and, hence, the variation

in the hydraulic gradient, take the conditions shown in Fig. 56, the head, lengths and sizes of pipe all being given.

Computing the discharge and the velocities by the methods and formulæ before given for loss of head, it is found that the discharge, Q, is 81.94 cu. ft. per second. Note that in computing the loss due to velocity head for determination of the total head lost, take the velocity through the discharge end of the system—not that at the entrance. In this case, the velocity head is taken for pipe (3).

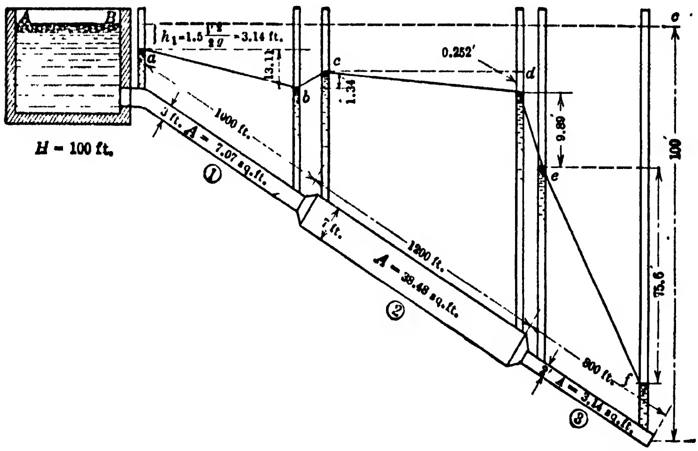


FIG. 56.—Hydraulic gradient for series of pipes of different sizes.

The areas of the several sections of the system are:

Pipe (1), = 7.07 sq. ft.
Pipe (2), = 38.48 sq. ft.
Pipe (3), = 3.14 sq. ft.

The corresponding velocities for a discharge of 81.94 cu. ft. per second are

$$\text{Through pipe (1), } \frac{81.94}{7.07} = 11.59 \text{ ft. per second.}$$

$$\text{Through pipe (2), } \frac{81.94}{38.48} = 2.131 \text{ ft. per second.}$$

$$\text{Through pipe (3), } \frac{81.94}{3.14} = 26.195 \text{ ft. per second.}$$

The drop in head due to entrance into pipe (1) at a velocity of 11.59 ft. sec. is

$$h_e = \frac{(11.59)^2}{64.4} \times 0.5 = 1.0477 \text{ ft.}$$

$$\text{Velocity head} = \frac{(11.59)^2}{64.4} = 2.0954$$

Hence, drop in head at entry to pipe (1) = 3.1431 ft. and the vertical distance from the line of water level in the reservoir to that in tube *a* is 3.1413 ft.

The drop in pipe (1) is $h = \frac{(11.59)^{1.853}}{3^{1.166}} \times 0.0005 \times 1000 =$ 13.110; which represents the loss in head from tube *a* to tube *b*. Also, the total loss in head at tube *b* is the sum of the three drops up to that point = 3.143 + 13.110 = 16.253 ft. From pipe (1) the water passes into pipe (2) of much larger cross-section, but as it is led in through a taper connection, there is no loss of head due to sudden change in pipe sizes.

The velocity head for the water entering pipe (2) is $\frac{(2.131)^2}{64.4} =$ 0.756 ft.

Velocity head in water emerging from pipe (1) = 2.0954 ft. Hence, change in pressure on entering pipe (2) =

$$2.0954 - 0.756 = 1.3394, \text{ or practically, } 1.34 \text{ ft.}$$

Since the velocity head in pipe (2) is reduced, the change in pressure is added to the pressure head in pipe (2), so that the head drop at the entrance of pipe (2) is

$$16.253 - 1.34 = 14.893 \text{ ft.}$$

The drop through pipe (2) is $0.0005 \times 1200 \times \frac{(2.131)^{1.853}}{D^{1.166}} =$ 0.252 ft. This makes the total decrease in head at the lower end of pipe (2) = 14.893 + 0.252 = 15.145 ft.

The velocity head at entrance of pipe (3) is $\frac{26.195}{64.4} = 10.655$ ft.

Velocity head in water on emerging from pipe (2) is 0.756 ft.

Additional velocity head required for entry of water into pipe (3) is 10.655 − 0.756 = 9.899 ft.

Hence, total drop in head to tube *e* is 15.145 + 9.899 = 25.044 ft.

Drop in head through pipe (3) is

$$0.0005 \times 800 \times \frac{(26.195)^{1.853}}{2^{1.166}} = 75.6 \text{ ft.}$$

Total drop in head = 25.044 + 75.6 = 100.644 ft., which is within 0.7 per cent. of the given head of 100 ft.

The water discharges at the velocity it had in the last section of pipe through which it passed. Hence, in this case, it emerges at a velocity of 26.195 ft. per second, corresponding to a head of 10.655 ft., which is the pressure at the lower end of the pipe.

The series of broken lines passing through the water surfaces in each of the vertical tubes is the hydraulic gradient for this series of pipes. Obviously, the hydraulic gradient is the graphical representation of the internal pressures in the pipes at any point.

It is important to observe that under no condition is it possible to abstract the total head within the limit of the pipe itself. Whatever the velocity within the last section of pipe through which it passes, the water emerges from the end of the pipe carrying in it that same velocity and the equivalent head.

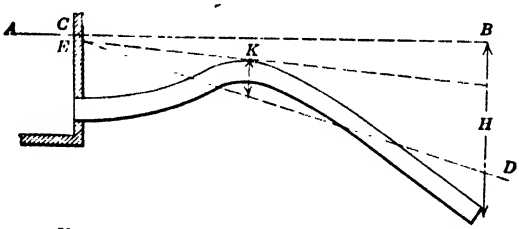


FIG. 57.—Pipe line above hydraulic gradient.

Obviously, the greater the area of the last section of pipe, the less will be the head lost in velocity of emergence. This is the reason why draft tubes are "flared," or tapered, so that the cross-section at the lower end is considerably greater than at the throat, where the tube joins the draft chest.

The pipe can not lie above the hydraulic gradient without a considerable loss of head or velocity, except in the case of the siphon, to be noted later.

In Fig. 57, consider ED as the hydraulic gradient from E to D which would obtain for a straight pipe. If the pipe is laid so that it rises above the normal hydraulic gradient, as indicated at K, the conditions now become changed. The difference in elevation between AB and K, fixes a new hydraulic gradient for this section of pipe, and the flow is proportional to this head. The total head from E to D does not influence the rate of flow,

except in the case of a siphon. That portion of the pipe from K to D acts merely as a channel conveying the water away from K, just as an open flume would. This condition, however, does not occur if the vertical distance from the top of K down to the gradient line is less than 25 ft., and means are provided to abstract the air from the upward bend as it accumulates. In this latter case, the bend forms a siphon, and its presence does not change the position or inclination of the gradient.

An air pump or ejector connected into the top of the bend is necessary to remove the air which, entrained in the water, is released in the vertex of the bend and destroys the vacuum which is required for siphon action.

Pipe Lines.—In determining the size of a pipe line or penstock, the character of load, the kind of water wheels which are to be supplied and whether the penstocks deliver water directly to the wheels or terminate in a settling basin, are all factors requiring consideration. Where the load on water wheels fluctuates greatly, it is best to choose a size of pipe that will give a good average efficiency which diminishes at the time of the load peak. The peak usually lasts but a short while and it is not wise to make an investment in a penstock on which interest and depreciation continue for 24 hr. in the day, to obtain a high efficiency for 1½ or 2 hr. per day.

With scroll-case water wheels and jet impulse wheels, the energy in the water due to its velocity is made use of in the production of power, which is not the case with ordinary reaction turbines in cylindrical casings. Therefore, for the latter kind of wheel, a high penstock velocity produces a loss of energy, which the first two types of wheels do not experience. Also, the greater the velocity of the water in the penstocks, the greater are the difficulties of speed control, as is pointed out elsewhere in this work. In any event, the greater the speed of the water, the greater is the loss of head, as has been shown.

Stresses in Pipes.—The forces acting to distort or rupture pipes are:

1. Bursting, due to internal pressure.
2. Collapse, due to formation of a partial vacuum in the pipe and consequent unbalanced external pressure.
3. Rupture, due to sliding on hillsides.
4. Breaking, due to flexure stresses when the pipe is carried on supports spaced along the line.

5. **Rupture, due to expansion and contraction with temperature changes.**

Bursting of Pipes.—The pressure tending to burst a pipe is

$P_b = pD$ lb. per foot length, P_b being the total internal pressure.
p = internal pressure, per square foot, $= 62.5H$.
D = diameter of pipe, in feet.

This internal pressure is resisted by the walls of the pipe, or, in the case of wood-stave and reinforced-concrete pipes, by the steel hoops.

Since there are two walls, or two cross-sections of each hoop for a section through the pipe, each side of the pipe must resist one-half the pressure.

$$\text{Tension in each wall} = \frac{pD}{2} \text{ lb. per foot length.}$$

Each wall must, therefore, have a cross-section of steel to resist this pressure, with the factor of safety adopted.

For rivetted steel pipe, the strength of rivetted joints must be taken into consideration. This, for single-rivetted, longitudinal joints, is about 70 per cent. of the strength of the plate, and for double-rivetted, lap joints, 85 per cent. Calling the percentage of joint strength ϕ, then, for steel penstocks,

$$\frac{pD}{2} = 12tS\phi \tag{99}$$

and

$$t = \frac{pD}{24S\phi} \tag{100}$$

D = diameter of pipe in feet.
t = thickness of metal in *inches*.
S = allowable stress in metal in lb. per square inch.

A good, safe value for S is 15,000, provided the pipe is never subjected to abnormal heads or shock from water hammer.

As an example, consider a 9-ft. diameter penstock with single-rivetted joints and subjected to a 100-ft. head.

$p = 100 \times 62.5 = 6250$ lb. per square foot.
$\phi = 0.70$.
$\frac{pD}{2} = \frac{6250 \times 9}{2} = 28{,}125$ lb. per foot length.
$$t = \frac{6250 \times 9}{24 \times 15{,}000 \times 0.70} = 0.223 \text{ in.}$$

This is, of course, too thin for durability and mechanical requirements. A 9-ft. pipe should have a thickness of not less than $\frac{5}{16}$ in. which is, practically, 40 per cent. greater than the thickness required for resistance to bursting.

With heads less than 150 ft., the thickness of metal required for strength is always less than that necessary for durability, and the arbitrary values of $\frac{1}{4}$ in. for pipes up to 5 ft. in diameter and $\frac{7}{16}$ to $\frac{3}{8}$ in. for larger size pipes are taken as the minimum thicknesses of the plates.

Unless provision is made to prevent shocks due to water hammer, penstocks are frequently subjected to stresses, which are greatly in excess of the normal stresses due to the static head, as is more specifically set forth in Chap. XI. The usual methods of providing against this are three in number, viz., relief valves, surge tanks, and air chambers.

Relief valves are of two kinds, namely, the plain pressure type and the mechanically operated kind. The pressure valve is simply a large valve made in the same fashion as the pop safety valve for steam boilers. Generally, a number of these valves are required in order to obtain a sufficiently great area of opening to give actual relief in case of surges in the penstock. As a rough approximation, the area of relief valves should be not less than one-third the area of the penstock cross-section, and more than this, under certain conditions discussed in Chap. XI.

The objection to valves of this kind is that they occasionally "stick" and do not open, and their manner of quick, spring-closing produces surges in the penstock pressure. They are, however, far better than no relief valve at all.

Mechanically operated relief valves are of two kinds. One form is actuated by a mechanism which operates the valve when the pressure in the penstock rises above normal, while the other kind is moved by the water-wheel governor. One of the varieties of the former type of valve is made as indicated in Fig. 58. It consists of a main relief valve which is arranged to close under pressure. The valve itself is a hollow cylinder, closed at one end by a conically shaped cap. This is free to move in a horizontal direction, and, in the position shown in the drawing, is closed. As it slides toward the left, there is a clear opening from the flange marked "Connection to penstock," down to the flange marked "Discharge to draft tube." The diameter of the annular-shaped valve seat is less than the total

diameter of the moving valve. Therefore, when the unit pressures on the two sides of the valve are equal, the total pressures will be unequal, the greater pressure being in the direction to move the valve to closed position. A small pipe,

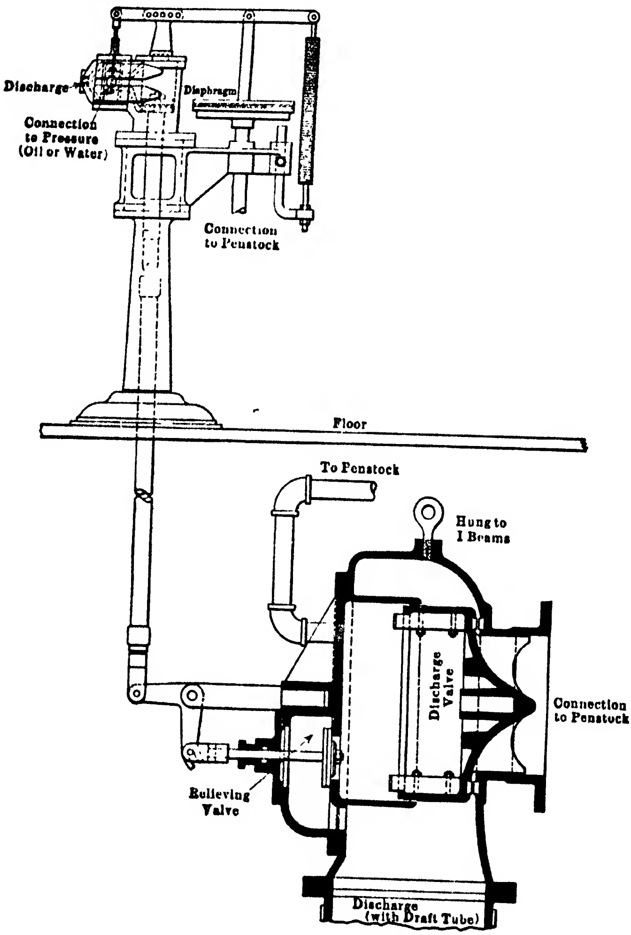


FIG. 58.—Pressure relief valve.

connecting with the penstock, leads to the left-hand side of the casing and keeps the inside pressure equal to the pressure on the front face of the valve which is, thereby, maintained in a closed position. An auxiliary valve, called the relieving valve,

the area of which is greater than the small pressure pipe leading to the penstock, is placed in the left-hand side of the casing, and when this valve is open, the internal pressure is relieved by the water discharging through it and passing into the discharge pipe. Therefore, when the relieving valve is open, the interior pressure falls, the pressure against the face of the valve becomes the greater, and the valve slides toward the left, giving a clear passage from the penstock to the discharge.

The relieving valve has its position fixed by a device which is responsive to pressure changes, and which is shown in the figure mounted on the floor above the valve. This is simply a diaphragm which has a pipe connection to the penstock, and the penstock pressure tends to move the diaphragm and, with it, the lever attached to the metal joint on it. This tendency to move is opposed by a tension spring which is also attached to the lever and tends to move it in a direction opposite to that in which the diaphragm tends to move. The area of the diaphragm and the tension of the spring are so adjusted that the force of the spring is greater than that of the diaphragm, under normal conditions of penstock pressure, and the lever is held in its downward position. The relieving valve is connected through a valve rod, bell crank and vertical operating rod, to a small piston working in a cylinder located at the upper end of the pedestal of the pressure control device. This piston is normally held downward by fluid pressure, either by oil from the pressure chamber of the water-wheel governor, or water pressure direct from the penstock. If the pressure on this control cylinder be released, it is obvious that the relieving valve will open, due to the internal pressure in the casing of the main valve. Upward movement of the pressure-controlled diaphragm operates a small relay valve which admits the liquid under pressure to the small operating cylinder, or allows it to discharge from the cylinder. Upward movement of this relay valve, caused by increase in penstock pressure, closes the supply port and opens the discharge port of the operating cylinder, thereby relieving the pressure in it. The relieving valve opens, the main valve then follows and the excess pressure in the penstock is brought down to normal. The converse of these operations then follows. When the relieving valve is closed, the main valve can not close immediately and produce shock, because of the velocity of the water moving through the opening.

Therefore, the movement to close is gradual. The rapidity of operation of these valves is considerably greater than would be supposed from this description, and they are reasonably satisfactory for the purpose they are intended to serve.

The best form of mechanically operated relief valve is one in which the water-wheel governor moves the valve to open it when the main turbine gates are moved toward closing position. As long as the movement of the main gates continues, opening of the relief valve also continues. When the governor comes to rest and the main water-wheel gates have been moved to their final position to accord with the change in load, the relief valve begins to return to its normal closed position, but it moves slowly, back to the point from which it was quickly moved by the governor, and the time element of the device is such that the column of moving water in the pipe is retarded so gradually that no appreciable rise in pressure occurs. This device is described in the discussion of "Speed Regulation of Water Wheels," Chap. XI.

Surge tanks are elevated tanks connected to the penstock by vertical pipes, the height of the top of the tank being greater than the elevation of the water in the reservoir from which the penstock draws its supply. Sudden increases of pressure in the penstock elevate the column of water in the tank while sudden decreases in pressure are compensated for by a momentary supply from the tank.

Since the height of the tank is somewhat greater than the total head on the power plant, such tanks are not well adapted for installations where the head is in excess of 200 ft.

Air chambers, as the name indicates, are large air-tight vessels which are directly connected with the penstock and are kept full of air which, normally, is under a pressure equal to the static head. The elasticity of the air compensates for variations in pressure in the penstock. They are not satisfactory in practice. It requires such large chambers to provide any reasonable degree of compensation for water hammer, that other means are cheaper and preferable.

Collapse of Pipes.—Where the profile of a pipe line is such that a long and steep incline occurs near the power station, and there is a considerable length of pipe, say 80 ft., or more, extending back beyond the bend which the pipe makes in turning downward, there is danger of collapse of the penstock,

unless it be designed to resist external pressure. This arises from the fact that a rapid opening of the water-wheel gates will allow that portion of the water column in the inclined portion of the penstock to accelerate rapidly, and this will cause a separation of the column of water at the bend, because the mass of water in the horizontal, or slightly inclined, portion can not accelerate as quickly as that in the steeply inclined section. Separation of a column of water means that a vacuum will be formed in the pipe, and the external air pressure may cause its collapse. Therefore, the strength of the penstock must be such as to resist this external pressure, or means must be provided to prevent its occurrence.

Collapse may be prevented by *air valves, or vent pipes,* placed at, or near, the bend where the penstock takes its steep, downward inclination. These air valves are simply a special form of spring closing check valve that opens inwardly and which is held in place by the springs and the pressure of the water.

Vent pipes are vertical pipes which are connected to the penstock and are open at the top, being, in effect, a form of very small surge tank. These must be sufficiently high to have their upper ends 4 to 10 ft. above the elevation of the reservoir from which the penstock receives its supply. Vent pipes are, therefore, not well suited to the conditions where the head on the penstock at the bend E is over 60 ft. (See Fig. 230.)

There is no rational formula for the computation of strength of a pipe to prevent collapse, and the conditions make it practically impossible to obtain any but an empirical formula.

No large, thin-walled metal tube is truly circular in cross-section when installed, but varies more or less from true roundness. Even if it were exactly circular when constructed, its own weight, and that of the contained water would cause it to flatten slightly and take an elliptical form, the vertical diameter being shorter than the horizontal diameter. This distortion makes the pipe easier to collapse, but without a definite known ratio between the major and minor axes of the flattened circle, no rational mathematical formula is possible.

Of empirical formulæ there are very few, and no experiments been conducted on pipes of greater diameter than 12 in., up to this time (1916).

The formula which appears most nearly to approach the facts is that of Carman, Carr and Stewart, which is

$$p_2 - p_1 = 50{,}200{,}000 \times \left(\frac{t}{d}\right)^3 \qquad (101)$$

p_1 = internal pressure in lb. per square inch.
p_2 = external pressure in lb. per square inch.
t = thickness of pipe wall in inches.
d = diameter of pipe in inches.

Based on this formula, the following table has been computed.[1]

TABLE 28.—PIPES OF VARIOUS THICKNESSES AND DIAMETERS WHICH WILL FAIL UNDER THE GIVEN PRESSURE DIFFERENCE

$p_2 - p_1$ lb. per sq in.	$\frac{t}{d}$	Least diameter of pipe which will collapse		
		$t = \frac{1}{4}$	$t = \frac{5}{16}$	$t = \frac{3}{8}$
14.7	0 0066	38 0	47.5	57.0
10.0	0.0059	47 5	53.0	64.0
9.0	0.0056	44.5	56.0	67.5
8.0	0 0054	46.5	58.0	70.0
7.0	0 0051	48.5	61.3	73.0
6.0	0.0049	51.5	64.0	77.0
5.0	0.0046	54.5	68 0	82.0
4.0	0.0043	59.0	73.0	88.0
3.0	0 0039	64.5	80.0	97.0
2 0	0.0034	74 0	92.0	111.0
1.0	0.0027	93.0	116.0	139.0
0 5	0 0021	117.0	149.0	176.0

From this table it is evident that appreciable differences between the outside and inside pressures must be avoided.

Since the change of pressure inside the pipe depends on the rate of opening the water-wheel gates, this subject is further discussed in Chap. XI on "Speed Regulation of Water Wheels."

Lock-bar Pipe.—Instead of rivetting the longitudinal seams in steel-plate pipe, the lock-bar joint is sometimes used. This joint is made by upsetting the abutting edges of the pipe so that they are thicker than at any other part of its surface, and these upset edges fit into grooves made on the two opposite edges of a longitudinal strip of steel which has a thickness equal to about

[1] Enger & Seeley, *Eng. Record*, vol. 69, No. 21.

2½ times the thickness of the plate. This longitudinal connecting strip with the edges of the plate inserted in the grooves, is squeezed together under heavy pressure, so that the metal of the holding strip is pressed firmly against the edges of the plate, thereby forming a strong mechanical and water-tight, longitudinal joint. Fig. 59 shows a cross-section of this joint. It is claimed that the strength of lock-bar joints is greater than that of ordinary rivetted joints, and the working strength may be taken at 90 per cent. of the strength of the solid plates.

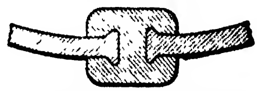


Fig. 59.—Joint of lock-bar pipe.

Welded Joints.—For high heads, where the thickness of the pipe becomes considerable, that is, 1 in. or greater, it is frequently found more economical to weld the longitudinal seams instead of rivetting them, and in high-head developments, welded pipe is frequently used. The connections between various sections of welded pipe are usually made with heavy flanges which are also welded to the ends of the pipe sections. Fig. 60 shows a form of flange connection which has been successfully used.

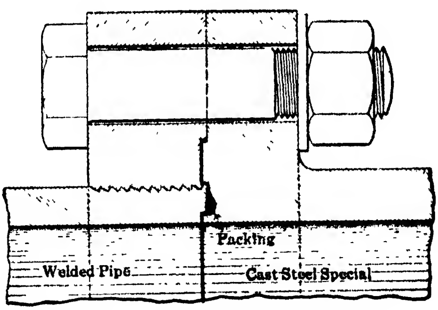


Fig. 60.—Joint used for penstocks.

Penstock Supports.—It often becomes necessary to support penstocks at an elevation above the ground surface, in which case, saddles made up as indicated in Fig. 61 are generally used. They may be of steel if the cost is less than that of concrete. Steel supports are made up of angles and gusset plates. These may rest on steel columns or concrete piers, as conditions may dictate.

To determine the proper distances apart of these supports, the penstock is treated as a continuous girder, except at the two

ends, where it is considered as a beam supported at one end and cantilevered at the other end.

When a penstock is supported at intervals, and bridges across from one support to the next, the stresses set up in the material of the pipe are those due to the tensile stress on the under side and the compressive stress on the upper side of the pipe produced by the flexure, due to the weight of penstock and the contained water therein.

The formula for allowable length between supports, where the pipe acts as a continuous girder, is

$$l = \sqrt{\frac{MS}{W}} \qquad (102)$$

At the ends of the pipe, or wherever expansion joints are placed, the pipe does not act as a continuous girder and, therefore, the

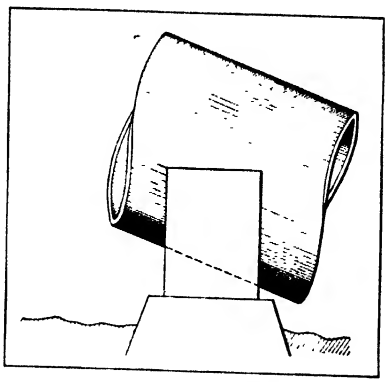


FIG. 61.—Concrete penstock cradle.

allowable span will be less. For these positions, the length of span will be

$$l = \sqrt{\frac{MS}{1.2W}} \qquad (103)$$

S = Maximum fibre stress in lb. per square in.
l = length between supports, in feet.
M = section modulus = $\dfrac{\text{moment of inertia of section}}{\text{radius of pipe, in inches}}$.
W = weight of pipe and water contained in it, per foot length.

The moment of inertia for tubes, or any hollow cylinder, about a diametrical axis perpendicular to its length is

$$I = \frac{\pi(D_1^4 - D_2^4)}{64} \tag{104}$$

and the section modulus is

$$M = \frac{\pi(D_1^4 - D_2^4)}{32D_1} \tag{105}$$

in which D_1 = outer diameter, in inches and
D_2 = inner diameter, in inches.

Formulæ 102 and 103 are derived as follows:

The bending moment for continuous beams is $\frac{Wl^2}{12}$ pound-feet = Wl^2 pound-inches, in which W = weight in pounds per foot length, and l = length of span in feet. If S = the maximum fiber stress in pounds, per square inch, and M is the section modulus,

$$Wl^2 = MS \tag{106}$$

$$S = \frac{Wl^2}{M} \tag{107}$$

$$l = \sqrt{\frac{MS}{W}} \tag{102}$$

The bending moment for beams of which one end is simply supported and the other end is a portion of a continuous girder, is

$$\frac{Wl^2}{10} \text{ lb.-ft.} = 1.2Wl^2 \text{ lb.-in.}$$

whence

$$l = \sqrt{\frac{MS}{1.2W}} \tag{103}$$

from which it is clear that for the same conditions of loading and unit stress in the material of the beam, the length between spans is $1 - \frac{1}{\sqrt{1.2}}$, or 10 per cent. less than for a continuous girder.

As an example of the use of this formula, consider the following conditions:

Diameter of penstock............	7 ft.
Thickness of plate	½ in.
Head on penstock.	100 ft.
Allowable stress in steel........	10,000 lb. per square inch

The actual working stress is taken at 60 per cent. of the allowable stress in the steel, to compensate for the weakness of the rivetted girth joints, the strength of which is about 60 per cent. of that of the steel plate. The spacing between the supports, at the ends of the penstock and the next adjacent ones, should be only 90 per cent. of the spacing as found from formula 103.

Then S = 60 per cent. of 10,000 = 6000 lb. per square inch.

$$M = \frac{\pi(85^4 - 84^4)}{32 \times 85} = 2788.$$

$W = W_1 + W_2$, in which W_1 = weight of water, per foot length, and W_2 = weight of penstock, per foot length.

Area of penstock cross-section $= \frac{\pi \times (7)^2}{4} = 38.5$ sq. ft.

Weight of water, per foot length, $= 62.5 \times 38.5 = 2406$ lb. $W_2 = \pi Dt \times 43.5$, in which, 43.5 is the weight of 1 sq. ft. of steel plate, 1 in. in thickness, including lap and rivets, and t = thickness of steel plate in inches.

W_2, for this case, $= \pi \times 7 \times 0.5 \times 43.5 = 478.5$, or practically, 478 lb. per foot.

$$W = W_1 + W_2 = 2884 \text{ lb.}$$

$$l = \sqrt{\frac{6000 \times 2788}{2884}} = 76.2 \text{ ft.}$$

This value is for distance between supports over which the penstock acts as a continuous beam. At the ends the distance between supports is 90 per cent. of this amount, or 68.6 ft.

It is the opinion of the author that if supports for penstocks are placed as far apart as these formulæ indicate, the total reaction at the supports will be too great and, possibly, will cause the pipe to collapse. In view of the lack of definite information concerning the resistance of pipes to collapse, this statement can be taken as an opinion only. Based on it, however, the actual span should never exceed 60 per cent. of that which the pipe will safely stand when considered as a girder. If conditions of crossing ravines or streams should require that the span be as great as the pipe will safely stand, heavy angle-iron reinforcing rings should be rivetted around the circumference of the pipe at the points of support, in order to prevent possible deformation.

Also, the pipe should be given an upward camber of approximately 4 per cent. of the span.

Anchoring.—Penstocks must be firmly anchored on hillsides, and whether horizontal or inclined, they must be anchored at the end where they connect with the turbine cases; otherwise, there is danger of the expansion and contraction of the penstock pulling the turbine out of line and thereby making the units inoperative. Actual instances of this kind have come before the author's notice.

The method of anchoring on hillsides varies, but one of the best is to rivet two angle-iron rings around the outside of the pipe at the point of anchorage, the section of which angles should be from 3 by 3 to 4½ by 4½, depending on the size of pipe,

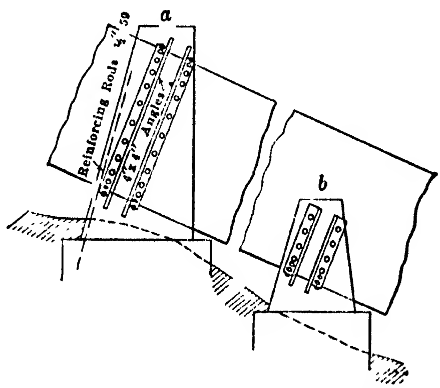


Fig. 62.—Concrete anchoring cradles.

the angle it makes with the horizontal, and the distance between successive anchorages. The pipe is then temporarily supported and a mass of concrete, resting on a good foundation, is cast around the pipe and the rings. This concrete saddle should extend upward a distance of from 8 to 12 in. above top of pipe, should project about an equal amount on either side and extend, axially, along the pipe a distance of from 2 to 3 ft. Where the profile of the pipe line is such that frequent anchorages are necessary, the two angle-iron rings may extend only halfway around the pipe, and the concrete saddle cast to a height equal to one-half the pipe diameter and not completely surrounding the pipe. These forms of anchorage are depicted in Fig. 62.

Another method of anchoring is by means of steel bars which are connected to the penstock by lugs on heavy plates, which

plates are rivetted to the penstock, one being placed on each side. The bars pass through the lugs and are held by nuts on the end of the bar which is threaded to receive them. The anchor end of the bars is fastened by a pin joint to an iron anchor, which latter is a vertically set iron bar, sunk into a drill hole in the rock and firmly grouted in. This anchor carries the mating half of the pin joint that works with that on the end of the bar connected to the lug, as shown in Fig. 63.

This form of anchorage is particularly suitable for remote districts where the transportation of concrete materials is difficult and expensive.

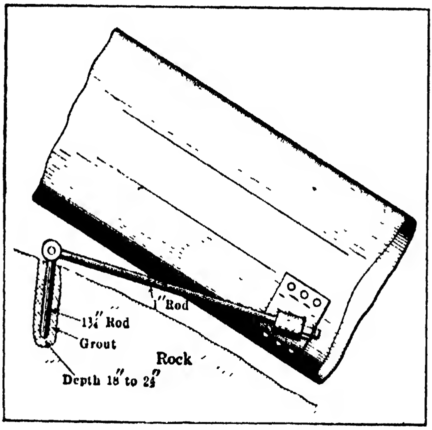


Fig. 63.—Steel rod anchor.

Expansion Joints.—In the case of long penstocks, that is where the length exceeds 150 ft., expansion joints should be provided to compensate for the changes in length due to the changes in temperature. There are many forms of expansion joints, but the simplest and best for low and moderate heads is made up as indicated in Fig. 64.

As shown, this is made up of a circular ring that is formed of a channel, or of a 2 by 2 in. square iron bar, and on either side of which is rivetted a flat disk of steel plate. These disks should be as thin as the head to which they are subjected, will allow. Centrally located holes are cut in the disks and the holes flanged, the flanges being formed either directly from the metal of the disk itself, or made by rivetting angle rings around the edge of the holes.

These flanges form a connection for the penstock ends, which are fastened thereto. When the length of the penstock changes, due to change in temperature, the disks simply buckle in or out like the bottom of an oil can. The inside diameter of the ring should be from 32 to 48 in. greater in diameter than that of the penstocks, the smaller figure being suitable for pipes up to 5 ft., and the larger one for pipes up to 10 ft. in diameter; and in between these sizes the diameter of the expansion joint ring varies proportionately with the penstock diameter. The number of expansion joints necessary in a line of steel pipe depends on the

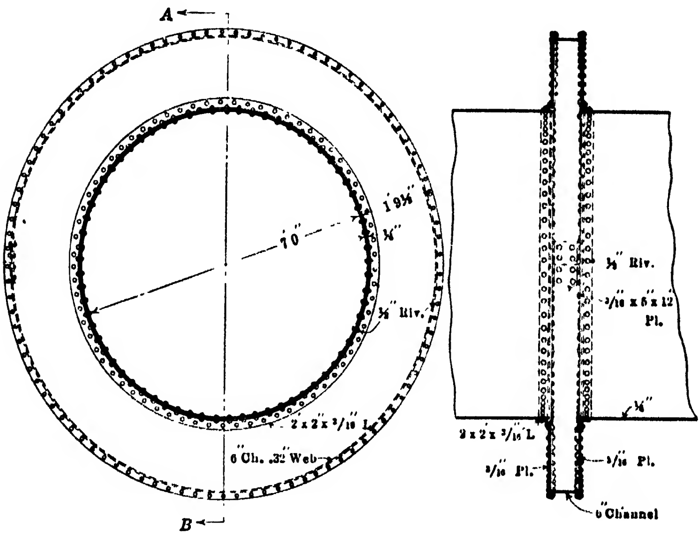


FIG. 64.—Expansion joint.

variations in the temperature that the pipe will be subjected to and, also, on the path of the pipe. If the pipe is straight, all the expansion must be compensated for. If the pipe line is crooked, the bends in it will partially, or wholly, take care of the changes in length. No rules can be given for the placing of expansion joints in curved pipe lines. Only experience and judgment can be relied on in such cases. For straight pipe lines, allow a coefficient of longitudinal expansion of 0.0000068 per degree F. Thus, a pipe 1000 ft. long, and subjected to a temperature range of 60°, changes its length.

$1,000 \times 60 \times 0.0000068 = 0.0408$ ft. $= 0.4896$ in. or, practically, ½ in.

An expansion joint, such as shown in Fig. 64, will allow a movement of each side diaphragm of from ⅜ to ⅝ in., depending on the size of the diaphragms. Hence, the total movement of the two diaphragms is from ¾ to 1¼ in. One expansion joint in the middle of a 1,000-ft. pipe line would be ample to take up all changes in length that might be anticipated if the pipe could slide easily in its supports, or on the ground. Owing to the resistance to motion, the expansion joints should be spaced not farther apart than 350 ft.

Shipping and Erection.—Penstock sections, up to 60 in. in diameter, can be shop-rivetted, shipped and handled in lengths up to 30 ft., and the field rivetting required is only that of the circumferential joints at 30-ft. intervals. Above 60 in., and up to 96 in., the sections are best handled in 10-ft. lengths loaded on a car, on end. Sizes larger than this are usually shipped from the works in the form of bent plates with rivet holes punched, and the entire rivetting is done in the field.

An air compressor with tank should be provided in every case, and air hammers used for the field work. If the rivetting is done by hand hammers, the cost will greatly exceed that done with air hammers.

Painting.—After erection, the metal should be well cleaned and painted. Some engineers recommend the use of an air blast for cleaning, but this is an unnecessary and expensive process. The first coating should be of boiled linseed oil, after which two coats of some good protective paint should be applied. This paint may be any standard metallic paint or some of the trade preparations of liquid cement. If the penstock is kept always well painted, and the water passing through it carries no sand or abrasives, its life will be much longer than that which sheet steel usually has—say from 40 to 50 years of service for a ⅜-in. thickness of metal.

Drains.—Penstocks must be arranged to drain all of the water out of them. Usually, no special provision has to be made for this purpose, as the pipe generally slopes continuously down from the forebay to the water wheels. In some cases, however, where long pipe lines follow the contour of the ground, there will be vertical bends which form low points in the line, and these will not drain, completely, to the discharge end of the

pipe. In such cases, drain pipes with valves, should be placed in the lowest points of the pipe.

Costs.—The cost of penstocks varies, of course, with the market price of steel plate, but the following are fair average values covering the shop-rivetted sections with all necessary field rivets delivered on the ground (1912–1913).

Three and one-half to 4¼ cts. per pound, for long straight runs, and 4½ to 6 cts. per pound, for bends and expansion joints, where the tonnage is considerable and the distance of transportation from the shop to the water power site is short.

Where the distance of transportation is long, and there are difficulties of distribution of the pipe sections along the line that the pipe is to follow, the cost will run from 4 to 5 cts. per pound, for straight runs, and from 5½ to 7 cts. per pound, for elbows and expansion joints.

Pipe Branches.—When branches are to be taken from the pipe line they should be made with their axes at an angle of from 45° to 60° with the axis of the main supply pipe. There is a considerable loss in head when abrupt changes in the direction of flow of the water occur, as when branches are taken off with their axes at right angles to the axis of the main supply pipe. There is an appreciable saving in pressure head effected by making the branches at an acute angle with the axis of the penstock. The size of the penstock should decrease after passing each branch, the amount of decrease in the area being equal to the area of the branch pipe.

Reinforced-concrete Penstocks.—Pipes of concrete, reinforced by steel hoops to resist the internal pressure, are now (1916), coming into commercial use. Such pipes possess certain advantages over steel or wood pipes, the principal ones being that the coefficient of frictional resistance to flow is less than for steel pipes; they are permanent and require no expense for maintenance or repairs if properly constructed; the materials can be easily transported in unit quantities of any desired size and weight, which makes the distribution of materials possible without derricks, pulleys or tackle, even where the country is mountainous and the handling of heavy weights difficult; the cost is usually less than that of equivalent steel pipes, and not greatly in excess of the cost of wood-stave pipe. Unless carefully made, thoroughly water-proofed, and strongly reinforced with steel, concrete pipes, under pressure, will leak and crack.

The materials are sand and cement with the addition of gravel or broken stone. The proper proportions of mixture and methods of determining them are as given in Chap. VII. Hydrated lime should be added to the concrete to make it waterproof, or some standard waterproofing mixture should be used. The amount of hydrated lime to be added is from 8 to 10 per cent. of the amount of cement used, the proportion being by weight.

In computing the strength and spacing of the reinforcing hoops, it should be remembered that the stress allowed in the steel should be low—not above 12,000 lb. per square inch, for twisted bars, or 10,000 lb. per square inch, for untwisted bars. The reason for this lies in the fact that while the steel is amply strong to resist much greater stresses, the concrete will crack if the steel undergoes any appreciable extension, even within its elastic limit. For pipes subject to light pressures—up to 20-ft. head—wire mesh makes the most satisfactory reinforcement, and for pipes to carry greater pressures, wire mesh should also be used, the necessary amount of reinforcement being added. The mesh prevents the formation of cracks and considerably improves the whole construction.

Thickness of concrete should be

Head	Thickness
0 to 10 ft.	5 in.
10 to 20 ft.	6 in.
20 to 40 ft.	8 in.
40 to 60 ft.	10 in.
60 to 80 ft.	14 in.
80 to 100 ft.	18 in.
100 to 150 ft.	20 to 24 in.

Formula for cross-section of reinforcing hoops is

$$62.5HD = \frac{2SA}{d} \tag{108}$$

$$A = \frac{62.5HDd}{2S} \tag{109}$$

For $S = 10{,}000$ lb. per square inch, $A = 0.003125HDd$ (110) in which

A = cross-sectional area of hoop, in square inches.
H = head on section of pipe, in feet.
D = inside diameter of pipe, in feet.
d = distance apart of hoops, in feet.

As an example, consider the design of a reinforced-concrete penstock, 8 ft. in diameter, under 18-ft. head at its upper end and 125-ft. head at its lower end:

Thickness of concrete under 18-ft. head = 6 in.

Take S = 10,000 lb. per square inch.

$A = 0.003125 \times 18 \times 8 = 0.45$ sq. in. steel, per foot length. If ½-in. round rods are used, having a cross-sectional area of 0.1963 sq. in., the spacing will be $\frac{0.1963}{0.45} = 0.436$ ft., or 5¼ in. For 125-ft. head, the spacing of the hoops should be about 4 in. = 0.333 ft., and the cross-section per hoop becomes

$$0.003125 \times 125 \times 8 \times 0.333 = 1.041 \text{ sq. in.}$$

In this case, a 1-in. square rod would give a sufficient area.

The thickness of concrete, at 125-ft. head, should be not less than 20 in. The thickness of concrete is to prevent cracking and leakage and is not depended on to give any tensile strength.

The hoops should be placed near the outside of the shell—from 4 to 8 in. from the outside surface.

Longitudinal construction bars must also be used which are placed, lengthwise, around the circumference of the pipe. These should be not less than 2 ft. apart, measured around the periphery of the pipe. The longitudinal bars should have the same cross-section as the hoop reinforcement bars, up to ¾-in. square. There is no occasion for making them any larger than this.

Expansion joints must be used on reinforced concrete penstocks. The joints described in Chap. VII are satisfactory for this purpose. Since reinforced-concrete pipe can not slide easily over the ground, or in saddles supporting it, good practice requires expansion joints at frequent intervals—certainly not over 100 ft. apart, and 40 to 50 ft. is a better spacing and less apt to cause girth cracks.

Wood-stave Pipe.—Within the past 10 years a large quantity of wood-stave pipe has been used for water-power installations. This pipe has a number of advantages over steel-plate pipe, which are: its low cost; its ability to act as a heat insulator and prevent freezing inside the pipes; it is not so subject to injury from settling or expansion and contraction due to extremes of temperature; if repairs are necessary, material can be obtained in any locality and repairs made quickly, without special tools or skilled labor; the friction loss in wood pipes is less than in steel, and

this friction loss does not increase with time as is the case with steel-plate pipes; the parts are light and easily transported into territory difficult of access, and, in many cases, this acts to further reduce the cost as compared with that of steel pipe.

The most complete and authoritative treatise on the subject of wood-stave pipe is a professional paper by S. O. Jayne, *Bulletin* No. 155, of the Department of Agriculture. Many of the data herein given have been obtained from this paper.

Continuous wood-stave pipe is built in place, of staves having radial edges and faces milled to form arcs of concentric circles, the inner circle having a radius equal to one-half the nominal diameter of the pipe. The staves are held together by steel bands

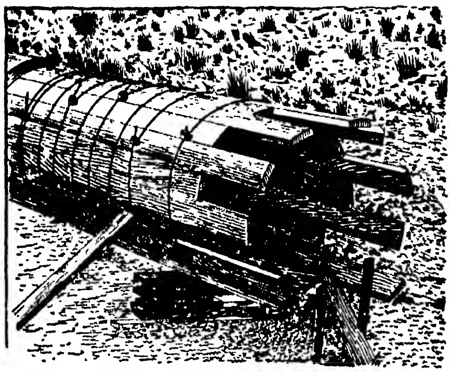


Fig. 65.—Construction of wood-stave pipe.

which encircle them, the ends of the bands being fastened in specially formed metal shoes. One end of the band is threaded and provided with a nut so that the bands may be drawn tight and compress the staves together. The butt joints between adjoining stave ends are made by the insertion of thin steel plates into grooves or saw kerfs, the kerf being cut transversely across the ends of the stave at the midpoint of its thickness.

Figure 65 shows a wood-stave pipe in the process of construction; the separate staves and the steel bands are clearly shown. As indicated, the circumferential joints do not go continuously around the pipe but are broken from stave to stave, making a stepped joint.

Figure 66 shows the details of the shoe, the ends of the steel compression band, and of the butt joints. These details are clear from the figure.

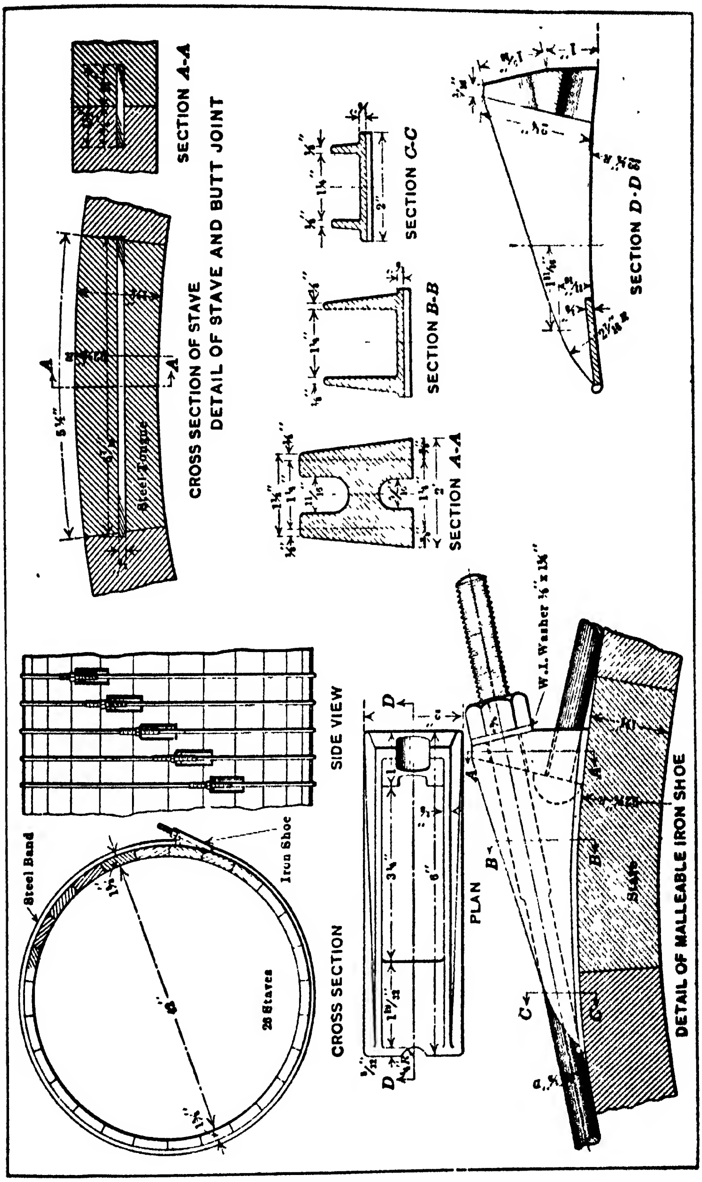


FIG. 66.—Details of 42-in. stave pipe.

Wood-stave pipe will give satisfactory service under heads of from 20 to 200 ft. While it has been made for greater heads, the construction seems hardly adapted to such high internal pressures. With heads below 20 ft., it is probable that the life of the staves will be reduced because the internal pressure will be scarcely great enough to keep the wood thoroughly saturated. Pipes have been made as large as 15 ft. in diameter, and one of the largest ever built, namely that for the North Western Electric

Fig. 67.—Wood-stave pipe line.

Co., of Portland, Ore., is 13 ft. 6 in. in diameter and a mile in length. Fig. 67 shows a portion of this pipe.

Loss of Head.—Figure 68 shows a diagram from which the loss in head, velocity and discharge may be taken directly. It is taken from the data and experiments of E. A. Moritz.[1]

The diagonals marked V represent the velocity, in feet per sec. The other set of parallel diagonals, which are numbered 4 in. to 72 in., represent pipe diameters, in inches.

[1] "Flow of Water in Pipes," *Eng. Record*, Dec. 13, 1913.

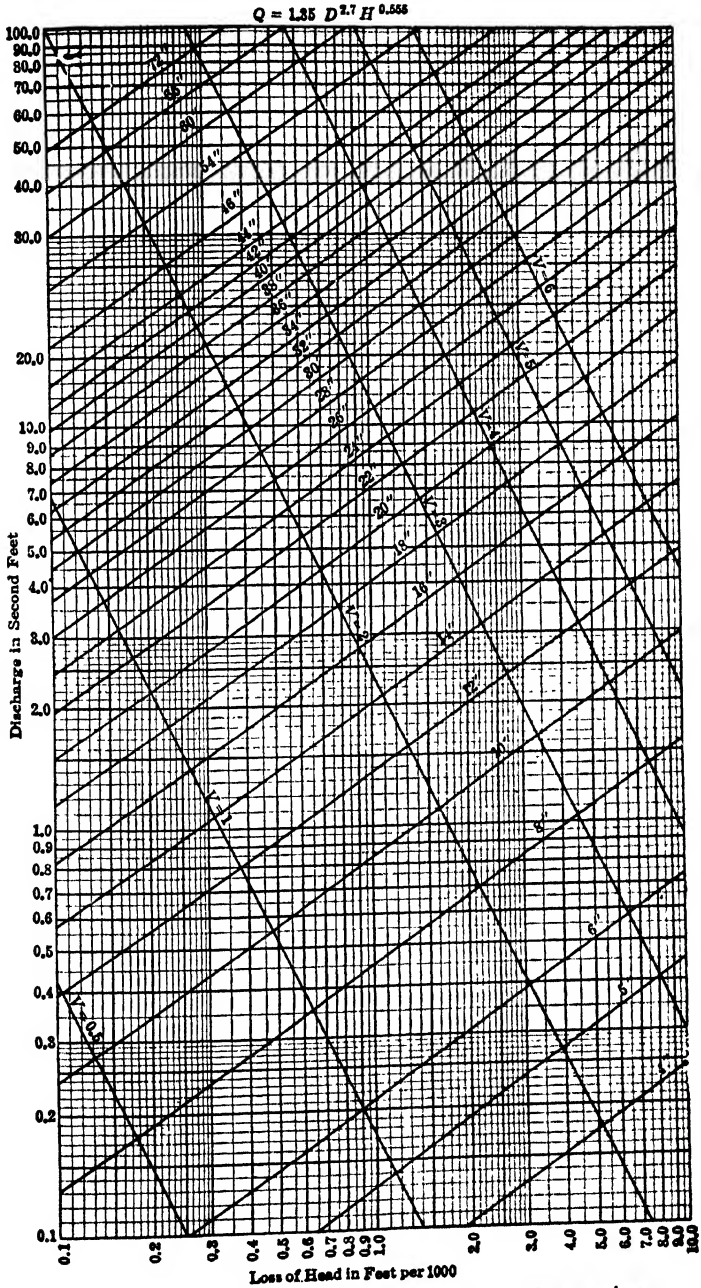


FIG. 68.—Diagram for the flow of water in wood-stave pipes.

To find the discharge and loss of head for any given size of pipe at a specified velocity, take the point where the diagonal of velocity intersects the diagonal representing the pipe diameter. Then, from this intersection, a horizontal line drawn to the left-hand vertical scale, marked "Discharge in second feet," will intersect this scale at a point representing the quantity of flow. Also, a vertical line drawn from the intersection of the two diagonals will cut the bottom horizontal scale at a point which will represent the loss in head.

Where the diameter or velocity have a value which lies between those represented by the diagonals, the desired values must be interpolated.

Staves.—In designing staves, economy dictates that the width and thickness be made such that stock lumber of standard sizes may be used. These are 2 by 4 in., 2 by 6 in., 3 by 6 in., 4 by 6 in., and 4 by 8 in. Staves for most pipes, for ordinary heads, and from 22 in. to 44 in. diameter, are milled from 2 by 6-in. stock, finished 1⅝ in. in net thickness. From 44 in. up to 60 in. of pipe diameter, staves 2 in. thick are commonly used, and in some instances for pipes 72 in. in diameter. For pipes from 5 to 8 ft. in diameter, staves are usually 2½ in. thick. For pipes to withstand extremely high pressure and for those of large size, the thickness of the staves should be increased accordingly, in order to insure safety against crushing or shear of the wood due to the greater tightness of cinching required. The width will be such as to cut with least waste from the stock sizes of lumber.

Western yellow pine, Texas pine, spruce, California redwood, and yellow fir have all been used for staves, but during recent years practically all pipes of this kind have been made either of redwood or fir, the other kinds of wood having proved to be less suitable for the purpose. At the present time, fir is used much more extensively than redwood. It is less durable than redwood when placed in the ground under unfavorable conditions, but in other respects is considered to be just as good or better, and it costs materially less than redwood.

The lumber for pipe should be of extra good quality. The following specifications for fir staves are typical requirements:

"Staves shall be made of live timber, sound, straight-grained, entirely free from all dead wood, rotten knots, dry rot, cracks, shakes, or any other imperfections or defects that might impair their strength or durability. Pitch pockets will be allowed,

provided they do not extend more than ¼ in. in diameter, and not occurring oftener than one in 4 ft. of stave. Sapwood is also allowed on the inside of the stave, provided it does not extend more than halfway through the stave at any point.

"Staves may be from 10 to 30 ft. in length, but not more than 10 per cent. shall be less than 14 ft. in length. Timber must be thoroughly seasoned, either by kiln or air drying, before being milled into staves.

"Another requirement, not common, however, is that staves shall be milled from flat or bastard sawed lumber, those in which the edge grain passes through the stave in a distance less than ½ in. more than the thickness of the stave will be rejected.

"The staves shall be dressed on both sides to true circles, and on the edges to conform to the radial lines of the pipe; all staves shall be of uniform thickness, and each stave of uniform width throughout its entire length; the end of the stave shall be cut square, and shall be fitted with a saw kerf for the insertion of a metal tongue; in depth the saw kerf shall be 1/16 in. less than half the width of the tongue, and its position must be the same in all staves."

Painting.—On exposed portions of new pipes, the United States Reclamation Service has used a paint consisting of 6 lb. of red oxid mixed with 1 gal. of boiled linseed oil. One gallon of the paint was sufficient for two coats on 125 sq. ft. of pipe. On top of the pipe where exposed to the sun, and where water from leaky joints runs down over it, this paint does not last long, much of it being gone in 2 years. Repainting while the pipe is in use, is usually not practicable, because oil paint will not adhere

TABLE 29.—MINIMUM SIZES OF PIPE FOR WHICH SPECIFIED BANDS ARE APPLICABLE

Size of band	S equals ¼ ultimate tensile strength	Band pressure per square inch	E equals band pressure per linear inch	Least external radius of pipe	Band pressure per square inch	E equals band pressure per linear inch	Least external radius of pipe
Inch	Pounds	Pounds	Pounds	Inches	Pounds	Pounds	Inches
⅜	1,650	650	122	13.5	750	140	11.8
7/16	2,250	650	142	15.8	750	164	13.7
½	2,950	650	163	18.1	750	187	15.7
9/16	3,725	650	183	20 4	750	211	17.65
⅝	4,600	650	203	22.6	750	234	19.6
¾	6,600	650	244	27.0	750	281	23.5

TABLE 30.—DIMENSIONS AND WEIGHTS OF STANDARD ONE-PIECE BANDS

[Dimensions in inches and weights in pounds.]

Size of band	Kind: Head	Kind: Nut	Threads: Diameter	Threads: Length	Threads: Per inch	Nuts: Short diameter	Nuts: Thickness	Washers: Diameter	Washers: Gage No.	Washers: Hole	Heads: Width	Heads: Thickness	Approximate length of rod required to form 1 head	Weight per 100 plain round rods 15 ft. long	Weights: Nuts per 100	Weights: Washers per 100	Weights: Heads per 100	Total weight per 100 bands 15 ft. long
3/8	Button	Square....	7/16	4	14	7/8	1/2	1 1/4	14	1/2	3/4	5/16	3/4	578.3	8.0	2.4	2.4	591.1
	Button	Hexagonal													7.0			590.1
	Square	Square....									9/16	9/32	13/16		8.0		2.5	591.2
	Square	Hexagonal													7.0			590.2
7/16	Button	Square....	1/2	4 1/2	13	7/8	1/2	1 3/8	12	9/16	3/4	3/8	7/8	766.5	7.5	3.9	3.7	781.6
	Button	Hexagonal													6.5			780.6
	Square	Square....									21/32	5/16	7/8		7.5		3.8	781.7
	Square	Hexagonal													6.5			780.7
1/2	Button	Square....	9/16	5	12	1 1/16	5/8	1 1/2	12	5/8	15/16	13/32	15/16	1,000.5	14.0	4.4	5.2	1,024.1
	Button	Hexagonal													13.0			1,023.1
	Square	Square....									3/4	3/8	1 1/16		14.0		6.0	1,024.9
	Square	Hexagonal													13.0			1,023.9
9/16	Button	Square	5/8	5	11	1 1/16	5/8	1 3/4	10	11/16	1	7/16	1	1,267.5	12.9	8.0	6.0	1,294.4
	Button	Hexagonal													11.9			1,293.4
	Square	Square....									27/32	13/32	1 1/32		12.9		8.2	1,296.6
	Square	Hexagonal													11.9			1,295.6
5/8	Button	Square....	11/16	5	11	1 1/4	3/4	1 3/4	10	3/4	1 1/16	7/16	1	1,364.5	24.3	7.7	8.7	1,605.2
	Button	Hexagonal													20.2			1,601.1
	Square	Square....									15/16	15/32	1 11/32		24.3		11.7	1,608.2
	Square	Hexagonal													20.2			1,604.1
3/4	Button	Square	13/16	5	10	1 3/8	7/8	2	10	7/8	1 3/16	1/2	1 1/16	2,253.0	34.0	9.7	13.3	2,310.0
	Button	Hexagonal													28.0			2,304.0
	Square	Square....									1 1/8	9/16	1 5/8		34.0		20.2	2,316.9
	Square	Hexagonal													28.0			2,310.9
13/16	Button	Square....	7/8	5	9	1 7/16	7/8	2 1/4	9	15/16	1 3/8	9/16	1 1/8	2,644.5	38.5	13.5	16.5	2,713.0
	Button	Hexagonal													30.5			2,705.0
	Square	Square....									1 7/32	39/64	1 1/4		38.5		25.7	2,722.2
	Square	Hexagonal													30.5			2,714.2
7/8	Button	Square....	15/16	5	9	1 7/16	7/8	2 1/4	9	1	1 1/2	5/8	1 3/16	3,066.0	36.5	17.5	20.3	3,140.3
	Button	Hexagonal													29.0			3,132.8
	Square	Square....									1 5/16	21/32	1 7/8		36.5		32.2	3,152.2
	Square	Hexagonal													29.0			3,144.7
1	Button	Square....	1 1/16	5	8	1 5/8	1	2 1/2	9	1 1/8	1 9/16	11/16	1 1/4	4,005.0	53.2	17.5	27.7	4,103.4
	Button	Hexagonal													44.5			4,094.7
	Square	Square....									1 1/2	3/4	2 5/32		53.2		47.8	4,123.5
	Square	Hexagonal													44.5			4,114.8
1 1/16	Button	Square....	1 1/8	5	7	1 13/16	1 1/8	2 3/4	7	1 3/16	1 11/16	3/4	1 5/16	4,521.0	75.6	20.0	33.0	4,649.8
	Button	Hexagonal													62.5			4,636.5
	Square	Square....									1 19/32	53/64	2 5/16		75.6		57.6	4,674.2
	Square	Hexagonal													62.5			4,661.1
1 1/8	Button	Square....	1 3/16	5	7	2	1 1/4	2 3/4	7	1 1/4	1 3/4	13/16	1 7/16	5,068.5	101.0	19.3	40.5	5,229.3
	Button	Hexagonal													87.0			5,215.3
	Square	Square....									1 11/16	27/32	2 13/32		101.0		67.8	5,256.6
	Square	Hexagonal													87.0			5,242.6

readily to wet material. The use of paint on exposed pipes under ordinary conditions probably adds very little to their life.

Bands.—In determining the size of bands, four is taken as the usual factor of safety. Bands less than 3/8 in. in diameter are not used. Table 29 shows minimum sizes of pipe for which bands of several sizes are applicable.

The particular style of band to use, one-piece or two-piece, oval-head or square-head, depends upon the size of the pipe, etc. Standard patterns of each, as made by one of the leading manufacturers, with weights and dimensions, are given in Table 30.

For bands, the usual specifications require soft steel of an ultimate tensile strength equal to 55,000 to 65,000 lb. per square inch; elastic limit not less than one-half the ultimate tensile strength; elongation in 8 in. not less than 25 per cent. and the bands are required to stand bending, cold, 180° around a diameter equal to that of the specimen tested, without fracture on either side. Such steel is similar in quality to that used for steam boilers.

It is usual to specify that bands shall be provided with not less than 5 in. of cold-rolled thread on the threaded portion. Each threaded end should be supplied with a standard hexagonal nut 3/16 in. thicker than the diameter of the band, and a plate washer of proper diameter and standard thickness.

For determining the spacing of bands many formulas have been developed and diagrams have also been prepared for graphical determination. The following formula has been commonly used.

$$d = \frac{S}{CPR} \qquad (111)$$

in which d equals distance between bands in inches.

S = maximum tensile strength of each band in pounds.

P = pressure of water in pounds per square inch in bottom of pipe.

R = internal radius in inches.

C = coefficient to allow for strain caused by swelling of wood, and includes safety factor of 4 to 5 for bands. Its usual value is 4.5 to 5.2.

The spacing of bands on some of the earlier pipes built, was as much as 16 in. or more, but at present, 10 in. is considered the maximum spacing permissible, and on some important recent

work the maximum was placed lower than this, even though the pressure did not require it.

There is a tendency for the ends of staves to spring out when subjected to high pressure and often, under light heads, where bands are further apart, if the pipe is exposed to the sun. In order to overcome this tendency it is now a common practice to specify additional bands at the joints, and to bring all joints within a longitudinal distance of 2 to 4 ft.

Coupling Shoes.—The designing of shoes is now left principally to the manufacturers, and selection may be made from a number of patterns. Cast-iron shoes were used principally during the earlier years of continuous stave-pipe building. They were heavy and easily broken, and on this account common cast iron has given place to malleable cast iron and steel. Malleable iron

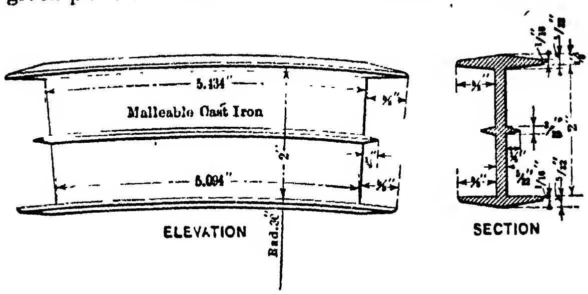


FIG. 69.—Kelsey joint.

for this purpose should be of the most tenacious character, capable of standing considerable hammering without fracture. The tensile strength should be not less than 40,000 lb. per square inch of section.

In designing butt joints, the use of thin steel clips inserted in saw kerfs is almost universal. The Kelsey joint (Fig. 69), consists of a malleable casting which takes the place of the metal clips and also fits tightly over the ends of the abutting staves. It appears to possess considerable merit. The cost is somewhat more than that of the thin metal clips, but it is claimed that the difference in cost is more than offset by the time saved in building the pipe and by eliminating expense of saw kerfs.

For the ordinary clips, No. 12 gauge steel or wrought iron, is used. As a rule they are 1½ in. wide and the length is ⅛ in. greater than that of the saw kerf, so that the ends may project $\frac{1}{16}$ in. at each edge of the stave.

Protective Coating of Bands.—The bands of continuous stave pipe are nearly always dipped or painted with some form of protective coating, and sometimes, the shoes also. For this purpose, there are numerous patented or trade preparations on the market. They consist usually, of asphaltum in combination with linseed oil or other ingredients for tempering and reducing, and, as a rule, are to be applied hot. Some manufacturers, however, recommend a cold dip instead of the hot, believing it to be equally effective.

Intakes and Outlets of Wood-stave Pipes.—Where a wood-stave pipe terminates, either in bulkheads or intakes, it is usual to make the joint by casting concrete around the ends of the pipe. Where this is done, there should be a number of additional steel bands put around the pipe both for the purpose of strengthening it and affording a better hold for concrete. In some instances, a section of cast-iron or steel pipe is set in the concrete and a junction is made between the metal and the wood sections. In other instances, where the concrete and wood are joined, space for caulking is provided by making the opening through the concrete slightly larger than the external diameter of the pipe. Either of the alternatives from the first plan given, makes it possible to replace or repair the end of the wood pipe with greater facility, though the caulked joint may be more difficult to keep watertight.

Connections with Other Kinds of Pipe.—Where curves are required which have too small a radius for wood-stave pipe, or where the heads are very high, or where wood pipe is to be joined to connections for water wheels, it becomes necessary to connect the wood stave to metal pipes. A common practice in joining wood and cast iron, or steel, is illustrated by Fig. 70. The wood pipe is made to overlap the metal pipe and the bands cinched up to make a tight joint. The usual lap is 12 to 18 in. but laps of as much as 4 ft. have been made.

A connection of this kind is criticized on the ground that it does not permit proper saturation of the wood pipe, where it overlaps the metal, thus leaving it subject to decay. It is considered better practice to insert the wood pipe into the metal pipe and caulk with lead and oakum. This usually requires a special coupling of cast iron or steel, having a thimble or flange which fits inside the wood staves to prevent them from being forced in by the caulking. If plain fittings are used, or the end

of the wood pipe is simply inserted inside the connecting end of the steel pipe, a heavy iron ring must be put inside the wood pipe to prevent the distortion of the latter when the caulking is done. This ring should be of plain hoop iron, varying from 1½ in. in width by ½ in. thick for 24-in. pipe up to 4 in. in width by ⅝ in. thick for 7-ft. pipe. Internal supporting hoops of angle-iron are objectionable because they constrict the area of the pipe.

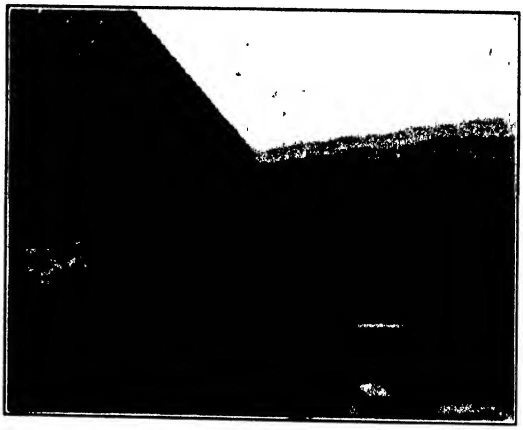


FIG. 70.—Steel angle in stave pipe showing method of joining them and of anchoring pipe on steep slope.

Supports.—If continuous stave pipes are built above ground they should be supported in "cradles," or "chairs." In the design and spacing of supports of this kind, the ideas and judgment of engineers differ and, as yet, there is no standard practice.

Cradles of the general type shown in *A*, Fig. 71, were used on several large pipe lines in Idaho, and they appear to be well designed. On some pipe lines, the 2 by 12-in. mudsills are continuous; on others, blocks 18 in. long are used. The use of short blocks instead of continuous sills is more economical of material, and requires less grading. Cradles of this type spaced 6 ft. center to center have been used under 54-in. and 100-in. pipe. A 48-in. pipe has been built on cradles 12 ft. apart, and while this spacing is unusually wide, the support appears to be ample.

Some large wood-pipe lines carried across rivers and ravines on bridges or trestles of steel, are supported by cradles also of

steel. The 84-in. pipe of a western irrigation company is, in places, supported on rock cradles set about 15 ft. apart.

Anchoring Pipes.—In order to secure pipes against water thrust at sharp horizontal curves, and to guard against the tendency to creep on steep inclines, anchorage in some manner is necessary. One method of anchoring a 44-in. pipe, and also

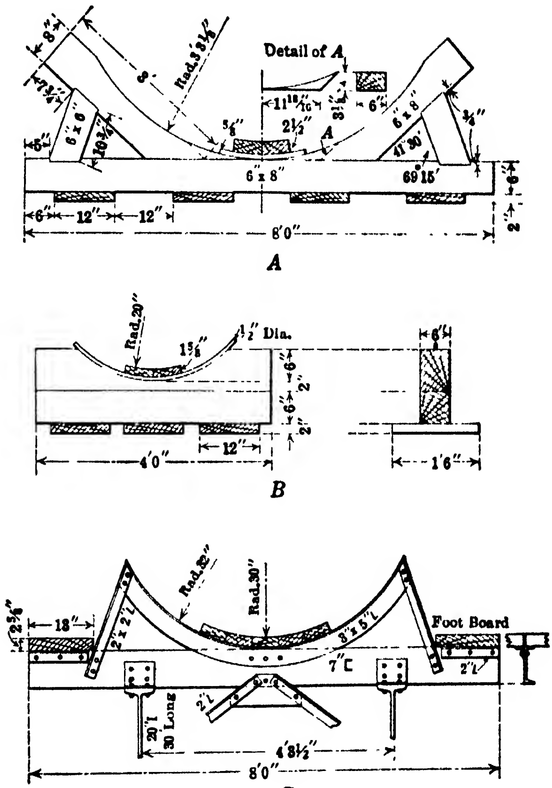


Fig. 71.—Cradles for carrying stave pipe.

making an angle too sharp for the curvature of wood pipe, is illustrated by Fig. 70. Another method is to build around the pipe a pier or mass of concrete or masonry to serve as anchorage.

Location of Continuous Stave-pipe Lines.—As a rule, a pipe line must follow, approximately, the variations of the ground

surface, but in both plan and profile, sharp curves should be avoided. Horizontal and vertical curves should not be placed in the same section of the pipe, and a tangent between curves is always desirable. A radius of 60 times the diameter of the pipe is usually taken as a measure of allowable curvature, though sharper curves are not uncommon.

A wooden pipe should be located so as to be entirely below the hydraulic gradient, and in making extensions, or in taking off branches from the pipe line already established, care should be taken not to lower the hydraulic gradient so as to leave the original pipe above it.

As to what the minimum distance below the hydraulic gradient should be, engineers differ in opinion. Assuming that pressure sufficient to keep the staves well saturated is necessary to prevent decay, some engineers advocate 50 ft. as the minimum, while others place it at 25 ft. With reference to the relation of pressure to durability of the wood, much may depend on other conditions of the location, particularly as to whether or not the pipe is placed in contact with the soil. If the pipe is placed in the ground or in contact with the soil, a pressure head of 50 ft. or more is desirable, but if it is kept free from contact with the soil, 15 ft. below the hydraulic gradient is sufficient.

Evidence based on the experience of the past 20 years appears to show that, in general, continuous stave-pipe lines should be located above ground and free from all contact with it, though contrary opinions are also held. If reasons appear sufficient to justify placing a pipe in the ground, it is best to insure a deep covering of a nature that will most nearly exclude air from the pipe, particularly if the water pressure is light. Gravel, shell rock, or other porous material is not satisfactory for backfilling.

Construction of Continuous Stave Pipe.—Where the pipe is to be built in a trench, the excavation is made from 1 to 2 ft. wider than the diameter of the pipe. Then the staves of the lower half of the pipe are set up in a U-shaped form made usually of 1½-in. gas pipe bent on a curve equal to the outside diameter of the pipe. Another piece of gas pipe, bent into a circle of a diameter slightly less than that of the wood pipe, with the ends overlapped and spread so that it will stand alone, is set on the lower staves already placed, and serves as a form for the upper part. If wooden cradles and two-piece bands are used, the lower section of the band, set in a cradle, is sometimes used as the bot-

tom form instead of the gas pipe. A few bands sufficient to hold the staves in place are then slipped on, and the final banding then completed, the spacing of each section being marked along the pipe according to tables or profiles in the hands of the foreman. During the progress of lining up and partially tightening the bands, the pipe is rounded out evenly and the staves are driven up to make the butt joints tight.

Wooden mallets are used for the "coopering," and in driving home the staves, iron-bound, hardwood blocks are used with sledge hammers.

The end driving must usually be done repeatedly as the bands are tightened, care being exercised not to bruise or injure the staves. Special braces or wrenches with long shanks and short leverage are generally used for this work, each builder, as a rule, designing his own tools. Curves are made by crowding or pulling the partially banded pipe to the desired position with jackscrews, or blocks and tackle.

A pipe-laying gang usually consists of from 8 to 16 men, the number depending on the closeness of banding and other conditions. The speed of construction depends on the size of the pipe, spacing of bands, curvature, and other factors specific to the locality. On a 48-in. pipe built at Clarkston, Wash., in 1906, 250 ft. was the most that was laid in 10 hr., and the amount ran down to as low as 50 ft. where work was difficult.

One hundred and fifty to 300 ft. of 34-in. pipe was made per day by a crew at Denver, Colo., the number of bands placed ranging from 700 to 1,000, while on 44-in. pipe 500 bands were placed per day. In 1910, a 48-in. pipe, 10 miles long was built for the Denver Union Water Co., in 75 days, with a force consisting of 150 men and 100 teams, and this included hauling 30,000 tons of material an average of 10 miles on wagons. This is considered to be very rapid construction for a pipe of this size laid in a trench averaging 7 ft. deep.

In building a long line of continuous stave pipe it is customary to employ several crews at convenient intervals of 1000 ft. or more. The different sections of pipe are joined by cutting staves to fit, allowing about ⅛ in. extra length, so that when sprung in place the end joints come tight.

Maintenance of Wood-pipe Lines.—Reasonably frequent inspection is advisable, and whenever leaks are found, or injuries of any nature are sustained, they should be repaired without

unnecessary delay. The continued impinging of a grit-laden jet from a small leak has been known to sever steel bands ⅝ in. in diameter.

Small leaks at the joints or seams of wood pipe are usually stopped with wooden wedges.

Under ordinary circumstances the repair of continuous stave pipe is not difficult. The removal and replacement of staves or portions of them is a matter of frequent occurrence. It is only necessary to remove a few bands to take out the defective stave, spring another into place and reband. If the pipe has been buried and the threads on the bands have become badly rusted, as they frequently do, any change in the position of the nut may necessitate the use of a new band, though if the body of the band is still serviceable a new thread may be welded on.

Where a pipe is above ground, any landslides coming into contact with it should be cleared away as a precaution against decay, particularly if it is at a point where the pipe is under light pressure. If supported in cradles the mudsills or footings should be renewed as decay progresses, in order to avoid injury to the pipe from settling.

Contrary to the theories commonly held 30 years ago, it has been found that the durability of wood pipe is usually dependent on the life of the wood rather than on the life of the bands. Only in rare instances have the bands failed first. Corrosion of the bands, being a chemical action, requires the presence of moisture and oxygen. It usually occurs most rapidly where pipes are buried and the backfill is wet, under conditions which, as a rule, are most favorable for the life of the wood. Corrosion is greatly accelerated by the presence of alkali in the soil. Under such conditions the bands almost invariably fail at the bottom of the pipe.

Cost of Stave Pipe.—The cost of wood-stave pipe will, of course, vary with local conditions, and it is only possible here to give some specific instances: 30-in. pipe of fir costs from $1.55 to $1.91 per foot length; 42-in. pipe from $2.20 to $2.85 per foot length; 48-in. pipe from $2.60 to $3.52 per foot length; 60-in. pipe from $4.25 to $6.30 per foot length; 84-in. pipe $9 per foot length.

CHAPTER VII

DAMS

Centers of Gravity.—To determine the forces acting on a dam or retaining wall, and the stability of the structure against sliding and overturning, it is necessary to find the position of the center of gravity of certain cross-sections of water and of masonry, or concrete.

For convenience, the following rules and formulæ are here given.

For a section like Fig. 72, or any quadrilateral having two parallel sides, bisect the parallel sides and join the middle points by a line. Thus, bisect AB at Z and CD at W and join these points by the line ZW. Extend each of the parallel sides, one in one direction, the other in the opposite direction, the amount of the

FIG. 72.—Center of gravity of trapezoid.

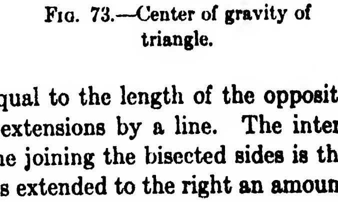


FIG. 73.—Center of gravity of triangle.

extension of each side being equal to the length of the opposite side. Join the ends of these extensions by a line. The intersection of this line with the line joining the bisected sides is the center of gravity. Thus, AB is extended to the right an amount equal to CD, while CD is extended toward the left by an amount equal to AB. The line XX joining the ends of these extensions intersects line ZW at G, which point is the center of gravity.

The center of gravity of a triangle is on the line joining the upper vertex with the middle point of the base and is one-third the

altitude of the triangle distant from the base. Fig. 73 indicates the location of the center of gravity of triangular sections.

The center of gravity of a figure like that shown in Fig. 74 may be obtained by dividing it into two parts such as *ABEK* and

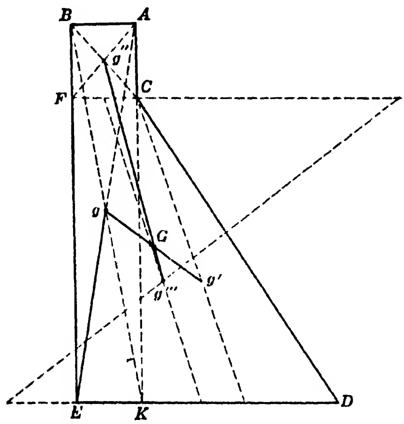


FIG. 74.—Center of gravity of irregular figure.

CKD. Locate the center of gravity of each part. That of *ABEK* is at g, while that of *CKD* is at g'. The center of gravity of the whole figure is on the line joining these two separate centers. Re-divide the figure into two other forms such as *ABFC* and *FCED*. Take their respective centers of gravity at g'' and g''' and join them by a line. The intersection G, of the two lines joining the two sets of centers of gravity, is the center of gravity of the figure.

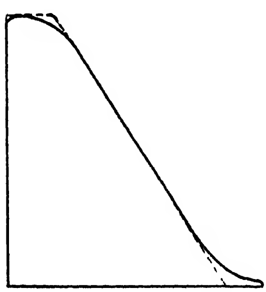


FIG. 75.—Equivalent trapezoid.

For contours like Fig. 75, it is sufficiently accurate to assume them to be trapezoids, the outline of the rear face being taken by prolonging the downstream face until it intersects the horizontal line drawn through the crest, and as indicated by the dotted lines.

The analytical formulæ for positions of centers of gravity are given in the following:

Trapezoid.—The center of gravity of any form of trapezoid, as in Fig. 76, is located at a height above the base equal to

$$\delta = \frac{h}{3}\left(\frac{b + 2a}{b + a}\right) \tag{112}$$

Its location, horizontally, is at the middle of the distance between the two non-parallel sides, at the height located by the formula, *i.e.*, the center of gravity G lies halfway between e and f in the figure. It, therefore, lies in the line drawn from the middle of one of the parallel sides to the middle of the other.

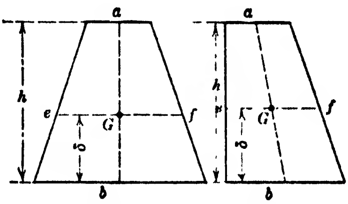


FIG. 76.—Center of gravity of trapezoid.

Formula for horizontal distance of center of gravity from either of the non-parallel sides is

$$d = \tfrac{1}{2}\left(b - \frac{b - a}{h}\delta\right) \tag{113}$$

where δ (*i.e.*, height of center of gravity above the longer parallel side) is computed by formula (112).

For figures such as Fig. 74, divide into any two convenient sections, for instance $A\ B\ E\ K$ and $C\ K\ D$, finding the center of gravity of $A\ B\ E\ K$ at g and that of $C\ K\ D$ at g^1. The center of gravity G of the whole figure then lies on the line gg', and at a distance along it determined by the condition that $A\ n$ must be equal to $A'\ (l - n)$, in which A and A' are the areas of the parts $A\ B\ E\ K$ and $C\ K\ D$, respectively.

n = distance from g to G.
l = length of line from g to g'.
$l - n$ = distance from g' to G.

From the above condition it follows that

$$n = \frac{A'l}{A + A'} \tag{114}$$

Parabola.—Figure 77 shows a parabola. The center of gravity for the area included between the curve and the axis, lies at a point $\frac{3a}{5}$ from the vertex and $\frac{3b}{8}$ from the axis, a being the horizontal and b the vertical coördinates of the end of the curve with the origin at the vertex. Also, the center of gravity of the portion included between the coördinates and outside the curve is located $\frac{3a}{10}$ from the vertex and $\frac{3b}{4}$ from the axis.

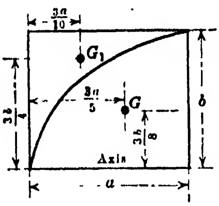


Fig. 77.—Center of gravity of parabola.

General Forms.—In general, the distance of the center of gravity from any chosen axis is equal to the sum of the moments of the areas about the axis, divided by the area of the whole figure. or, algebraically,

$$l_c = \frac{A_1 l_1 + A_2 l_2 + \ldots . + A_n l_n}{A_1 + A_2 + \ldots . + A_n} \tag{115}$$

As an example take the form shown in Fig. 78.

$A\ B\ C\ D\ E\ F\ H\ K\ M$ is an irregular figure having given dimensions. This figure is divided into symmetrical sections in any convenient manner. Assume this division to be as indicated by the dotted lines.

Length of line $BD = \sqrt{18^2 - 10^2} = 14.96$
Length of line $SK = 14.96 + 8 = 22.96$
Length of line $AS = 20 - 16 = 4$
Length of line $SM = 10 - 4 = 6$

The areas are as follows:

$$\begin{aligned}
A_1 &= 22.96 = 4 &&= 91.84\\
A_2 &= \tfrac{1}{2} \times 6 \times 22.96 &&= 68.88\\
A_3 &= \tfrac{1}{2} \times 10 \times 14.96 &&= 74.80\\
A_4 &= 20 \times 10 = && 200.00\\
\hline
\text{Total area, } A &= && 435.52
\end{aligned}$$

Taking the reference, axis XX through the upper horizontal line of the figure, the center of gravity of A_1 is at the distance $l_1 = \frac{22.96}{2} = 11.48$ from the axis.

Similarly,

$$l_2 = \frac{22.96}{3} = 7.65$$

$$l_3 = 8 + \tfrac{2}{3} \times 14.96 = 17.97$$

$$l_4 = 22.96 + \frac{10}{2} = 27.96$$

Then the moments are

$$\begin{aligned} A_1l_1 &= 91.86 \times 11.48 = 1054.55 \\ A_2l_2 &= 68.88 \times 7.65 = 526.38 \\ A_3l_3 &= 74.80 \times 17.97 = 1344.16 \\ A_4l_4 &= 200 \times 27.95 = 5592.00 \\ \hline \text{Sum of moments} &= 8517.09 \end{aligned}$$

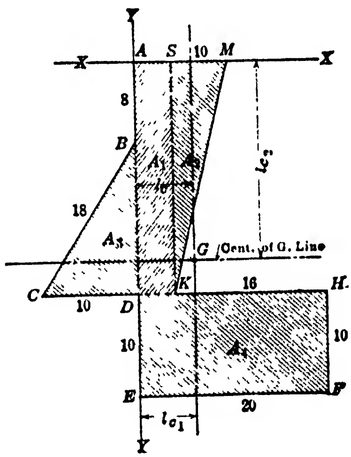


FIG. 78.—Center of gravity by moments.

The sum of the moments divided by the sum of the areas, *i.e.*,

$$\frac{8517.09}{435.52} = 19.55$$

is the distance of the center of gravity of the whole figure from the axis XX. The center of gravity lies on the line parallel to XX and 19.5 distant from it. This line is drawn in the figure and called "center of gravity line."

Taking now the axis YY, perpendicular to XX, and finding the moments of each area around it and dividing by the sum

of the areas, the center of gravity line referred to YY, is found. At its intersection with the center of gravity line referred to XX, is the center of gravity of the figure.

It is often convenient to pass the reference axes through the figure, along certain of the division lines, as in the case of the line Y–Y in the previous example. In such cases, all the moments on one side of the axis must be taken as positive quantities, while all those on the opposite side as negative quantities.

In the foregoing example the centers of gravity of A_1, A_2 and A_4 lie on one side of Y–Y and that of A_3 lies on the other side. Hence the moment A_3l_3 is negative, and in this case the distance of the center of gravity from Y–Y is

$$l_{cy} = \frac{A_1l_1 + A_2l_2 - A_3l_3 + A_4l_4}{A_1 + A_2 + A_3 + A_4} \qquad (116)$$

The distance of the vertical gravity line from the axis Y–Y, or l_{cy}, is found to be 6.126, when the proper quantities are substituted in the formula, and the center of gravity is seen to lie outside the figure.

These rules apply only to plane figures or volumes of a homogeneous substance having the same thickness; that is, a section of a material having uniform specific gravity and thickness.

If the material is homogeneous, but the thickness of the different portions of the section varies—as in hollow dams—then instead of the areas A_1, A_2, A_3, etc., their respective volumes must be used, so that each area must be multiplied by its thickness. Hence, the moment of each section around the axis chosen, becomes $A_1t_1l_1$, $A_2t_2l_2$, etc., t, being the thickness of the section. After summing up all the individual moments, this sum is divided by the sum of the products of A_1t_1, A_2t_2, etc., as the divisor, the result being the distance of the center of gravity of the whole section from the chosen reference axis.

If the material of which the several sections are composed varies—as in the case of a concrete dam overlaid with granite—the product for the moment of the separate sections will be $A_1t_1y_1l_1$, $A_2t_2y_2l_2$, etc., y being either the specific gravity or actual weight, per unit volume, of the material. The divisor for the sum of the moments is, of course, $A_1t_1y_1 + A_2t_2y_2$, etc. This last condition is seldom encountered.

It is to be noted that it is not necessary to know the height of the center of gravity for computing the forces acting on dams.

Its distance from either face of the dam is the only location required; that is, the distance of the center of gravity from the vertical axis, as Y–Y in Fig. 78. The gravity moment of the dam about any horizontal axis, depends only on the horizontal distance of the center of gravity from the axis chosen, and, in no wise, on its height.

Forces Acting on Dams.—In order to understand the several factors that enter into the subject of the design of dams, two principles of hydrostatics must first be known, namely:

1. The pressure exerted by a fluid is transmitted equally in every direction;

2. The pressure, per unit area, produced by a head of water, is proportional to the *height* of the head, and is independent of the total mass of water.

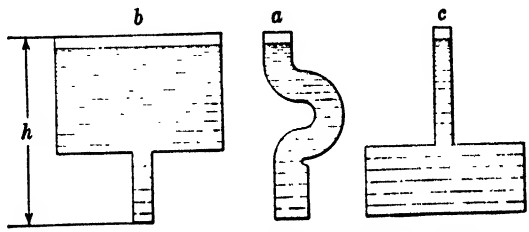


FIG. 79.—Illustrating unit pressure due to head.

In the three vessels, a, b and c, Fig. 79, the head "h" is the same. Likewise, the pressure, per square inch at the bottom of each of the vessels is the same, although the quantity of water in vessel b is many times greater than that in vessel a. Also, as the area of the bottom of vessel c is much greater than that in vessel b, the total pressure on the bottom of vessel c is much greater than the total pressure on the bottom of the vessel b, although the quantity of water in the two vessels is practically the same.

Dam Without Overflow.—Consider the case of a vertical wall with water against one side of it, as in Fig. 80, which shows, in outline, the section through a dam with the water backed up behind it.

The head at the surface of the water is zero, hence, the pressure is zero at the surface line. The pressure at the bottom is $62.5H$ lb. per square foot, H being the head or depth of water, in feet.

Since this pressure is transmitted equally in every direction, the horizontal pressure against the vertical surface is $62.5H$ lb. per square foot. Furthermore, as the pressure increases from 0 to $62.5H$ proportionally to the depth, the pressure at any point may be indicated by drawing the straight line OA from the point O of the surface line diagonally downward, the distance AB representing, to scale, the value $62.5H$. At any height, say at the line p_2, the value of the pressure is equal to the scalar value of the horizontal length p_2. In other words, the horizontal distance between the vertical and diagonal lines at any height,

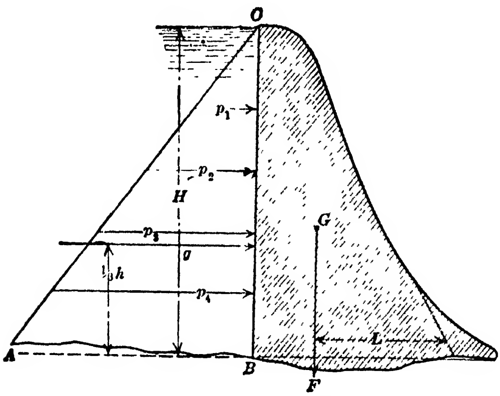


FIG. 80.—Water pressure against dam.

represents the pressure at that height. Since the pressure is zero at the top and 62.5 H lb. per square foot at the bottom, the *average* pressure over a unit area of the whole surface is

$$p = \frac{0 + 62.5H}{2} = 62.5\frac{H}{2} \text{ lb. per square foot.}$$

If the area of the vertical surface is aH sq. ft., a being the length and H the height, the total pressure tending to push the wall horizontally downstream is

$$aH \times 62.5\frac{H}{2} = 62.5\frac{aH^2}{2} \text{ lb.} \tag{117}$$

and the horizontal pressure on the surface for 1 ft. length, *i.e.* for $a = 1$, is

$$P = \tfrac{1}{2}62.5\, H^2 \text{ lb.} \tag{118}$$

The total pressure, per foot length, is also equal to the area

of the pressure triangle to the scale adopted for the vertical and horizontal ordinates, that is, $\frac{1}{2}H^2 \times 62.5$ lb. Since the pressure increases from zero at the surface, to the maximum at the bottom, and may be represented by the area of a triangle, the *center of pressure* is at the center of gravity of the triangle which, for any triangle, is located at one-third the altitude above the base.

The altitude is equal to H ft. and, therefore, the center of pressure is at $\frac{1}{3}H$ ft. above the bottom of the water, which is the length of the lever arm through which the total pressure acts to cause overturning.

Since the *overturning moment* is equal to the pressure multiplied by its lever arm, this moment is

$$M = \frac{62.5}{2} H^2 \times \tfrac{1}{3}H = 10.4H^3 \text{ lb.-ft.} \qquad (119)$$

From the foregoing it follows that if there is a water pressure acting on two sides of a vertical partition and the water on one side is at a higher level than that on the other, the net resultant force of the deeper body of water acting against the partition is not directly proportional to the difference in head of the water on the two sides, but is proportional to the difference in their squares; that is, the net pressure, per foot length, tending to move the wall horizontally is $\frac{1}{2}62.5(H_1^2 - H_2^2)$ lb., while the net overturning moment, per foot length, is proportional to the difference of their cubes and is $10.4(H_1^3 - H_2^3)$ lb.-ft., H_1 and H_2 being the respective heads on the two sides of the partition.

It is to be noted that the pressure is independent of the total volume of water on either side of the wall. If the water extended out horizontally, a distance of 10 miles from the wall on the side having the lower head, and were only an inch in horizontal length on the side of the higher head, the relative pressures as computed above would be unchanged. The head, alone, determines the pressure.

Dam with Overflow.—When a dam, or other vertical surface, having a water pressure against it, has water flowing over it, the head being greater than the height of the dam, the foregoing formulæ do not give the values of the forces acting.

If H = height of the dam, and h = the height of the water

above the crest of the dam, the force tending to push the dam downstream, per foot of length, is

$$P_1 = \frac{62.5}{2} H(H + 2h) \text{ lb.} \tag{120}$$

The overturning moment will be

$$M_1 = 62.5H^2(\tfrac{1}{6}H + \tfrac{1}{2}h) \text{ lb.-ft.} \tag{121}$$

These formulæ are derived as follows:

In Fig. 81 the line AB represents the pressure, per square foot, at the bottom of the dam due to the total head $H + h$, and is, to scale, equal to 62.5 $(H + h)$ lb.

The line CD represents the pressure per square foot at the top of the dam due to head h, and is, to scale, equal to $62.5h$ lb.

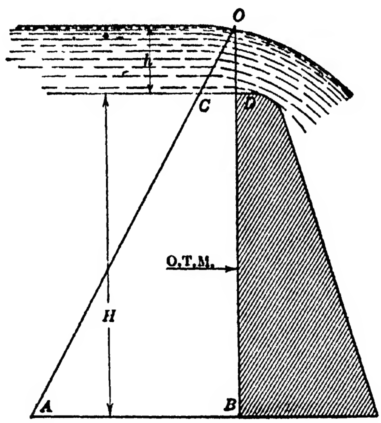


Fig. 81.—Forces acting on overflow dam.

The average pressure, per square foot, against the dam is the average of these two pressures, *i.e.*

$$\frac{62.5\,(H + h) + 62.5h}{2} = 62.5(\tfrac{1}{2}H + h) \text{ lb.} \tag{122}$$

The total pressure against the surface, H ft. high and 1 ft. wide, or the total pressure, per foot length of dam, is

$$H \times 62.5\,(\tfrac{1}{2}H + h) \text{ lb.} \tag{123}$$

which is the pressure tending to push the dam downstream.

The lever arm of the overturning moment is at that height above the base of the dam equal to the height of the center of gravity of the trapezoid *CABD*. The height of the center of gravity of a trapezoid is equal to $\frac{H}{3}\left(\frac{b+2a}{b+a}\right)$, in which, H is the altitude of the trapezoid, b the length of the base, and a the length of the top.

For trapezoid *CABD*, $H=H$; $a=62.5\ h$, and $b=62.5(H+h)$.

Height of center of gravity $= \frac{H}{3}\left[\frac{(H+h)+2h}{(H+h)+h}\right] = \frac{H}{3}\left(\frac{H+3h}{H+2h}\right)$

Multiplying the total pressure, per foot length by the lever arm thus found, the overturning moment, per foot length, is

$$62.5\ H\left(\frac{1}{2}H+h\right)\frac{H}{3}\left(\frac{H+3h}{H+2h}\right) = 62.5\ H^2\left(\frac{1}{6}H+\frac{1}{2}h\right)\text{ lb.-ft.} \tag{124}$$

Dam with Inclined Face.—The water pressure acting against any surface is always normal to the surface. Hence, when the upstream face of a dam is inclined, there is a component of the water pressure which tends to push it vertically downward, but the foregoing formulæ are subject to no modification except that imposed by the vertical component of the water pressure. In general, for all conditions where the water does not overtop the dam, the total pressure P, per foot length, normal to the face of the dam is (Fig. 82)

$$P = \frac{H^2}{2}\ \frac{62.5}{\cos\theta} \tag{125}$$

in which

H = depth of water.
θ = angle between face of dam and the vertical.

The horizontal and vertical components of the pressure are, respectively:

$$P_h = P\cos\theta = \frac{H^2}{2}\times 62.5 \tag{126}$$

$$P_v = P\sin\theta = \frac{H^2}{2}\times 62.5\tan\theta \tag{127}$$

Obviously, for a vertical face, P_v reduces to zero.

The overturning moment is

$$M = 10.4H^3$$

For dams in which the upstream faces are sloping and which are overtopped by the water, as in *d*, Fig. 82, the formulæ become

$$P = \frac{62.5H}{2 \cos \theta} (H + 2h) \tag{128}$$

$$P_h = P \cos \theta = \frac{62.5H}{2} (H + 2h) \tag{129}$$

$$P_v = P \sin \theta = \frac{62.5H}{2} (H + 2h) \tan \theta \tag{130}$$

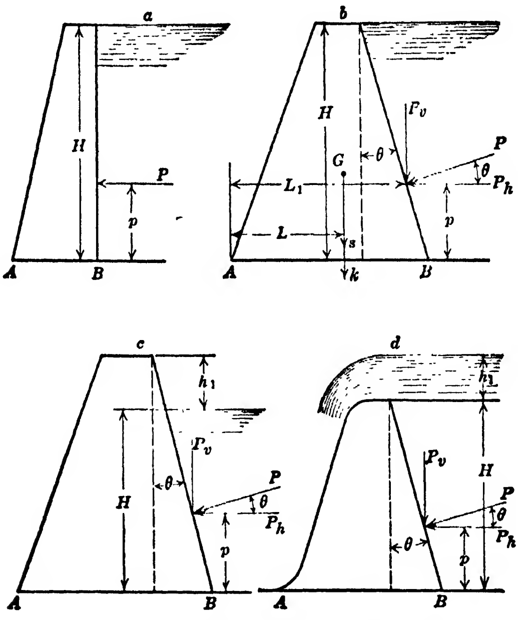


FIG. 82.—Overturning moments against dams.

and the overturning moment is

$$M = 62.5H^2 \left(\frac{H}{6} + \frac{h}{2}\right) \tag{131}$$

in which

H = height of dam, in feet

and

h = height of water surface above crest of dam, in feet.

The moment of the vertical pressure, acting through a lever

arm which has a length depending on the thickness of the base of the dam, opposes the overturning moment and, therefore, adds to the stability of the dam against overturning. Hence a sloping upstream face tends to increase the stability of a dam, and also to increase its resistance to sliding, as will presently be set forth.

Uplifting Force.—Unless the base of a dam is well bonded to the foundation, so that the joint between them is water-tight, water, under a pressure head H (or $H + h$), will enter the joint and produce an upward pressure under the base of the dam equal to 62.5 H per square foot.

It is usual to assume that if water can percolate between the base of the dam and the foundation, it can, more easily, and under less pressure than that due to H, find its way along the joint and

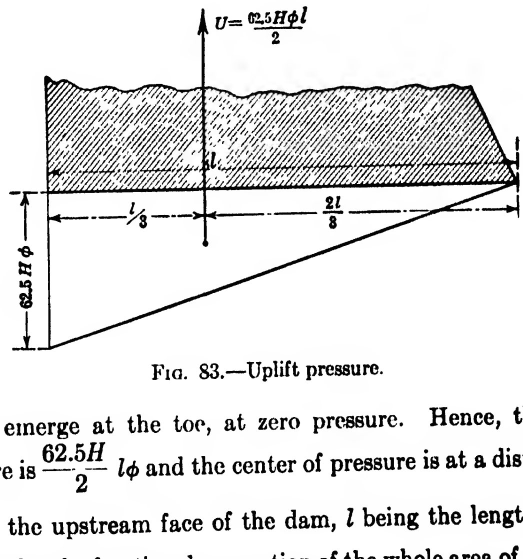


Fig. 83.—Uplift pressure.

finally emerge at the toe, at zero pressure. Hence, the total pressure is $\frac{62.5H}{2} l\phi$ and the center of pressure is at a distance, $= \frac{l}{3}$ from the upstream face of the dam, l being the length of the base, and ϕ the fractional proportion of the whole area of the base to which the water can penetrate. Fig. 83 shows, graphically, the variation in pressure under the base and its resultant, which is

$$U = \frac{62.5H\phi l}{2} \text{ per ft. length.} \tag{132}$$

For reasons given elsewhere, under the caption, "Uplift Pressure Under Dams," the author considers the effect of such possible uplift negligible except for sand foundations. In the

exact formulæ which are given in this chapter, and also in the paragraph on "Resultant of Forces Acting on Dams," the method of treating this uplift is included for the computation of forces acting on dams on sand foundations, and also for the benefit of such designers as may still believe it of importance in masonry dams on rock foundations.

Ice Pressure.—Whenever the level of the impounded water in an artificial lake sinks down to, or below, the crest of the dam and freezes, the expansion of the liquid when entering into the solid state, imposes a horizontal pressure against a vertical faced dam.

How great this pressure, has been variously guessed at—for this is about all the attempts to fix a value for it, amounts to.

What the ice pressure against the crest of a dam is, has never been determined. Various figures of from 4000 to 40,000 lb. per foot length have been proposed. My own opinion inclines to 3000 to 6000 lb. per foot length.

Whatever figure is adopted, it must be remembered that the pressure at the crest, due to overfall of water, can not be exerted at the same time as the ice pressure. In computing the cross-section of a dam, it is well to compute one section to resist all the forces including overfall, then compute another, omitting overfall and including ice pressure. Whichever section is the larger should be the one adopted. For bulkhead sections, there is, of course, no overfall and the ice pressure is a factor of some importance.

Ice pressure can be transferred to the point of center of pressure due to the water, viz., at $\frac{H}{3}$ ft. above the base, and added to that of the water pressure which exists when the level of the water is just at the top of the dam. Its value, thus transferred, will be equal to 3 times its actual pressure.

Thus, if the water pressure is 150,000 lb. and the ice pressure 4000 lb. per foot length, the total pressure, taken as acting through the center of pressure for the water will be $150{,}000 + 3 \times 4000 = 162{,}000$ lb.

Vacuum Produced by Overfall.—Whenever the water discharged over the crest of a dam has a natural path which lies some distance away from the downstream face, or in other words, when the water tends to leap beyond the spillway face, and there is no means for air to reach the spillway surface, a vacuum

is formed between the sheet of water (nappe), and the face of the dam, which draws in the nappe and forces it to adhere, or tend to adhere, to the face of the dam. This means that a force which, at times, may be considerable, is set up, tending to push the dam downstream, and also, to cause overturning. This vacuum is never produced when the spillway is properly designed, because the water can not then leap beyond the boundary of the spillway face. It may be corrected in existing structures by admitting air to the spillway surface under the nappe, at the two ends of the spillway, using short lengths of 2- or 3-in. pipe fastened to the face of the spillway near the crest, so that they are parallel to the axis of the dam and project under the nappe 3 or 4 ft. and out into the air a short distance.

Simple obstructions fastened on the spillway surface near the crest at intervals of 40 to 60 ft. will accomplish the same result—say three 4 by 4-in. timbers 3 ft. long spaced 3 ft. apart down the spillway. Any device whereby the nappe is broken to admit air will destroy the vacuum. In a reinforced-concrete dam, it is customary to leave open the bolt holes through the spillway made by form-holding bolts.

If the curve of the spillway be designed in accordance with the laws of a falling mass of water, no additional precaution is necessary.

Resultant of Forces Acting on Dams.—In addition to the forces set up by the water pressure, as before set forth, the weight of the dam acting vertically downward, provides another force, which must be combined with the water forces to find the resultant force acting on the dam.

The resultant is found by the usual method of composition of forces by parallelograms and is practically obvious from the Figs. 84, 85, 86 and 87.

Figure 84 shows a dam having a vertical face, the force P acting horizontally. $P = \frac{62.5H^2}{2}$ lb. per foot length for dams not overtopped with water, or, $P = 62.5H\left(\frac{H}{2} + h\right)$ lb. per foot length for dams H ft. high with h ft. of water flowing over them, The weight acts as if concentrated at the center of gravity. denoted by G in the figures. If the lines representing P and G are prolonged until they intersect—in whatever direction the prolongation may be necessary for intersection—and the point

of intersection, *O*, taken as the common point from which the two forces act, the parallelogram of forces can be drawn.

Continue both *P* and *G* in their directions of action, laying them off so that $OP = P$, and OW = weight, in pounds, of 1 ft. length of dam, and on these two force lines, form a parallelogram, as shown. The diagonal *OR* will be the resultant force, its magnitude and direction both being thus found.

The position of the point at which the resultant intersects the base is important. It must fall within the middle third of the base: that is, if the thickness through the dam at the base, be divided into three equal sections, the resultant of the forces

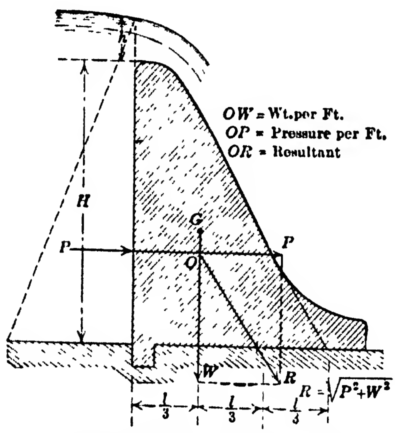


FIG. 84.—Resultant of forces on dam.

acting must intersect the base line somewhere within the middle section. Otherwise, the pressure on the foundation and the lower courses of masonry at the downstream side, may be excessive and there will be tension on the upstream side of the dam—a condition which is undesirable in any masonry structure. This is discussed further under the caption, "Foundations of Dams."

If the resultant force does not cut the base within the middle third the dam is too thin and must be re-designed.

Figure 85 shows the composition of forces acting when the face of the dam is inclined upstream. The method of locating the resultant is obvious from the drawing.

Figure 86 shows a condition frequently encountered, namely,

that of a dam having an inclined upstream face and surmounted by a gate, flash-board, or cylindrical drum having a height above the crest of the dam $= h_1$.

Take first, the pressure, p, acting on the gate (or drum), which is $p = \frac{62.5h_1^2}{2}$. The center of pressure will be $\frac{h_1}{3}$ ft. above the crest of the dam. Then find the value of the force P acting on the inclined portion of the dam. This will be the same as for an inclined dam overtopped by h_1 ft. of water, or

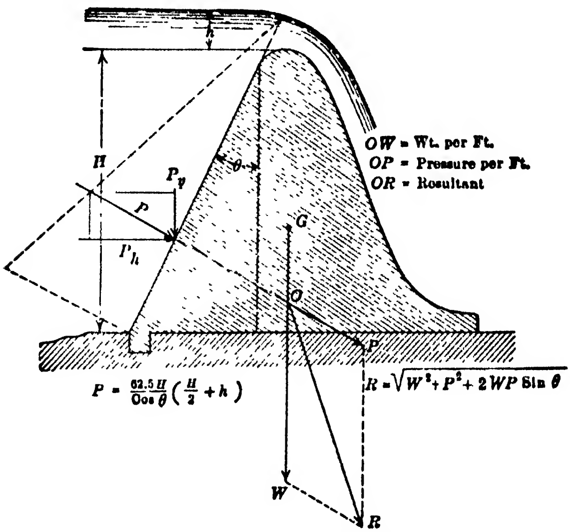


Fig. 85.—Resultant of forces acting on inclined face dam.

$62.5H\left(\frac{H}{2} + h_1\right)\cdot$ The center of pressure will be located at the center of gravity of the trapezoid, of which $H + h_1$ is the base, h_1 the top and H the height, or $\frac{H}{3}\left(\frac{H + 3h_1}{H + 2h_1}\right)\cdot$ The direction of P is normal to the face of the dam.

Having now the directions, magnitudes and locations of the two forces p and P, the lines representing them must be prolonged until they intersect. In this case it is necessary to continue the lines back from the face of the dam until they intersect at O'. Then lay off p and P to scale (represented by $O'p'$ and $O'P'$) in the direction of action of the forces, which

is always toward the downstream side of the dam. The resultant, $O'r = r$, is the resultant of all the water forces acting against the dam. Prolong $O'r$ until it intersects the vertical line of force due to weight, GW. At the intersection O of these two forces, lay off Or and OW equal to $O'r$ and W respectively, and construct the parallelogram. The resultant OR shows the direction and magnitude of the combined forces acting on the dam.

Where forces other than the water pressure on the face of the dam must be resisted, such as ice pressure at the top, and uplift pressure, due to water finding its way underneath the base and

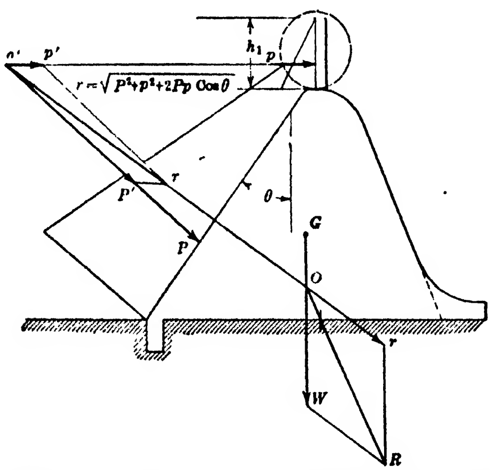


FIG. 86.—Resultant of forces acting on vertical and inclined faces of dam.

exerting an upward pressure, they may be combined graphically by the application of the parallelogram of forces.

For ice pressure it is usually, simpler to transfer its equivalent force to the center of water pressure, add the equivalent force to the water pressure, and treat the combined force as if it were simply a single pressure, exerted at height above the base equal to the height of the center of the water pressure.

Obviously, if the ice pressure at the top of the dam $= p_i$, and is exerted at a height H above the base, the force to add to the water pressure as an equivalent of p_i is $p_i \times \dfrac{H}{\frac{H}{3}} = 3p_i$ in which

$\frac{H}{3}$ = height of the center of pressure of the water against the face of the dam. After combining these two forces, the resultant of these and the uplifting pressure may be combined and a second resultant obtained, after which, this resultant may be combined with the gravity force to get the final resultant and its direction of action. Thus, in Fig. 87, which shows a dam 72 ft. high, with a 60-ft. base, the ice pressure at the top is taken at 20,000 lb. per foot length. There is no force due to overflow head from water passing over the spillway, as this condition can not exist at the same time as an ice sheet pressing against the top of the dam.

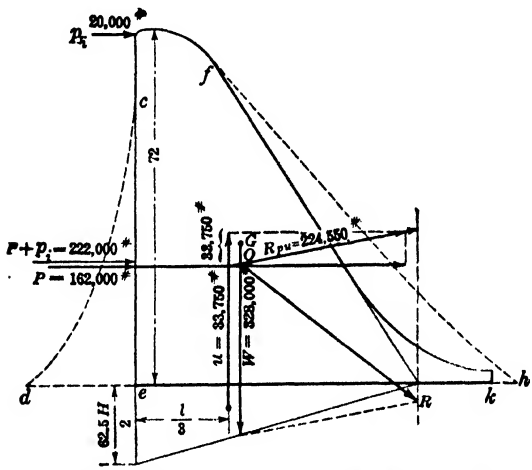


FIG. 87.—Resultants including ice pressure and uplift.

Transferring the ice pressure to the point $\frac{H}{3}$, its value becomes $3 \times 20{,}000 = 60{,}000$ lb. per foot length of dam. This excessive ice pressure is here assumed to bring out more clearly, in the drawing, the effect of ice.

The water pressure is $\frac{(72)^2 \times 62.5}{2} = 162{,}000$ lb. per foot length, and the sum of the water and ice pressures is $162{,}000 + 60{,}000 = 222{,}000$ lb. The average uplift over the whole front of the dam, is taken at $0.25 \times 62.5\ (H + h) = 1125$ lb. per square foot, while it diminishes to zero at the toe, making an average pressure under the whole base of 562.5 lb. per square foot. For

a 60-ft. base, the entire uplift per foot length of dam = 562.5 × 60 = 33,750 lb.

The center of action of this force lies at a point $\frac{l}{3}$ from the front of the dam. Combining this uplift with the combined horizontal forces, the resultant, R_{pu}, is found. It has a value = 224,550 lb. and intersects the vertical line which represents the gravity force, at O. Constructing the second parallelogram on the gravity force line and the resultant R_{pu}, the resultant, OR, of all the forces, is found. Obviously, it intersects the base much too near the toe for this section to be safe, and, for the forces taken, the dam would have to be considerably thicker at the base.

The uplifting pressure acts only on the base of the dam. Hence, the upper sections may be computed, just as if this force did not exist. This means that the bottom section only must be made thicker, as indicated by the dotted lines *cde* on the rear face, or *fhk* on the front face.

In determining resultants, remember to lay out the force lines from their intersection in the directions in which the forces act. It is, sometimes, easy to reverse the parallelogram and obtain erroneous results unless some care is exercised in this respect.

The value of the resultant and the point at which it cuts the base line can, also, be determined analytically.

$$R = \sqrt{(W - U)^2 + (P + p_i)^2} \tag{133}$$

R = numerical value of the resultant.
W = weight of dam, per foot length.
U = total uplift pressure, per foot length of dam.
P = total water pressure, per foot length of dam.
p_i = ice pressure, per foot length of dam.

Also,

$$\beta = \frac{l(2\Delta + kl)}{6\Delta} \tag{134}$$

β = distance from downstream toe of dam to point where resultant cuts the base.
Δ = $W - U$.
l = length through base.
k = $p - J$ = net pressure, per square foot, of dam on foundation at upstream side.
p = pressure on foundation at upstream side, per square foot when no uplifting force is acting.
J = pressure per square foot, under the dam at the upstream side due to uplift.

Likewise, if q, the pressure, per square foot of section on, foundation at the downstream side, is known,

$$\beta = \frac{l(4\Delta - ql)}{6\Delta} \tag{135}$$

The manner in which p and q are found is explained in the succeeding paragraph.

Usually, the uplift pressure U, or J, is negligible, and (134) formula becomes

$$\beta = \frac{l(2W + pl)}{6W} \tag{136}$$

Stresses on Foundations of Dams.—Let Fig. 88 represent the lower portion of the cross-section of a dam. The downstream side HKF is curved to discharge the water horizontally. The portion HKE is considered as the boundary of the dam on which the stresses are imposed, the curved, bottom portion being regarded simply as a deflector of the downcoming sheet of water.

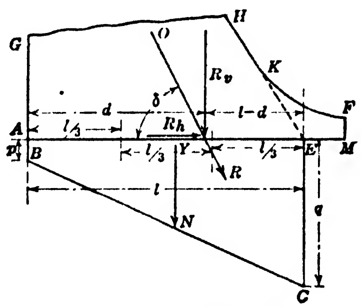


Fig. 88.—Foundation stresses.

The length through the cross-section of the dam at the base, from the upstream to the downstream face, and marked l in the figure, ends at E so far as the computed foundation stresses are concerned. Of course, there is some strength in the projecting curved toe that assists in distributing the weight of the dam over the foundation, but this factor is indeterminate and should be regarded simply as a slight addition to the factor of safety.

The line OR is the resultant of the forces acting on the dam, made up of the water forces and action of gravity. This resultant intersects the base of the dam at a point d ft. from the upstream face and $l - d$ ft. from the downstream face, and makes an angle δ with the base line, l being the length through the base.

Resolving R into vertical and horizontal components, R_v = the downward pressure to be resisted by the foundation, while R_h is the horizontal pressure to be resisted by the strength against shear. From the previous discussion of "Resultants of Forces Acting on Dams" it may be seen that

For vertical-face dams

$$R_v = P \sin \delta \tag{136}$$

$$R_v = W \tag{137}$$

For inclined face dams

$$R_v = W + 62.5H\frac{(H + 2h)}{2} \sin \theta, \tag{138}$$

$$W = \text{weight of dam, per foot length.}$$

The stress on foundation at the downstream side, is

$$q = \frac{2R_v}{l}\left(\frac{3d}{l} - 1\right), \text{ lb. per sq. ft.} \tag{139}$$

Stress at the upstream side, is

$$p = \frac{2R_v}{l}\left(2 - \frac{3d}{l}\right), \text{ lb. per sq. ft.} \tag{140}$$

Hence, if $d = \frac{2l}{3}$, the stress at the upstream side = 0,

If d is greater than $\frac{2l}{3}$, the stress at the upstream side is negative, and there is tension in the masonry. These conditions are shown as follows.

The total reaction of the foundation must equal the total weight imposed on it, and the distribution of pressure over the foundation must either be uniform (where the resultant intersects the base in the middle), or change uniformly from one face to the other, because the dam is a rigid body, and as such can not impose abrupt changes of pressure on its foundation, if the latter is regarded as sufficiently elastic to distribute the stress over the whole under surface of the dam.

While the accuracy of this latter hypothesis has been questioned, it appears rational, and, in any case, is the only one now known, on which any analysis can be based. Practically all the important dams in existence have been designed with this assumption as the starting point.

Referring to Fig. 88, the reaction of the foundation at the

upstream face is less than at the downstream face, because the center of pressure R_v, is nearer the downstream than the upstream face. The length of the line $AB = p$, is the smaller pressure and the length of the line $EC = q$, is the greater, and the pressure increases gradually, from B to C, as indicated by the straight line drawn between these two points. At any intermediate point, as N, the reaction of the foundation is shown by the length of the vertical line erected at N and equal to NY.

Obviously, the average pressure over the whole foundation (per foot length of dam) is $\frac{p+q}{2}$, and the total reaction is the average pressure multiplied by the area over which the pressure acts, which area, for 1 ft. length of dam, is equal to the length through the base, or l.

Hence,

$$R_v = \left(\frac{p+q}{2}\right)l \tag{141}$$

But $\left(\frac{p+q}{2}\right)l$ = area of the pressure trapezoid $ABCE$. Hence, the moment of the trapezoid about any point must be equal to the moment of the weight R_v about the same point.

Taking A as the point of rotation.

Moment of $R_v = R_v d$.

Moment of trapezoid $ABCE$ =

$$\left(\frac{p+q}{2}\right)l \times \frac{l}{3}\left(\frac{p+2q}{p+q}\right) = \frac{l^2}{6}(p+2q)$$

$$R_v d = \frac{l^2}{6}(p+2q)$$

$$p + 2q = \frac{6R_v d}{l^2}$$

Also, from equation 141,

$$p + q = \frac{2R_v}{l}$$

Whence

$$q = \frac{2R_v}{l}\left(\frac{3d}{l} - 1\right) \tag{139}$$

Solving for p from the foregoing equations, this is found to be

$$p = \frac{2R_v}{l}\left(2 - \frac{3d}{l}\right) \tag{140}$$

Figure 89 shows the diagram of foundation reaction for the position of intersection of resultant with the base at the downstream end of the middle third, or $d = \frac{2l}{3}$, making $p = 0$. For this condition,

$$q = \frac{2R_v}{l} \tag{142}$$

Figure 90 shows the diagram of foundation reaction for intersection of the resultant with the base line at a point $\frac{3}{4}l$ from the upstream side.

For this condition,

$$p = \frac{2R_v}{l}\left(2 - \frac{9}{4}\right) = -\frac{R_v}{2l}$$

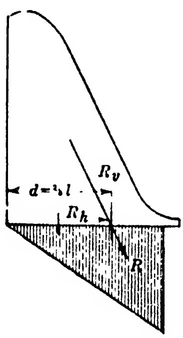


Fig. 89.—Foundation reaction for resultant at end of middle third.

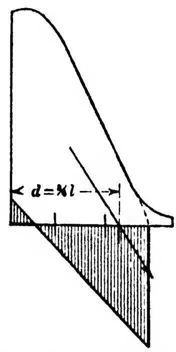


Fig. 90.—Foundation reaction for resultant outside middle third.

which is the *tension* in the masonry at the foundation, which continues to the top, gradually diminishing to zero.

Also,

$$q = \frac{2R_v}{l}\left(\frac{9}{4} - 1\right) = \frac{2.5R_v}{l}$$

In general,

$$p = \frac{\Delta}{l}\left[1 - \frac{6e}{l}\right] \tag{143}$$

and

$$q = \frac{\Delta}{l}\left[1 + \frac{6e}{l}\right] \tag{144}$$

e being the distance from the *middle* of the base to the point where the resultant cuts the base.

l = length of base.

$\Delta = W - U$, W being the weight of the section, and U the total uplift pressure of the section.

It may be noted that U is generally negligible, and $\Delta = W$.

Also, if either p or q is known, the other can be found from

$$p = \frac{2\Delta}{l} - q \tag{145}$$

And

$$q = \frac{2\Delta}{l} - p \tag{146}$$

Note that when p is a tension and negative,

$$q = \frac{2\Delta}{l} - (-p) = \frac{2\Delta}{l} + p \tag{147}$$

The value of q is always the maximum pressure, per square foot, on the foundation, and this must be kept within the limits of safe bearing pressures of the material on which the dam may be built.

These are as given in the following table:

TABLE 31.—SAFE PRESSURES, TONS PER SQUARE FOOT

	Tons per square foot
Granite	28
Sandstone	15
Limestone	11
Shale, hard	8
Shale, soft	3
Hard-pan	3
Blue clay	3

It is not to be assumed that these values are limiting because of any specific factor of safety. They simply represent present practice. As a matter of fact, any ordinary natural stone in its native bed will safely support any artificial masonry structure that can be built upon it.

Ordinary blue clay has withstood loads of 10 tons per square foot safely. Of course, if limestone is much fissured and honeycombed with cavities and cracks—as all limestone in stream beds is liable to be—the full bearing strength can be obtained only by grouting the foundation, as elsewhere described. As it is always necessary to grout a limestone stream bed to prevent percolation under a dam, the bearing pressure of this material may be taken at its full value of 11 tons, or more, per square foot.

Where the dam is constructed on a soft, easily eroded material, the spillway toe must be extended downstream in the form of an apron, or mat, to protect the foundation against erosion from the swirls of the water pouring over the spillway. The general form and proportions of these aprons are given in the paragraph on "Foundations of Dams." The mat or apron described under the caption "Foundations on Sand" is heavier and longer than would be needed for any but sand and soft clay foundations. No general rule can be given for proportioning these mats.

Effect of Under-pressure.—The conventional and accepted ideas concerning the entrance or percolation of water under the base of a solid dam are:

1. The full pressure, due to the head of water backed up by the dam, is exerted under the base, at the upstream edge of the joint.

2. The pressure is zero at the downstream edge of the joint.

3. The diminution in pressure is uniform through the base of the dam from the upstream to the downstream side.

4. The total upward force exerted by the pressure taken over the whole base, is added to the reaction of the foundation against the base of the dam.

5. The center of upward pressure is located at a point $\frac{l}{3}$ from the upstream edge of the dam at the base line, and the net force has therefore, a lever area about the toe $= \frac{2l}{3}$.

Based on these assumptions, many failures of dams have been ascribed to leakage through the foundation, and the necessity urged of designing dams heavy enough to resist this added overturning moment, as well as the other normal ones due to water and ice pressures. How such a weird fancy could have ever obtained the importance of a standard engineering rule of design, is beyond the writer to understand, being contrary to every law of physics, mechanics and common sense.

In order for the full pressure to be exerted under the base of the dam at the upstream edge of the joint it would be necessary for the whole dam to be without bond to, or contact with, the foundation. A good bond is always attempted in construction, and it would require special care to so build a dam that water, even in an infinitely thin film, could enter between the base and the foundation over any appreciable area. It might be that a

very short length would be without a tight base joint, and, as gravity dams are computed for a single foot length, as if each were independent of the other, this possible short section would seem unsafe. The adoption of a 1-ft. sectional length for purposes of computation and analysis is dictated by convenience only. Certainly, if sporadic openings, each a few inches wide and aggregating 10 ft. in length, should occur in the base joint of a dam 500 ft. long, the average uplifting pressure would be only 2 per cent. of the maximum that would exist if the whole base were acted on. Even if the whole 10 ft. were concentrated in one section, the adhesion of the ends of this section to the rest of the structure on either side, would serve to distribute the uplift pressure through the whole mass of the dam.

But assumption (4) is the one for which least excuse exists. This assumes that a structure having a certain weight rests on a supporting surface and that a water pressure interposes itself between the whole area of the base of structure and that of the support and then adds itself to the existing reaction of the support.

This would mean:

(*a*) That the base of a dam could rest only on a film of water and not touch against the foundation.

(*b*) That the water pressure, plus the reaction of the foundation against the base of the dam is the total pressure against the base and the total reaction of the foundation, notwithstanding the fact that, according to the theory of an interposed film of water, the dam would not rest on the foundation, but on a water cushion, and the only foundation reaction possible would be that of the water.

(*c*) That water having a given pressure would spread apart surfaces between which a much greater pressure exists, and add its pressure to that greater one which first existed.

Having pointed out the fallacies of the existing theory, it now remains to show what the actual conditions are.

To make clear the theory, assume, first, an artificial condition of a dam having a series of channels running transversely through the base from the upstream to the downstream side. These channels may be very flat in cross-section, being, in effect, open, horizontal cracks having a definite known width.

If these channels are uniformly distributed over the whole length of the dam, computations can be made for a single foot length as applicable to the whole structure. Assume that

these channels occupy 25 per cent. of the area of the base of the dam, and that the remaining 75 per cent. is solidly sealed to the foundation, so that there is absolutely no percolation. Assume, further, that the cross-section of the channels is varied from the upstream to the downstream side, so that the velocity of the emerging water is small, and the pressure inside of the

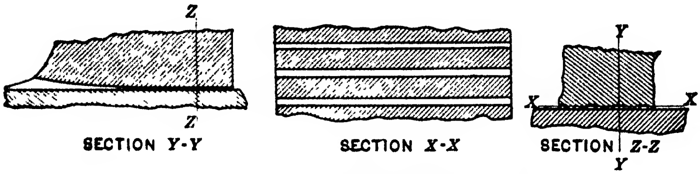


FIG. 91.—Dam with channels through base.

channels, practically zero, at the toe. The cross-sectional variation is, of course, effected by changing the vertical dimensions of the channels. The horizontal dimension is kept constant. This hypothetical structure is shown in Fig. 91. Obviously, it will be the equivalent of a dam having underflow through the base, which has a unit pressure equal to the full

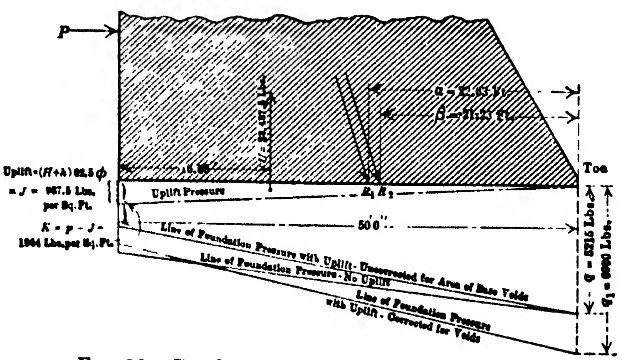


FIG. 92.—Resultants of forces and base pressures.

water pressure due to the head at the upstream side, and zero pressure at the downstream side.

Assume the dam to have a vertical face with height, $H = 50$ ft. Depth of water over the crest $= h = 10$ ft., making the water pressure at the base $= 60 \times 62.5 = 3750$ lb. per square foot. Thickness through base $= 50$ ft.; thickness through crest 8 ft. See Fig. 92, which shows the lower portion of this dam.

The weight of a 1-ft. length of this dam, if made of material weighing 140 lb. per cubic foot, will be $50 \times \frac{(8 + 50)}{2} \times 140 =$ 203,000 lb.

If the dam rests solidly on its foundation the average pressure, per square foot, $= \frac{203{,}000}{50} = 4060$ lb.

If, however, the solid portion of the foundation is only 75 per cent. of the total area of the base, the average pressure on the foundation is, actually, $\frac{4060}{0.75} = 5413$ lb., per square foot. For purposes of computation, it is convenient to use the value 4060 lb. per square foot, as if the whole base had a bearing on the foundation, and transform the final result to its proper value.

For the foregoing conditions, and neglecting the uplift pressure, the resultant will intersect the base at a point 22.63 ft. from the downstream toe. This is best found graphically, though it can be located, algebraically, by the formula,

$$\beta = \frac{2l}{3} - \frac{a^2}{3(l + a)} - \frac{Pz_1}{W} \tag{147}$$

$\beta =$ distance from downstream toe to point where resultant intersects the base.

$z_1 = \frac{H}{3}\frac{H + 3h}{H + 2h} =$ height of center of pressure.

$P =$ water pressure on face of dam, for 1-ft. length $= \frac{62.5(H + 2h)H}{2}$.

$W =$ weight of section, for 1-ft. length.

$l =$ thickness through of the section.

$a =$ thickness through crest.

Substituting the proper values, β is found to be 22.63 ft., which locates the position where the resultant intersects the base.

The pressure, per square foot, at the upstream side (assuming solid contact over all the base) is $p = \frac{\Delta}{l}\left(1 - \frac{6e}{l}\right)$, in which $\Delta =$ net weight acting on foundation and $e =$ distance from the middle of the base to the point where the resultant cuts the base.

$$e = \frac{l}{2} - \beta = 25 - 22.63 = 2.37 \text{ ft.}$$

Hence,

$$p = 4060\left(1 - \frac{6 \times 2.37}{50}\right) = 2905 \text{ lb. per square foot, taken}$$

over the whole area, or $\frac{2905}{0.75} = 3873$ lb. per square foot, of actual base support. At the downstream edge, the pressure is q and is equal to $2 \times 4060 - 2905 = 5215$ lb. per square foot of total base area.

If now, an uplifting force be applied under the dam, due to the water in the channels, the water pressure underneath the dam at the upstream side, will be $62.5 \times 60 = 3750$ lb. per square foot. The under pressure at the downstream side is assumed at zero, hence, the average uplifting pressure is $\frac{3750}{2} = 1875$ lb. per square foot. The total area against which this uplift pressure can act, per foot length of dam, is 50×25 per cent. $= 12.5$ sq. ft., and the total uplift pressure is $12.5 \times 1875 = 23{,}437.5$ lb. per foot length of section.

The center of under pressure is $\frac{l}{3} = 16.66$ ft. from the upstream face of the dam. This force is most easily combined with the existing forces, graphically, by methods previously given.

The new position of the resultant, after the application of the uplifting force, may also be found, algebraically, from formula (134), which is

$$\beta = \frac{l(2\Delta - kl)}{6\Delta}$$

in which

β = horizontal distance from downstream toe to position where new resultant cuts the base.

$\Delta = W - U$.

$k = p - J$.

U = total uplifting force, per foot length, $= \frac{62.5\phi l(H + h)}{2}$.

ϕ = fractional proportion of total base area under which water can penetrate.

W = weight of section, per foot length.

J = pressure, per square foot of base, at upstream side due to uplift $= 62.5(H + h)\phi$.

p = pressure, per square foot of base, due to weight that exists at upstream side when no uplifting pressure is applied.

From the graphical construction, the intersection of the resultant with the base is found to have moved to a point 21.23 ft. distant from the toe. The same result is found from formula (134) by substituting values as follows:

$l = 50$, $W = 203{,}000$, $p = 2905$ lb. per square foot, $J = 0.25 \times 3750 = 937.5$ lb. per square foot, $U = 23{,}400$, $\Delta = 203{,}000 - 23{,}400 = 179{,}600$.

Then

$$\beta = \frac{50[2 \times 179{,}000 + (2905 - 937.5)50]}{6 \times 179{,}600} = 21.23 \text{ ft.}$$

Total weight on foundation $= \Delta = W - U = 179{,}600$ lb.

Average pressure, per square foot of area, of base $= \dfrac{179{,}600}{50} = 3591$ lb.

Pressure at upstream side is,

$p_1 = \dfrac{\Delta}{l}\left(1 - \dfrac{6e}{l}\right) = 3591\left[\left(1 - \dfrac{6(25 - 21.23)}{50}\right)\right] = 1964$ lb. per square foot, taken over whole area of base.

Also, the weight on the foundation at the downstream side is $2 \times 3592 - 1964 = 5220$ lb. per square foot, based on the dam resting over the whole foundation. Since, however, it rests on only 75 per cent. of the base area, the actual pressure, per square foot, on the down stream side is $\dfrac{5220}{0.75} = 6960$ lb. per square foot.

This may also be computed by deducting the value of the unit pressures of the uplifting force at the toe and the upstream side from the unit pressures of the gravity force at those points, to find the net pressures acting on the solid portion of the foundation.

The previous pressure of 2905 lb. at the upstream side, is now reduced to $2905 - 937.5 = 1967.5$ lb. per square foot, of total base area $= \dfrac{1967.5}{0.75} = 2622$ lb. per square foot, of actual contact area, 937.5 lb. being the water pressure under the base, per foot length of dam, when the voids are equal to 25 per cent. of the base. The previous pressure at the toe of 5215 lb., remains unchanged, the value of the uplift pressure at the toe being zero. The average pressure over the whole foundation is

$\frac{1967 + 5215}{2} = 3591$ lb. per square foot of total area, or $\frac{3591}{0.75}$ $= 4789$ lb. per square foot, of actual contact area.

The total pressure for 1-ft. length, 50-ft. base $= 50 \times 3591 =$ 179,550 lb., which corresponds to the total weight on the foundation, per foot length, of $203{,}000 - 23{,}437 = 179{,}563$ lb., which checks with the previous calculation within the limits of the decimal places to which the quantities are carried.

In the case where the existing pressure p on the base at the upstream side of the dam is very small, or zero—which latter is

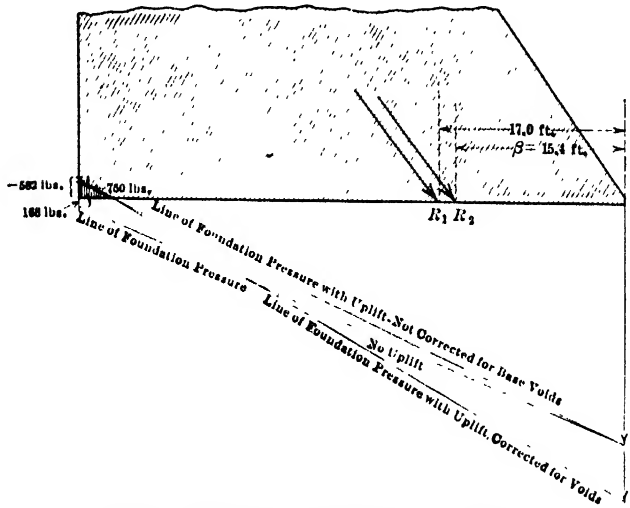


FIG. 93.—Resultants of forces and base pressures.

the case when the resultant intersects the base at the downstream end of the middle third—the uplift pressure, J, being greater than the pressure p, tension is produced at the upstream side. Consider the dam shown in Fig. 93, with a base length of 50 ft. and having its resultant intersect the base at c, very near the end of the middle third, the distance of c from the toe, or β, being 17 ft. The pressure on the foundation at the upstream side of the joint is $\frac{210{,}000}{50}\left[1 - \frac{6(25 - 17)}{50}\right] = 168$ lb. per square foot. $210{,}000 =$ weight of section, and $\frac{210{,}000}{50} = 4200$, is the average pressure, per square foot, over entire base. Pres-

sure at downstream side of joint is $2 \times 4200 - 168 = 8232$ lb. per square foot, taken over entire base. Assume that an uplift pressure be now applied, and that the area of voids penetrated by the water, or ϕ, is 20 per cent. of the whole base. Head of water 60 ft. Then the uplift pressure at the upstream side will be $60 \times 0.20 \times 62.5 = 750$ lb. Deducting this from the existing pressure of 168 lb., there remainh -582 lb. $= k$, the negative sign indicating that the pressure is negative and is, actually, tension. That is, the uplift pressure at the upstream side not only counteracts the pressure due to the existing weight, but exerts an unbalanced force tending to separate the dam from the foundation.

In this case, part of the uplift pressure reduces the weight of the dam, acting on the foundation, and part sets up a tensile stress in the upstream side of the masonry. This gives k, in formula (134), a negative value. Assuming $\phi = 20$ per cent. and $H + h = 60$ ft.

$$U = \text{total uplift pressure} = \frac{50 \times 0.20 \times 62.5 \times 60}{2} = 18{,}750 \text{ lb.}$$

$$\Delta = W - U = 210{,}000 - 18{,}750 = 191{,}250 \text{ lb.}$$

$$\beta = \frac{50[2 \times 191{,}250 + (-582 \times 50)]}{6 \times 191{,}250} = 15.4 \text{ ft.} = \text{distance from}$$

toe to where resultant cuts the base.

$$e = \frac{50}{2} - 15.4 = 9.6 \text{ ft.}$$

Average pressure on base, is

$$\frac{\Delta}{l} = \frac{191{,}250}{50} = 3825 \text{ lb. per square foot.}$$

Pressure on base at downstrean toe is

$$q = \frac{2\Delta}{l} - k = 2 \times 3825 - (-582) = 8{,}232 \text{ lb. per square foot.}$$

But q and k are based on the pressure (or tension) being spread over the whole area of the base, while they are, in reality, applied to only $1 - \phi$ per cent., of the base area. In this case, $1 - \phi =$ 80 per cent. Actual tension at upstream side $= \dfrac{582}{0.80} = 727.5$ lb. per square foot. Pressure at downstream side $= \dfrac{8232}{0.80} =$ 10,290 lb. per square foot.

In the foregoing examples, the values of ϕ taken were made great to illustrate the principles set forth. In practice, the value of ϕ could not reach 20 or 25 per cent. except under extraordinary conditions.

These methods of analysis differ from the usually adopted ones, only in:

(*a*) The assignment of the coefficient ϕ to the uplift pressure, and which coefficient represents the ratio of area of the voids underneath the base of the dam to the whole base area.

(*b*) The use of the coefficient $\frac{1}{1 - \phi}$ to determine the unit pressures of the dam on the foundation. Some arbitrary coefficient, to determine the uplift pressure, has been in general use, but it has been applied as a factor which was meant to give the ratio of the water pressure at the up stream edge of the base, to the average water pressure under the base, and, while it worked somewhat in the same fashion as the rational area relationship, it was vague, and from its very vagueness was made unnecessarily large—becoming, in reality, a factor of ignorance.

To Recapitulate.—1. Water enters under the base of a dam only through construction or contraction voids, which voids would exist even if the dam were above water, and dry.

2. The only uplift pressure possible is that due to the under pressure, multiplied by the area of the voids cut by a horizontal plane through the base.

3. The water pressure under the base, if existent, may, or may not, diminish with approximate uniformity from the upstream to the downstream side. This pressure variation is peculiar to each individual structure.

4. The unit pressures of a dam on its foundation are proportional to the whole base area, minus the area of the voids cut by a horizontal plane through the base.

5. The horizontal cross-section of the voids in the base of any gravity dam, can not be sufficiently great to be an appreciable proportion of the total area, no matter what the construction. In an ordinarily well-built dam they should be negligible and, certainly, under 5 per cent. Probably in no single short section of, say, 2 ft. length, would the area of the voids exceed 40 per cent.

6. The effect of the uplifting force is substantially negligible and any failure of the dam, by overturning, must be assigned to other causes.

To these statements there may be general assent so far as they relate to the dam itself, and its actual base contact with the foundation, but it may be alleged that the voids, fissures and breaks in the foundation might be sufficiently great in number and area to allow the production of a dangerous uplift pressure. To any such reasoning, the reply is that it is not proper engineering to compensate for a fault in one element by an over-strengthening of some other one. If the foundation is faulty, correct the fault by thorough grouting. This is cheaper than building a costly dam of excessive volume and base thickness. Furthermore, a bad foundation, unfilled and unstrengthened will finally lead to disaster, even though the dam be thick and heavy. The natural recourse, in the case of bad foundations, is to build a hollow, steel-reinforced, concrete structure, as is shown later.

Classification of Types of Dams.—In general, there are three broad classes of dams.

One is that in which the weight of the material, of which the dam is composed, resists the forces set up by the water pressure. This type is called the "gravity dam," though some writers persist in calling the hollow, inclined-deck dam, a gravity dam.

The second is the hollow dam having an inclined upstream face which depends, principally, on the vertical component of the water pressure to hold it in place.

The third is the kind which is curved in plan and resists the forces, set up by the water pressure, by acting as an arch.

There may be mixed types, combining two or all three of these broad elements.

Gravity Dam.—Referring to Fig. 84, the center of gravity of the cross-section is at G. Consequently, the weight of the dam may be considered to be concentrated at G and acting vertically downward. Considering 1 ft. length of dam, the area of the cross-section of the dam, in square feet, will give the number of cubic feet, per foot length. This number of cubic feet, multiplied by the weight of the material, per cubic foot, gives the value of the vertical force acting through G.

The weight, per cubic foot, of masonry varies, but is in practical accordance with Table 32.

Calling ω the weight, per cubic foot, and A the area of the cross-section of the dam, in square feet, the weight, per foot length, is ωA.

TABLE 32

	Lb. per cubic foot
Cut stone laid in mortar, granite.........	170
Cut stone laid in mortar, sandstone......	150
Cut stone laid in mortar, limestone.......	160
Cyclopean masonry....................	140
Concrete............	140

Resistance to Sliding.—The force which tends to shove the dam downstream is $\frac{1}{2}62.5H^2$ lb. per foot length of dam. If the dam be considered as merely resting on its foundation, with nothing to hold it against the water pressure but the frictional resistance between the base of the dam and its foundation, the resistance to sliding, per unit length of dam, will be $\phi\omega A$, in which ϕ = coefficient of friction.

For the conditions which usually obtain, ϕ = (approximately) 0.65 to 0.75. Taking the smaller figure, if the factor of safety against sliding be 2, then

$$0.65\omega A = 2 \times 62.5 \frac{H^2}{2}$$

and, therefore,

$$A = \frac{62.5H^2}{0.65\omega}$$

Thus, for a dam of concrete having a vertical face 50 ft. high, only resting on the foundation,

$$A = \frac{2500 \times 62.5}{140 \times 0.65} = 1717 \text{ sq. ft.}$$

This indicates an excessive cross-section and shows that a dam having a vertical face must be sunk well into the foundation so that there must be shear of the dam or its foundation before sliding can occur.

For a dam having a sloping upstream face, the weight on the foundation is increased, due to the vertical component of the water. Thus in b, Fig. 82, the force due to the weight of the cross-section of the dam, per foot length, is given by the vertical line Gs acting through the center of gravity G. The force due to the vertical pressure of the water, P_v, is equal to $\frac{1}{2} \times 62.5H^2 \tan\theta$ which, added to the force Gs, gives the force sK, making the total vertical pressure

$$W_t = \omega A + \tfrac{1}{2} \times 62.5H^2 \tan\theta \tag{148}$$

The total resistance to sliding then becomes

$$\phi(\omega A + \tfrac{1}{2} \times 62.5H^2 \tan\theta)$$

This value should be not less than twice the pressure which would cause sliding, *i.e.*

$$\phi(\omega A + \tfrac{1}{2} \times 62.5H^2 \tan \theta) \geq 2(\tfrac{1}{2} \times 62.5H^2)$$

whence

$$A \geq \frac{62.5H^2}{\phi\omega}(1 - \tfrac{1}{2}\phi \tan \theta) \tag{149}$$

For example, if a dam were 50 ft. high and the slope of the upstream face were 30° and the coefficient of friction 0.65, the cross-sectional area to prevent sliding, with a factor of safety of 2, should be not less than

$$\frac{62.5 \times 2500}{0.65 \times 140}(1 - \tfrac{1}{2} \times 0.65 \times 0.5773) = 1395 \text{ sq. ft.}$$

This area is also excessive for the cross-section of a 50-ft. dam, as will appear from formulæ given later.

The approximate cross-section for a 50-ft. dam, having a top thickness of 8 ft., is about 1250 sq. ft. Hence, for a factor of safety of 2, the slope of the upstream face for a dam 50 ft. high and made of concrete, must be not less than 51½°, if the dam is not sunk into the rock and simple sliding friction only, is depended on to resist the horizontal thrust of the water.

This value of θ is derived as follows:

An approximate formula for the area of a section H ft. high, is

$$A = H\left(\frac{a}{2} + 0.333H\right) \text{sq ft.}$$

in which

$$a = \text{thickness of dam at the top.}$$

equating this to the value of A required to give a factor of safety of 2 against sliding

$$A = H\left(\frac{a}{2} + 0.333H\right) = \frac{62.5 \times H^2}{\phi \times 140}\left(1 - \frac{\phi \tan \theta}{2}\right)$$

whence

$$\tan \theta = \frac{2}{\phi} - s\left(\frac{a}{H} + 0.666\right) \tag{150}$$

s = specfic gravity of material of which dam is made. For concrete

$$s = \frac{142}{62.5} = 2.28$$

$$\tan \theta = \frac{2}{\phi} - 2.28\left[\frac{a}{H} + 0.666\right]$$

For $\phi = 0.65$, $a = 8$ ft. and $H = 60$ ft.

$$\text{Tan } \theta = \frac{2}{0.65} - 2.28\left(\frac{8}{60} + 0.666\right) = 1.253.$$

Therefore, $\theta = 51\frac{1}{2}°$ (approximately).

Of course, A does not vary, strictly, as H^2, but it is so nearly proportional to it that the minimum value of θ, as found from equation (150), will change but little, for various heights of dams, unless the value adopted for ϕ is changed.

A slope of 51° is excessive for any except certain types of wood-frame dams. Therefore, it is clear that any commercial form of masonry dam must have its base sunk into the rock, or hard-pan, which forms the bearing surface, so that actual shear must take place before sliding is possible.

For unreinforced concrete, or masonry, a safe shearing stress is 50 lb. per square inch, or 7200 lb. per square foot.

Hence, the thickness, T of the base, for vertical face dams of concrete, must satisfy the equation

$$\frac{1}{2} \times 62.5H^2 = 7200T \tag{151}$$

whence

$$T = \frac{62.5H^2}{2 \times 7200} = 0.00434H^2 \tag{152}$$

For a dam 50 ft. high, therefore,

$T = 0.00434 \times 2500 = 10.85$ ft., which is less than one-fourth the thickness required to resist overturning.

The formulæ and example show that if a dam is properly designed to resist the other forces acting on it, and has its base sunk into the foundation earth, or rock, it is amply strong to resist sliding, for H less than 200 ft.

The foregoing formulæ apply only to dams that have no water flowing over the crest. Since most dams constructed for power purposes, have water overtopping them, the proper values for sliding pressure as given in equation (123) should be used in all computations.

Wherever horizontal joints are made, they should be stepped off, or broken, in some effective manner so that the full value of the resistance to shear of the material, can be secured.

Resistance to Overturning.—Referring to Fig. 94, the center of gravity is at G and the horizontal distance from this point to the vertical line through the toe of the dam, is L, as shown. The moment, resisting overturning, is usually taken as equal to

the weight of the dam multiplied by the lever arm having a length equal to the horizontal distance from from the center of gravity to the toe. Considering 1 ft. length of dam, the weight is ωA, in which A is the area of the cross-section of the dam and ω is the weight of the masonry per cubic foot. If L is the horizontal distance from the center of gravity to the toe, the moment due to the weight is $L\omega A$.

The overturning moment for dams not overtopped by water is $10.4H,^3$ and, for a factor of safety of 2,

$$L\omega A = 2 \times 10.4H^3 \qquad (153)$$

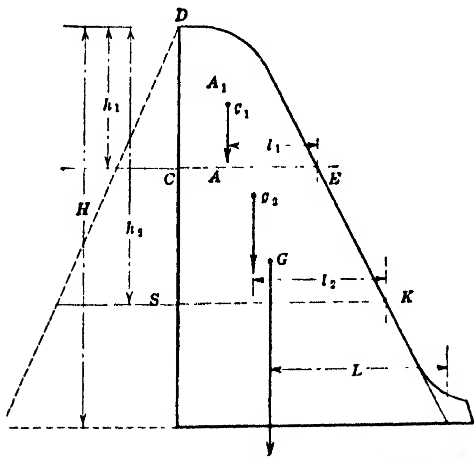


Fig. 94.—Computation of strength of dam through horizontal sections.

Taking the average value of ω at 140 lb.,

$$LA = 0.1486H^3 \qquad (154)$$

For dams with inclined upstream faces, the vertical component, H_v, of the water pressure, acts against the face of the dam at the center of gravity of the pressure triangle, or trapezoid. It has a lever arm from its point of application back to the line of the toe, equal to the horizontal distance represented by L_1 in *b*, Fig. 82.

The resistance to overturning then becomes

$$L\omega A + H_vL_1 \text{ or } L\omega A + \tfrac{1}{2} \times 62.5H^2L_1 \tan \theta \qquad (155)$$

in which, θ = angle between the face of the dam and a vertical line.

If a dam has an inclined upstream face and a maximum height of water over its crest h, with a factor of safety of 2 against overturning, the equation becomes

$$2 \times 62.5H^2 \left(\tfrac{1}{6}H + \tfrac{1}{2}h\right) = L\omega A + \tfrac{1}{2} \times 62.5H^2L, \tan\theta \quad (156)$$

whence

$$A = \frac{62.5\,H^2}{2L\omega}\left[\frac{2H}{3} + 2h - L, \tan\theta\right] \quad (157)$$

In computing the cross-section required to resist overturning, it is not sufficient to take into account only the moment of the entire cross-section of the dam about the toe.

To proportion a dam properly it is sometimes necessary to make computations for several sections—not less than three, and usually, at vertical intervals of 10 ft. This is done by dividing the figure into the number of horizontal sections desired. Fig. 94 is divided into three sections, as shown, the first being from the top down to the line CE, the second from the top down to the line SK, and the third from top to bottom, including the entire structure. Considering the first section, the overturning moment of the water is $10.4h_1^3$, h_1 being the depth of water down to line CE. Let the area of section of the dam, DEC, included between the upper edge and the line CE, be equal to A_1, and the horizontal distance between its center of gravity and the rear face of the dam where CE intersects it, equal to l_1, the weight per cubic foot of material being ω, then the resistance of the upper section to overturning, about the line CE, is $l_1\omega A_1$. This must be greater than the overturning water pressure $10.4h_1^3$. Similarly, the overturning water pressure about line SK is $10.4h_2^3$, h_2 being the depth of water down to line SK. The resistance to overturning is $l_2\omega A_2$, in which A_2 is the area of section from the top down to line SK, and l_2 is the horizontal distance of the center of gravity of this section from the rear surface of the dam. In the same manner, the total water pressure and resistance to overturning of whole dam are computed.

All the computations should show the dam amply strong at every point. If any section taken shows too small a resisting moment, the dam must be thickened at that section until the calculations show it to be safe.

The foregoing analysis is in accordance with most works on the subject of dams. It is partly misleading in that it assumes

an enormous mass as turning on a thin corner at the toe, which is impossible. In the case of actual overturning, the toe would begin to crush under the heavy unit pressures imposed on it and as the dam continued to turn over, the masonry at the toe would continue to crush, so that the rotation would take place about a roughly rounded toe and the lever arm would be, progressively, diminished.

It, however, is the only way known for computing the resistance to overturning, and when the resistance to crushing is considered, the formulæ are probably not greatly in error.

Design of Gravity Dams.—The cross-section of a gravity dam may be designed by trial and error methods. However, there are a few simple, approximate formulæ, from which the section of a dam may be quickly computed, and the approximate section, thus found, corrected by trial and error, with much less labor than is required if the calculations are made without a basic outline from which to work. There are also, general, exact formulæ, which are given later herein.

The height of the dam should be divided into vertical sections, and intermediate base lines drawn through each section. All that portion of the dam lying above the first (top) section should be treated and analyzed as a dam having a height equal to the height of the water level above the first base line. The second section (next to the top) should be treated in the same manner, remembering that the second section comprises the first or top section as a portion of its cross-section. Continue in this way to the bottom or natural base line of the dam.

There should be at least three sections taken on any dam, and more where the height of the dam exceeds 30 ft. A good height of section is 10 ft. If the height of the dam exceeds 30 ft. but is not a multiple of 10, the last or lowermost section should be the one having the height that is not exactly 10 ft.

The formulæ for a gravity section having a trapezoidal form and a vertical upstream face, in which the resultant force will cut the base just at the downstream end of the middle third, are as follows:

For the top section.

$$l = \frac{H}{\sqrt{s}} \text{ if the top is rectangular.} \tag{158}$$

$$l = \frac{H}{1.08\sqrt{s}} \text{ if the top is parabolic.} \tag{159}$$

l = distance from front to rear face, *i.e.*, thickness of dam in feet.

H = depth of section below crest.

s = specific gravity of material of which dam is constructed

$= \frac{\omega}{62.5} = 2.24$ for concrete.

These formulæ apply only to cases where the dam is not overtopped with water.

For dams having a thickness of water, h, over the crest, the approximate formula for the uppermost section of a parabolic topped dam is

$$l = \frac{H_e}{1.08\sqrt{s}} \tag{160}$$

in which

$$H_e = \sqrt[3]{H^3 + 3H^2h} \tag{161}$$

H = height from base line of section to top.

For concrete, formula (160) reduces to

$$l = 0.666\ H_e. \tag{160a}$$

For overflow dams, always use the value for H_e in the formulæ for thickness through a section.

These formulæ are all approximate only. Also, they are for dams without uplift under the base, or ice pressure at the top.

Practically all power dams have a rounded top over the greater portion of their length, which acts as a spillway, or weir, while a smaller length, higher than the spillway section, is square-topped, and technically known as the "bulkhead" section. The curve of the top of the dam should be that of the underside of a free nappe when the maximum head of water is passing over the crest.

An arc of a circle is used in practice, for both the parabolic curve at the top and the curve in the toe at the bottom. While a circular arc is the proper curve for the toe of the dam, it is advisable to make the top curve a true parabolic section.

The path of the nappe is parabolic, and the ordinates of the curve are computed from the table and formulæ previously given in the chapter on "Weirs and Orifices."

The approximate equation

$$x^2 = 1.78yh \tag{162}$$

will, however, give a curve that is sufficiently close for every practical purpose.

h = height of water over the crest of dam.
x = horizontal abscissæ of curve.
y = vertical ordinates of curve.

If the velocity of approach is above 4 ft. per second, this should be included in the formula. In this case,

$$h_1 = h + \frac{Va}{2g}\left(0.25\sqrt{h} + Va\right) \tag{163}$$

Va being the velocity of approach. This value of h_1 should be substituted for h in formula 162.

The vertex of the parabola is at the highest point on the crest, and the x values are measured horizontally in a downstream direction, while the y values are measured vertically downward from the horizontal line at the elevation of the highest point of the crest.

The radius of the circular arc which forms the bottom of the spillway at the toe seems to be a matter of individual preference on the part of the designer.

Good practice makes the radius $= \frac{H}{2}$, and the center is located at this height, vertically above the end of the toe, as shown in Fig. 95. The end of the toe will be a distance equal to $R \cos \theta$, downstream from the tangent point on the rear face of the dam, R being the radius adopted, and θ being the angle the line of the downstream face makes with the vertical. The tangent point will be at a height, measured along the downstream face from the toe, equal to

$$\sqrt{a^2(1 + \tan^2 \theta)}, \tag{164}$$

in which

$$a = T + \frac{H}{2}(1 - \sin \theta) \tag{165}$$

T = vertical thickness of end of toe, *i.e.* is the vertical distance from the base line to the top surface of toe at its downstream end.

Frequently, the toe is shortened and not carried out to a point where the water is discharged horizontally. This results in a small saving, and where the spillway discharges into a stream

bed of fairly good rock, there is no objection to shortening the toe.

As an example of the use of these formulæ, take the design of the cross-section of a dam to fulfill the following conditions.

H = height of crest above foundation = 60 ft.
h = maximum height of water over spillway = 12 ft.
Va = velocity of approach of water, at maximum discharge = 10.1 ft. per second.

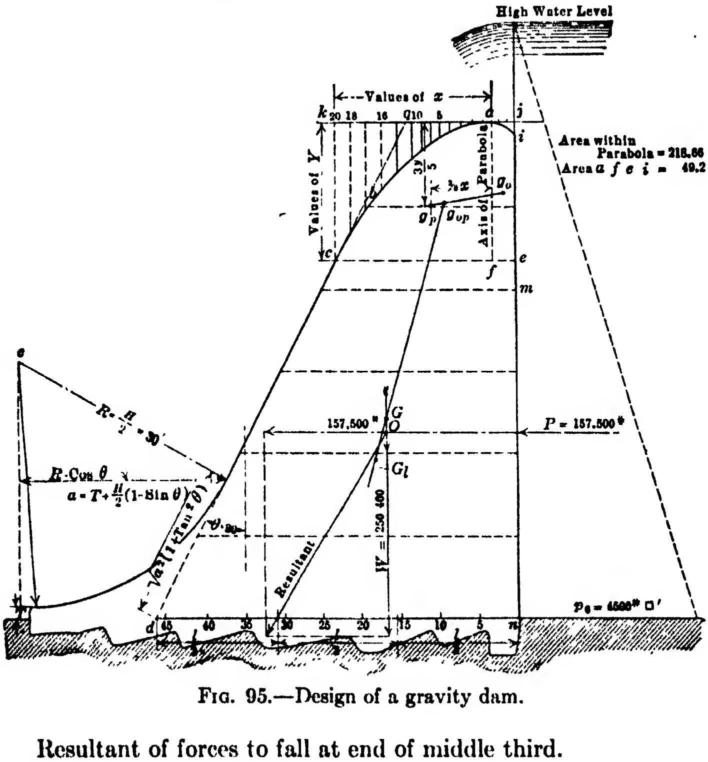


Fig. 95.—Design of a gravity dam.

Resultant of forces to fall at end of middle third.
Factor of safety, approximately = 2.
Material—concrete.

ω = weight, per cubic foot, of material = 140 lb. per cubic feet.

s = specific gravity of material = $\frac{140}{62.5}$ = 2.24.

Total pressure acting on dam, per foot length =

$$\frac{(60 + 2 \times 12)\ 60 \times 62.5}{2} = 157{,}500 \text{ lb.}$$

Figure 95 shows the section of this dam.

First, round off the upper, upstream corner, so that the highest point of the spillway is 3 ft. back from the face of the dam at *a*. This rounding should begin 18 in. below the level of *a* as indicated.

Next, compute the parabola for the top section.

From formula (163),

$$h_1 = 12 + \frac{10.1}{2g}\left(0.25\sqrt{12} + 10.1\right) = 13.768.$$

From formula (162), $x^2 = 1.78 \times 13.768 \times y = 24.42y$

For $x = 1$ ft., $y = \frac{1}{24.42} = 0.041$ ft.

For $x = 2$ ft., $y = \frac{4}{24.42} = 0.1636$ ft.

For $x = 3$ ft., $y = \frac{9}{24.42} = 0.368$ ft.

For $x = 4$ ft., $y = \frac{16}{24.42} = 0.654$ ft.

For $x = 5$ ft., $y = \frac{25}{24.42} = 1.02$ ft.

In this manner, the following additional values of y are found:

For x =	6	7	8	9	10	12	14	16	18	20
y =	1.47	2.01	2.62	3.31	4.09	5.9	8.02	10.5	13.26	16.4

These values are plotted in Fig. 95, beginning at the point *a*, which is the vertex of the parabola. The resulting curve is *abc*, as shown.

At a depth of 10 ft. below the crest, the thickness through the dam is 18.6 ft., which thickness is fixed by the parabolic curve. The thickness required to cause the resultant of all the forces to fall within the middle third of a section, whose base line is 10 ft. below the crest is,

$$l_1 = 0.666\ H_{e1}, \text{ where } H_{e1} = \sqrt[3]{(10)^3 + 3 \times 10^2 \times 12} = 16.74$$

$l_1 = 0.666 \times 16.74 = 11.16$ ft. Hence, the dam is $18.5 - 11.16 = 7.34$ ft. thicker at the 10-ft. level than strength requires. This, however, is not a loss of masonry. The extra weight helps to produce a higher gravity moment for the whole dam, and will, therefore, permit the masonry to be made thinner at some lower section.

The thickness through the bottom section at the foundation base line may now be approximated.

$l_6 = 0.666 H_{e6}$. $H_{e6} = \sqrt[3]{(60)^3 + 3 \times (60)^2 \times 12} = 70.2.$
$\therefore l_6 = 0.666 \times 70.2 = 46.75$ ft.

This length is laid out and from the end of the base line thus found, at point d, a straight line is drawn to point c, at which the parabolic section is stopped. The point of ending the parabola is arbitrary. Usually it is carried down until the line, cd, to the downstream end of the base, is tangent to the curve, the line and curve connecting smoothly. $x = 1.6h$ is, usually, a sufficient distance to extend the curve.

The area of the cross-section from the topmost point of the crest down to the end of the parabolic curve, or the area $abcfa$, is $\frac{2}{3}xy = \frac{2}{3} \times 20 \times 16.4 = 218.66$ sq. ft. This follows from the fact that the area included between a parabola and its axis is $\frac{2}{3}xy$. The area $afej$ is $3 \times 16.4 = 49.2$ sq. ft., neglecting the rounded corner. Total area down to line $ce = 267.86$ sq. ft.

Area below line $ce = \left(\frac{23 + 46.75}{2}\right) 43.6 = 1520.5$ sq. ft.

Total cross-sectional area of whole dam $= 1788.36$ sq. ft.

Total weight, per foot length $= 1788.36 \times 140 = 250{,}400$ lb.

The center of gravity is located with sufficient accuracy by prolonging the tangent forming the downstream face until it intersects the horizontal line drawn through the top of the crest, and treating the figure as a trapezoid.

However, in this case, partly for greater accuracy and partly to show how the rules for finding centers of gravity are used, the exact method will be employed.

Taking first, the parabolic section included between the curve abc and its coördinates, af and ak, the center of gravity is $\frac{3 \times 20}{8} = 7.77$ ft. horizontally from the axis af of the parabola and $\frac{3 \times 16.4}{5} = 9.84$ ft. vertically, from upper horizontal line ak, and lies at g_p (see Fig. 77). Considering $afei$ as a parallelogram, its center of gravity lies at g_o, its geometrical center.

Area, parabolic section $= 218.66$.

Area, rectangular section $= 49.20$.

Length of line $g_p g_o = 9.33$ ft., by scale.

Then g_{op} the center of gravity of all that portion lying above the line ce, lies on line g_pg_o and at distance, x, from g_p such that $218.66x = 49.2\,(9.33 - x)$ (Equation (114)).
Whence

$$x = 1.715$$

which locates g_{op}.

Center of gravity of trapezoid $cdne$ is found graphically. It lies at G_l. The center of gravity, G, of the whole dam lies on the line joining G_l and g_{op}.

Length of line $G_l\,g_{op}$, from scale $= 32$.

Area, upper portion $= 267.86$.

Area, trapezoidal portion $= 1520.5$.

Distance of G from $g_{op} = x$

$$267.86x = 1520.5\,(32 - x)$$
$$x = 27.21 \text{ ft.}$$

which locates G.

Combining the water and gravity forces, the resultant is found, and it cuts the base just inside the middle third.

Each section can now be separately analyzed to discover if the dam is too thin at any point in its height. For this type of dam with only the water and gravity forces acting, any further analysis is unnecessary because the topmost section has been found thicker than necessary and the bottom, or whole section, as heavy as the middle third requirement demands. The whole section, from the top down to the base, being bounded by straight lines, it is obvious that each section will show an excess thickness, this excess increasing with the height above the base.

The total resistance to overturning is
$250{,}400 \times 30 \doteq 7{,}512{,}000$ lb.-ft., 30 being the horizontal distance from the center of gravity of the section to the downstream end of the base.

The overturning moment is $62.5 \times (60)^2 \left(\frac{60}{6} + \frac{12}{2}\right) = 3{,}600{,}$-000 lb.-ft.

Factor of safety $= \frac{7{,}512{,}000}{3{,}600{,}000} = 2.08$ which is 4 per cent. greater than the factor of safety required.

It is to be noted that where the resultant falls just at the end of the middle third, the factor of safety against overturning,

(neglecting the effect of the toe) is about 2. While it is customary to neglect the effect of the toe, it, unquestionably, adds considerably to the strength against overturning. Although its cross-section is small, and it will break through under a comparatively small force, the break will not occur at the exact face of the dam, but at some distance downstream from it, so that it adds to the lever arm of the gravity force and increases the moment against overturning.

Exact Formulæ for Solid Dams.—The exact formulæ for the thickness of a gravity section at any point in its height, are as follows:

Thickness of section required to make the resultant cut the base at the downstream end of the middle third of the section taken, is

$$l = \sqrt{\frac{A + B + 0.096p}{C} + \left(\frac{D}{2C}\right)^2} - \frac{D}{2C} \qquad (166)$$

Values of the symbols are:

For vertical upstream face.	For inclined upstream face.
$A = H(H + 3h)$.	$H(H + 3h)(1 + \tan^2 \theta)$.
$B = sa^2$.	$sa(a + 2H \tan \theta)$.
$C = s - \phi$.	$s - \phi$.
$D = sa$.	$sa + \tan \theta[2(H + 2h) - sH]$.

l = thickness of section.

H = height of section.

h = height of water level above crest.

s = specific gravity of material = $\frac{\text{weight per cubic foot}}{62.5}$ = 2.24 for concrete.

ϕ = per cent. of total area of base, under which it is assumed water can penetrate, and expressed as a decimal fraction.

a = thickness of crest, in feet, as arbitrarily adopted.

θ = angle the upstream face of the dam makes with the vertical.

p = force due to ice pressure, per foot length of dam.

Note that when h has a value, $p = 0$, and when p has a value, $h = 0$.

Formula for thickness of section for a given factor of safety against overturning.

$$l = \sqrt{\frac{F}{E}\left[A - 0.096p\right] + \frac{J}{E} + \left(\frac{K}{2E}\right)^2} - \frac{K}{2E} \qquad (167)$$

The values of these symbols are:

For vertical upstream face.	For inclined upstream face.
$A = H(H + 3h)$.	$H(H + 3h)$.
$E = 2(s - \phi F)$.	$2(s - \phi F)$.
F = factor of safety.	factor of safety.
$J = sa^2$.	$sa^2 + \tan\theta H[2sa + \tan\theta\,(H + 3h)]$.
$K = 2sa$.	$2sa + \tan\theta[3(H + 2h) - sH]$.

a, s, p, θ, H and h are as just previously given.

Formula for factor of safety of a given section against overturning

$$F = \frac{2sl^2 + Kl - J}{A + 0.096p + 2\phi l^2} \tag{168}$$

For strict accuracy, the formula should take account of the rounding and parabolic form of the top. Also, the exact value of E is $2\left(s - \phi F\left(\frac{H + h}{H}\right)\right)$. Since, however, ϕ is arbitrarily assumed, it is simpler to take a value of ϕ equal to $1 + \frac{h}{H}$ times the value actually required.

The factor of safety assumed may, or may not, bring the resultant of all the forces within the middle third. Hence, a computation should be made with formula (166), and the value for length of base, as given by it, compared with the length of base required for the desired factor of safety, and that which is found to be the greater, adopted.

When gravity dams are of considerable height, say 100 ft. or more, the pressures near the base may be too great for safety against crushing, even though the resultant may fall within the middle third, and the factor of safety against overturning large, considering the section on the basis of mass only.

The minimum thickness through any section, such that the dam may be safe against excessive crushing stresses, is given by the formula

$$l = \sqrt{\frac{6PZ + a^2\omega H}{S}} \tag{169}$$

$$P = \text{pressure against one foot length of dam} = \frac{62.5H(H+2h)}{2}$$

$$Z = \text{height of center of pressure} = \frac{H}{3}\cdot\frac{H + 3h}{H + 2h}$$

ω = weight of cubic foot of material = 140 lb. for concrete.
S = pressure material will sustain safely, per square foot.

H, and a are as given in preceding formulæ.

Values for l and H are for any point in the height of the dam.

As an example to illustrate the use of these formulæ, and also to compare the value of l computed from them with that obtained by using the simpler, approximate formulæ, take the same example as before given, namely,

$H = 60$ ft. $h = 12$ ft. $a = 8$ ft. $\theta = 0$.
$p = 0$. $s = 2.24$. $F = 2.08$. $\phi = 0$.

To find length of base so that the resultant cuts it just at the end of the middle third. Equation 166.

$$A = 60(60 + 3 \times 12) = 5760$$
$$B = 2.24 \times 8^2 = 143.4$$
$$C = 2.24 - 0 = 2.24$$
$$D = 2.24 \times 8 = 17.92$$
$$l = \sqrt{\frac{5760 + 143.4 + 0}{2.24} + \left(\frac{17.92}{2 \times 2.24}\right)^2} - \frac{17.92}{2 \times 2.24}$$
$$\doteq \sqrt{2642 + (4)^2} - 4 = 47.56 \text{ ft.}$$

By the approximate formulæ, l was found to be 46.75 ft., or 0.81 ft. less than given by the accurate formula, which is a difference of about 2 per cent.

To find the thickness through the base for a factor of safety against overturning equal to 2.5, of a dam having the following constants:

$H = 50$ ft., $h = 10$ ft., $\phi = 0.24$, $\theta = 30°$.
$a = 8$ ft., $s = 2.24$, $sa = 17.92$, $p = 0$.
$\tan\theta = 0.5774$.

Then, from equation (167),

$$A = 50\,(50 + 3 \times 10) = 4000.$$
$$E = 2\,(2.24 - 2.3 \times 0.24) = 3.28.$$
$$J = 2.24 \times 8^2 + 0.5774 \times 50\,[2 \times 17.92 + 0.5774\,(50 + 2 \times 10)] = 143.4 + 28.87\,[76.26] = 2345.$$
$$K = 2 \times 17.92 + 0.5774\,[3\,(50 + 20) - 2.24 \times 50)] = 35.84 + 0.5774\,[98] = 92.42.$$
$$l = \sqrt{\frac{2.5}{3.28}\,[4000 + 0] + \frac{2345}{3.28} + \left(\frac{92.42}{2 \times 3.28}\right)^2} - \frac{92.42}{2 \times 3.28} = \sqrt{3764 + (14.09)^2} - 14.09 = 48.85 \text{ ft.}$$

Figure 96 shows the approximate cubical contents of vertical faced gravity sections, per foot length, for different heights of dam and overfall. Of course, the quantity of masonry in any section, for a given factor of safety, will vary with the thickness of crest assumed and the specific gravity of the masonry. Hence, the curves shown are only general guides for preliminary computations.

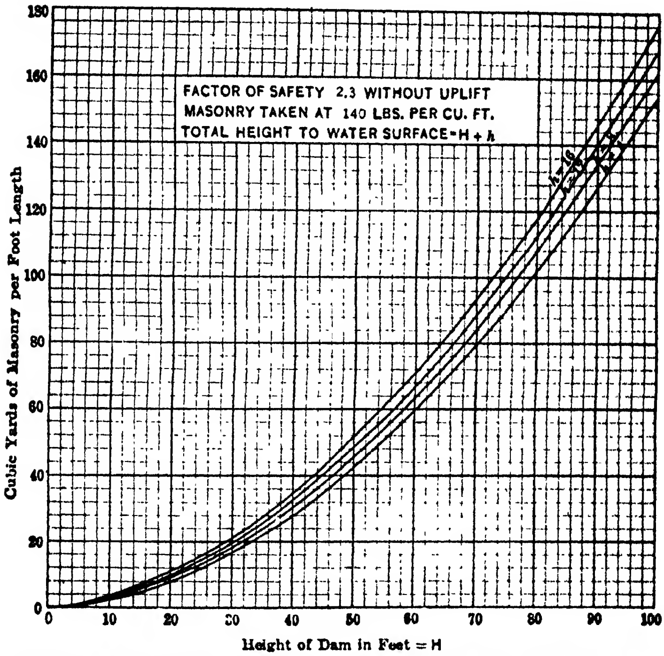


Fig. 96.—Approximate cubic contents of solid dams with vertical upstream face. Spillway section.

It now remains to be seen whether the pressures on the foundations in the two preceding examples are within safe limits.

From equation (139), the maximum foundation pressure at the toe of the dam is

$$q = \frac{2R_v}{l}\left(\frac{3d}{l} - 1\right) \text{ lb. per sq. ft.}$$

l = length through base of dam.
R_v = net vertical pressure of dam, per foot length.
d = distance of resultant from upstream face.

For the first example, $R_v = W = 250{,}400$ lb.

$$l = 47.56 \text{ ft.}$$

$$d = \frac{2l}{3}.$$

$$q = \frac{2 \times 250{,}400}{47.56}\left(\frac{3 \times \frac{2l}{3}}{l} - 1\right) = 10{,}440 \text{ lb. per square foot.}$$

This is well within the limits of safe pressure for any kind of rock foundation.

In the second example, the total value of the vertical forces acting is $w + W - U$, in which U is the uplift pressure.

w = the weight of the water on the inclined deck =

$$62.5 \tan \theta\, H\left(\frac{H}{2} + h\right)$$

$$= 62.5 \times 0.5773 \times 50\left(\frac{50}{2} + 10\right) = 63{,}142 \text{ lb. per foot length.}$$

W = weight of dam

$$= 140 \times 50 \times \frac{(8 + 48.85)}{2} = 198{,}975 \text{ lb. per foot length.}$$

Total vertical force is

$$W + w = 262{,}117 \text{ lb. per foot length.}$$

$$U = \frac{0.24 \times 50 \times 62.5 \times 48.85}{2} = 18{,}350$$

Net weight on foundation $= W + w - U = 243{,}767$, or, in round numbers, 243,800 lb.

By constructing the diagram of forces, it is found that the resultant intersects the base at a distance = 16.65 ft. from the toe and 32.2 ft. from the upstream face.

$$\frac{3d}{l} = \frac{3 \times 32.2}{48.85} = 1.9774.$$

$$q = \frac{2 \times 243{,}800}{48.85}(1.9774 - 1) = 9{,}756 \text{ lb. per square foot}$$

of base.

Of the base, the actual area in contact with the foundation is $1 - 0.24 = 76$ per cent.

$$\text{Pressure per square foot} = \frac{9{,}756}{0.76} = 12{,}839 \text{ lb. per square foot.}$$

This pressure is not excessive for any natural rock.

It is feasible and economical to design a dam with an overhanging upstream face, provided the tension that would be set up in the toe when the reservoir is empty, be not too great. If this tension is kept within the limits of the working tensile strength of masonry or concrete—say 4000 to 6000 lb. per square foot—it is safe to so design a dam, particularly as the condition of empty reservoir will not ever exist after the dam is completed and the lake filled.

The old and conventional school of dam designers has fixed as one of the conditions of design, that the resultant must fall within the limits of the middle third when the reservoir is empty. Since this is a special and unusual condition and no disaster could follow injury to the dam, if any should occur, no expenditures for labor and material are justified to produce a structure any more stable than one in which tension exists in the down-stream face within safe limits, when unsubmerged. The Hauser Lake dam shown in *a*, Fig. 99, is an example of a large gravity section designed with an overhanging, upstream face.

Bulkhead Sections.—In cases where dams are higher than the level of the water they impound, or where the end of a dam is carried up to a height greater than that of the spillway section, and the top of which will always be higher than the level of the water, the thickness at any point can be computed by the foregoing formulæ, whether approximate or exact, if h be put equal to zero, and H = height of water surface above base of section. Fig. 97 shows the usual form of bulkhead section. The thickness, a, is from 4 to 6 ft., and is constant down to the point, b, the distance, k, being such that the overturning moment of the water at elevation b, causes the resultant of the water and gravity forces to intersect just within the middle third.

Thus, for a bulkhead section, which is to be 15 ft. higher than the spillway section, and the maximum height of water over crest of the spillway = 12 ft., the maximum height of the water against the bulkhead will be 3 ft. less than the height of the bulkhead itself. Hence, the thickness through the bulkhead, 30 ft. below

its top, from approximate formula (158), would be

$$l = \frac{H}{\sqrt{s}} = \frac{27}{1.5} = 18 \text{ ft.}$$

A more exact solution is given by the formula

$$l = \frac{l_1}{\sqrt{1 + 2r^2 - 3r^3}} \tag{170}$$

in which,

$$l_1 = \frac{H}{\sqrt{s}}$$

$$r = \frac{a}{l_1}$$

a = thickness of top section.

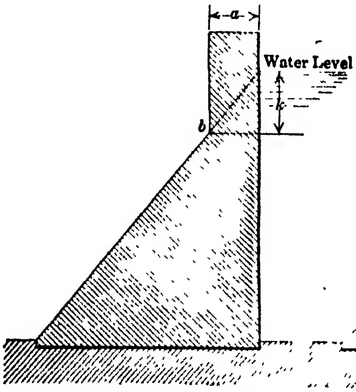


Fig. 97.—Bulkhead section.

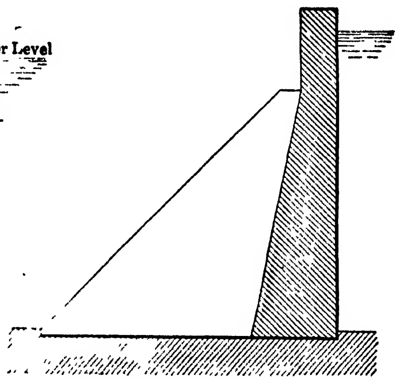


Fig. 98.—Buttressed bulkhead section.

For the foregoing example

$$l_1 = \frac{27}{1.5} = 18$$

If $a = 5$ ft.

$$r = \frac{5}{18} = 0.277$$

$$r^2 = 0.07673$$

$$r^3 = 0.02125$$

$$l = \frac{18}{\sqrt{1 + 0.15246 - 0.06375}} = 17.26 \text{ ft.}$$

In some instances, bulkheads are constructed as shown in

Fig. 98, the wall being of a practically constant thickness and supported against overturning by buttresses, placed at intervals of from 10 to 18 ft. on centers. It is, however, more economical to make the wall of increasing thickness with depth, and self-supporting at any point in its length.

In designing buttressed walls to resist overturning, the weight of the wall, over the length between buttresses, is taken, with the moment of its weight around the toe of the buttress, as one factor in resistance to overturning. To this is added the moment of resistance to rotation of one buttress around its toe. Opposed to the sum of these two moments is the overturning moment of the water, taken, not over a single foot length, but the total length of span between buttresses. From the foregoing analyses of dams, the method of computing the resistance to overturning and the overturning moment is clear, and need not again be explained here.

Bulkheads will frequently be exposed to ice pressure at the normal water level, while the spillway of a power dam seldom, or perhaps never, will have ice pressure against its crest.

Examples of Gravity Sections:

Figures 99 and 100 each show several typical sections of gravity dams. From these, the reader can obtain considerable data as to the various types of masonry dams, their proportions, stability and factors of safety.

Cutoff Walls.—With any form or kind of dam it is necessary to construct a cutoff wall at the upstream side, unless the foundation is of solid granite, in which case the cutoff wall may be omitted.

A cutoff wall is simply a vertical wall, sunk in a narrow trench that is excavated across the stream bed parallel to and, usually, underneath the upstream edge of the dam. Its function is to effectually seal the stream bed so that no underflow can take place through seams, cracks, fissures or between the bottom face of the dam and the foundation.

Cutoff walls vary in depth and thickness to suit the particular local conditions. They should be carried down deep enough to get the bottom well below any defective strata, or any suspected of not being water-tight. The thickness of the walls is made only great enough to resist percolation of water through them under a given head. The element of strength does not enter

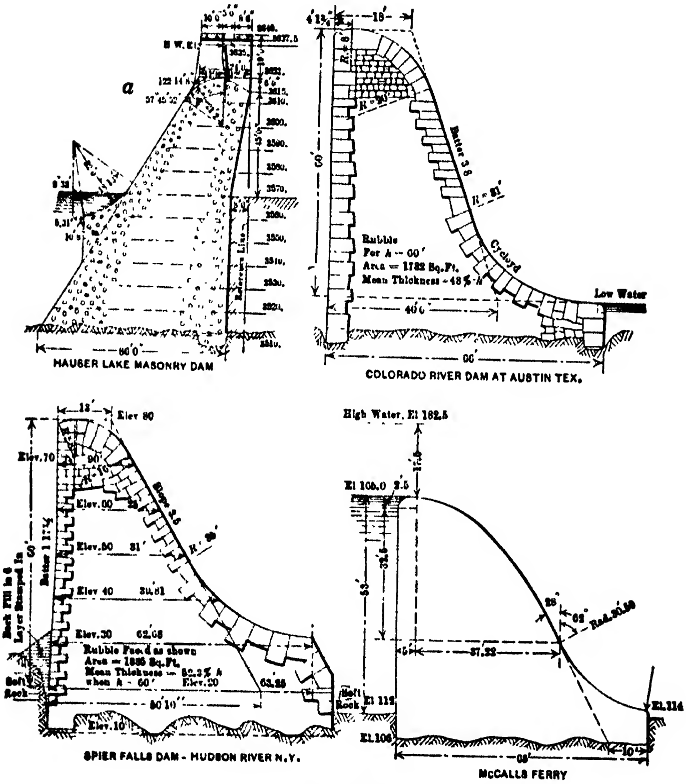

HAUSER LAKE MASONRY DAM
COLORADO RIVER DAM AT AUSTIN TEX.
SPIER FALLS DAM - HUDSON RIVER N.Y.
McCALLS FERRY

FIG. 99.

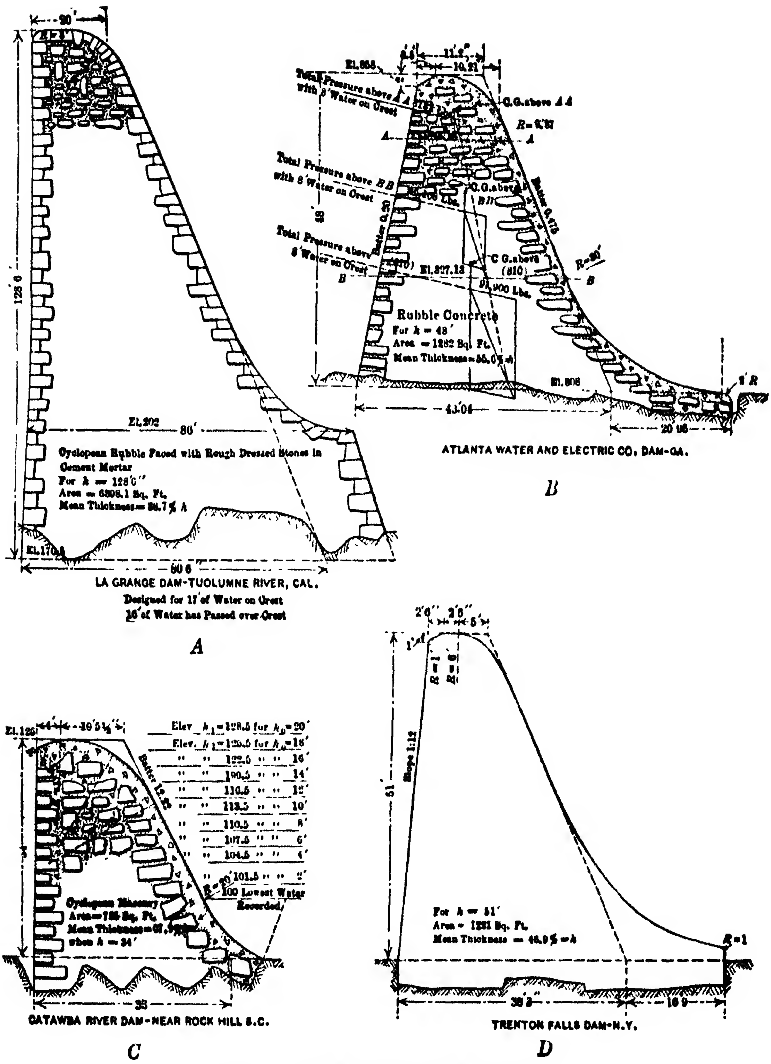


Fig. 100.—Typical sections of solid dams.

into the design as they rest solidly against the downstream face of the trench, and are in compression, only.

For safety against percolation, the thickness of the wall should be 0.3 to 0.5 in. thick, per foot head.

Where the foundation is particularly seamy and fissured—as is always the case for a limestone bottom—drill holes should be made in the bottom of the trench after it is completely excavated. These holes should be 2 to 4 in. in diameter, spaced 3 to 20 ft. apart, and from 8 to 20 ft. deep depending on the character of rock encountered. They should be blown out clean with compressed air, and filled with grout under pressure. This is easily and cheaply done with a standard grouting machine as described in the paragraph on "Foundations."

After the bottom of the trench is well grouted, the cutoff wall is placed, and after it solidifies, earth, stones and gravel are rammed in behind the downstream side. Sometimes it is better and cheaper to smooth up the downstream wall of the trench and use it as one side of the form, casting the concrete directly against the side of the trench.

Engineers frequently build cutoff walls along the downstream toe of the dam, in addition to the upstream wall. This is an unnecessary expense if the upstream wall is properly constructed, except in the case of clay and sand foundations, which could easily be eroded by the action of the water falling over the crest of the dam and setting up eddy whirls at the toe. For no kind of stone foundation is any, other than the upstream wall, necessary.

Arch Dams.—The gravity dam, as its name implies, resists the action of the water pressure by the gravity action of its mass, and the strength of the material does not contribute toward the strength of the dam, except in so far as the strength is required to support the weight of the structure. The waste of material, which results when its weight only is useful, has caused engineers to develop forms of dams, in which the strength of the material will contribute to, or form the chief factor in, their stability.

The arch dam is one of this type, and it is simply a dam which reaches from one side of a stream to the opposite one in the arc of a circle instead of straight across from bank to bank. The convex side is, of course, placed upstream. The pressure of the water is resisted by the arch action of the structure.

This means that dams of this kind must have unyielding walls at either end to act as skewbacks and resist the stresses that are transmitted to the ends of the arch.

The dam will also act, partly, as a gravity section. A vertical wall having a given weight and thickness at the bottom will offer a certain resistance to sliding and overturning, due to its gravity action alone, as has been previously described. Authorities are not, however, in accord on this subject. Some maintain that dams which are curved in plan should be designed as gravity sections, regardless of the curvature. Others maintain that they should be designed as arches solely, regardless of the gravity action. It is the general belief that both the arch and gravity effects work together to resist the water pressure, but their respective limits of resistance, in conjunction with each other, appear to be unknown. Whether the limit as a thin gravity section is reached, and the arch action begins only after the dam begins incipient overturning, or whether it is stressed as an arch with the stress reduced by the moment of its resistance to overturning is not definitely known. The fact remains that there are many thin dams, curved in plan, which would overturn if they were straight, and recent, multiple-arch structures, made of concrete, prove conclusively that arched dams may be depended on to stand safely, acting simply as arches.

Neglecting the gravity action, and also the effect of the thickness at the top, arbitrarily assumed, the formula generally used for the thickness through the section at any point in its height is

$$l = \frac{62.5HR}{S} \tag{171}$$

H = depth below water surface in feet.
R = radius of arch to upstream side in feet.
S = allowable compressive stress in the material lbs. sq. ft.

This formula applies only to sections having a vertical, upstream face, the batter being on the downstream side.

Obviously, the greater the value of R, the larger the value of l, so that the thickness of the dam increases with its span. With a radius of 500 ft., the thickness is approximately as great as that of a gravity section, so that nothing is gained by making the dam curved in plan, where the distance apart of the banks

is 500 ft. or more. On the contrary, there is a loss due to the excess of length of the curve as compared with the length of a straight dam of the same span.

Where a considerable length of gap is to be spanned, the length may be divided into several short portions, and an arch

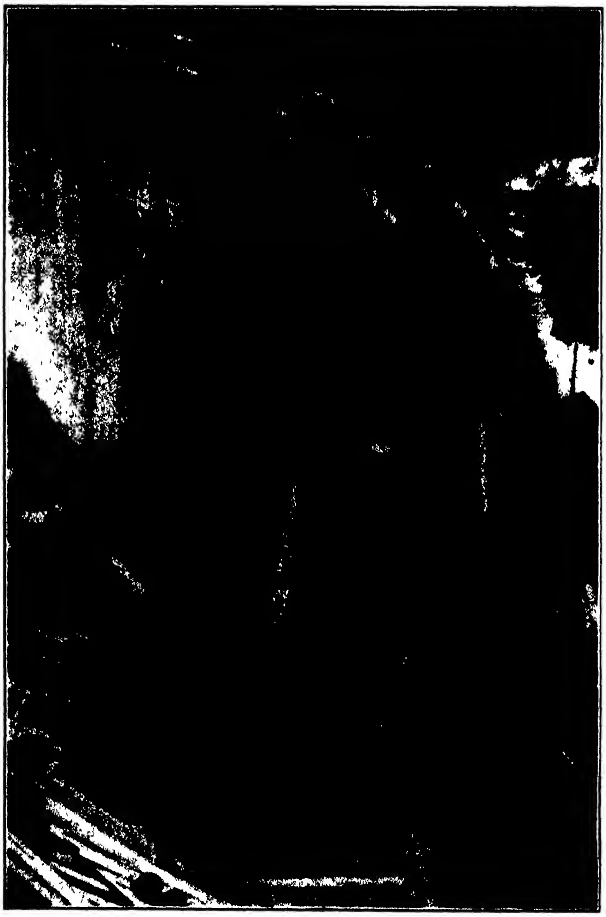


Fig. 101.—Salmon Creek arched dam.

having a short radius, put in each portion. At the points where the ends of adjacent arches join, buttresses are built which take up the thrust, downstream, of the arches, the side or trans-

verse thrust being transmitted from arch to arch until finally taken up in the side banks of the stream.

Figure 101 shows Salmon Creek dam, which is curved in plan.

The Halligan dam is an excellent example of the arch type of dam. It is shown in Fig. 102 and its cross-section. in Fig. 103. It is 62.3 ft. high at the crest of the spillway section, and 72 ft. high at the bulkhead section. At the top of the bulkhead the width is 2 ft. and at the bottom the thickness is 27 ft. Its

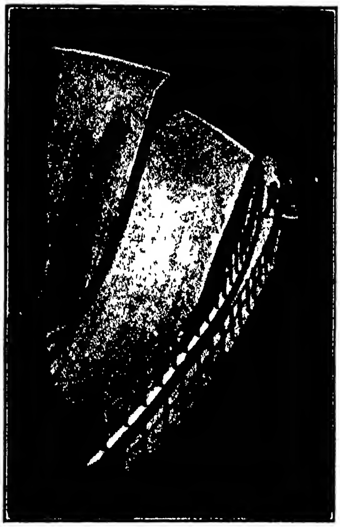


FIG. 102.—The Halligan arched dam.

radius is 324 ft. and length 350 ft. Hence its "rise" is 46.2 ft. and its chord length, 334 ft.

According to the formula, the compressive stress in the masonry at the bottom of the section for arch action only, would be

$$S = \frac{62.5 \times 72 \times 324}{27} = 54{,}000 \text{ lb. per square foot.}$$

The stresses were computed on the basis of the dam acting as an arch, and, also, as a cantilever beam. That is, a section 1 ft. long, extending from top to bottom, is considered as a vertical beam anchored at the bottom and, acting as a cantilever, takes a portion of the thrust. The remaining stresses are resisted by the arch action. While this "combination" theory does not

appear rational to the author, the dam has been under pressure since 1910, and is, by now, a presumably permanent structure.

In order to provide a proper width of spillway crest, an overhanging lip was constructed on the upstream side of the spillway section, as is indicated in Fig. 103.[1]

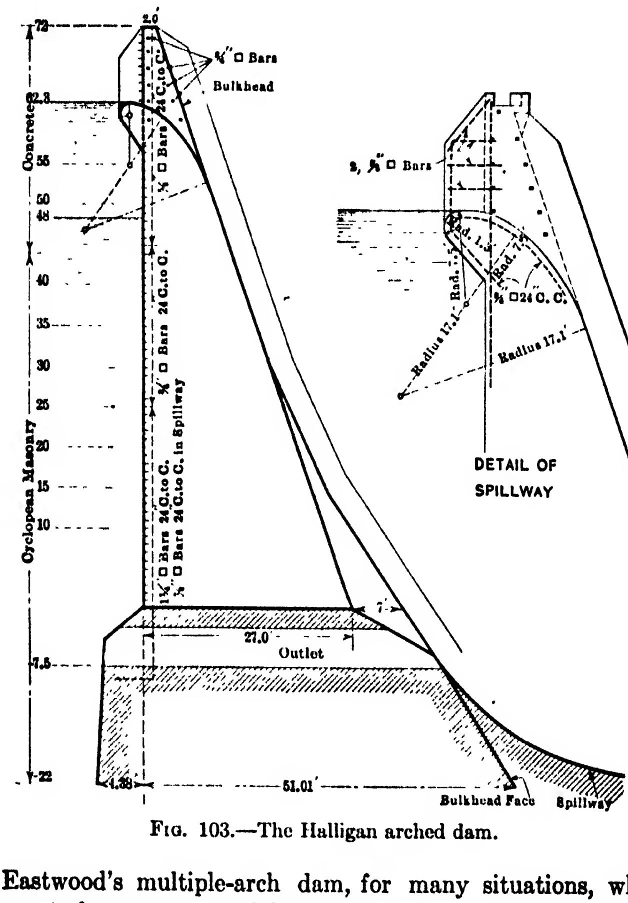


FIG. 103.—The Halligan arched dam.

Eastwood's multiple-arch dam, for many situations, where the cost of concrete materials is high, and form lumber abundant, is one of the lowest cost dams that can be constructed. A portion of the Hume Lake dam, designed on the inclined multiple-arch principle, is shown in Fig. 104, and a vertical section through it is given in Fig. 105. The arches are short, have a small radius,

[1] The Halligan Dam, Houston, *Proc.* Am. Soc. Civ. Eng., October, 1911,

and span from buttress to buttress, as shown. Also, they are inclined on the upstream side so that the weight of the water

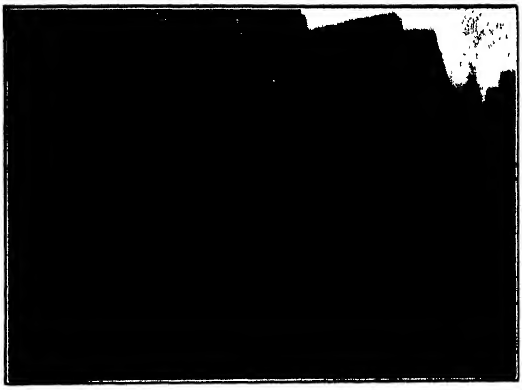


Fig. 104.—Portion of Hume Lake dam showing spillway gate openings.

will tend to hold down the dam and thus diminish the length of base required for the buttresses.

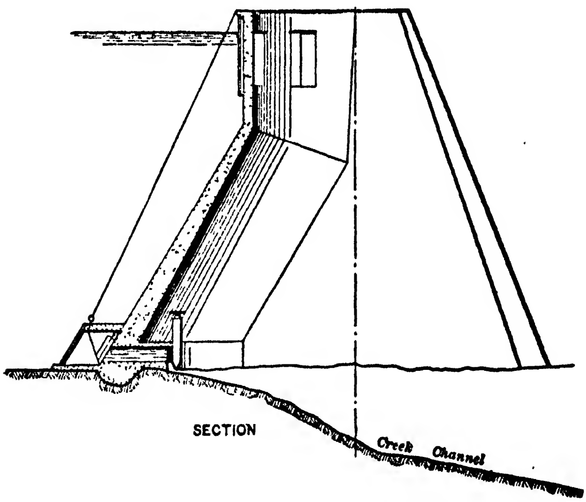


Fig. 105.—Cross-section of Hume Lake dam.

This design is not suitable for conditions where any considerable depth of water will pass over the crest of the dam.

Foundations of Dams.—There is such a wide difference in the character of earths, clays and rock of the same general class, that any discussion of foundations must, necessarily, be wanting in definiteness. Curious and unexpected conditions are sometimes revealed when borings and excavations are made, and each case is specific to the particular site being worked. Hence, this short discussion must not be regarded as anything other than a few suggestions of more or less importance.

If any unusual conditions are encountered, experienced judgment is the only safe resource.

As has before been pointed out, the most important factor in the stability and permanence of dams is the character of the foundation on which they rest. Where the stream bed is of *granite*, there is usually no particular difficulty in securing a reliable foundation. Drill holes should be made into the rock from 6 to 12 ft. in depth and 2 in. or more in diameter in order to be sure that no defects exist. Occasionally it will be found that what appears to be solid bed rock, is merely a boulder underlaid with mud or earth.

Removing all loose rock and blasting out a shallow trench to receive the bottom of the dam, are the only steps necessary to prepare a foundation on granite.

Limestone, however, is seldom encountered in a solid and safe state, and wherever the stream bed is of limestone, it is necessary to make a number of drill holes over the surface on which the dam will rest. These holes should be spaced at intervals of about 12 ft., and blown out thoroughly, with compressed air, the pressure being from 50 to 80 lb. per square inch. It will generally be found that air entering one hole will blow out of one, or more, of the adjacent holes, showing a fissure or cavity with which the holes connect. The drilling will usually be done in a cofferdam, where the elevation of the rock surface is below the level of the surrounding water level of the stream, and water will emerge from the drill holes after they have reached a certain depth, depending on the disposition of the strata. In order to close up all the small cavities and seams, grout must be forced into the holes until all of the fissures and openings in the underlying rock are thoroughly filled and the whole mass solidified. The usual mixture of grout comprises one-half cement and one-half sand, mixed with sufficient water to make a liquid that will flow freely. The connection for forcing the grout into the hole

is made by cementing a short pipe nipple, or sleeve, over each hole. The nipple, or sleeve, is set over the hole and cement packed around it by hand, until any water which emerges from the hole passes through the metallic coupling. In this way, after the cement has solidified, a screw connection is made for the reception of any pipe it may be desired to connect therewith.

The grouting machine is a well-known device, and consists simply of a steel cylinder which, normally, stands vertical, the bottom being conical in form. The top is closed by a manhole of the usual kind. Pipe connections, with valves are provided, so that compressed air may be admitted to either the bottom or the top of the cylinder. In operation, the sand, cement and water are poured into the cylinder through the manhead, a small amount of compressed air is admitted to the bottom, and this, blowing through the mixture for 15 or 20 sec., mixes the ingredients thoroughly. The manhead is then closed and a flexible pipe leading from the bottom of the cylinder has the free end screwed into the coupling fixed in the drill hole. Air is then admitted, at low pressure, to the upper end of the cylinder, and the grout thus driven into the hole. If the quantity of grout in the cylinder exceeds the amount which can enter the hole, the air pressure is gradually raised until the maximum of 70 or 80 lb. to the square inch is reached. The machine is allowed to stand 2 or 3 min. with this pressure on the hole, after which the air is cut off, the pressure on the interior of the cylinder is relieved through a blowoff valve in the bottom, which also clears it of the remaining unused grout, and the flexible pipe then disconnected from the hole. If the amount of grout which the hole will receive exceeds the contents of the cylinder, other charges are prepared and forced into the hole, always under a low pressure, that is, from 5 to 8 lb. per square inch. This work must proceed continually, and with sufficient rapidity to prevent the grout from setting during the process. When several charges have been forced into a hole, and it will receive no more grout, the air pressure must be gradually increased until the maximum pressure is finally attained. It will frequently be found that after the hole has apparently been filled under the lower pressure, higher pressures will force in additional grout, indicating that under the lower pressure some of the narrower seams had not been filled. This is an operation which requires care, skill and patience. In some cases, grout which is forced into one hole will finally begin

to emerge from adjacent holes. In this event, it is well to let a small amount come out of the surrounding holes, and then screw caps or plugs on to the couplings which have been cemented to them and continue the grouting.

It is probable that water will come up through some or all of the drill holes, in which case, it is customary to screw a piece of iron pipe to the metallic connection at the surface of the drill hole, the length of pipe being such that its upper end is at a higher elevation than the surrounding water outside the cofferdam. This, effectually, stops any further flow. The grout is then forced down through this vertical pipe. After the first operation of grouting the holes, which are drilled on approximately, 12-ft. centers, additional drill holes should be made about halfway between those of the original group. In many instances, the drill will pass through hardened grout, showing that the seams or cavities underlying the location of the new drill holes have already been filled. In any case, all the new holes should be grouted.

If any portion of the stream bed appears to be particularly honeycombed, as indicated by the comparative quantity of grout which may be forced into the drill holes, the number of holes made over this area should be greater than in the other portions of the foundation site.

Each individual case has to be drilled in accordance with the conditions that are found, and the foregoing is all given simply as a general suggestion as to methods of procedure. As the engineer acquires personal experience from the initial stages of the work he will soon be able to recognize the good and bad portions of the site and proceed accordingly.

No dam should be built on a limestone foundation without an adequate cutoff wall at the upstream edge of the dam. The bottom of the wall should be well below, practically all, permeable strata. In the bottom of the trench excavated to receive this wall, drill holes should be made, varying from 10 to 30 ft. in depth, and on 6-ft. centers, and these should be grouted, as has been described.

In the case of hollow reinforced-concrete dams, there is no need to grout the entire stream bed covered by the dam. The grouting in the cutoff wall trench, and in the trenches which receive the buttresses, will be sufficient.

Sand Foundations.—The best discussion of the subject of sand foundations for dams is contained in a paper by Koenig.[1] Dams may be successfully built on sand foundations provided that water be prevented from flowing through the body of the sand on which the dam rests, and the stream bed be protected against wash on the downstream side, so that the toe may not be undermined. The complete prevention of any flow through the sand is practically impossible, but the path of the water may be so lengthened, and its passage so retarded, that the velocity can never be sufficiently high to move any particles of sand. If these conditions are met, the foundation will be permanent. If, however, a stream of even infinitesimal size should move with sufficient velocity to carry with it any of the particles of sand, the size of the stream would increase and, in a short time, the whole structure would be undermined and destroyed.

Obviously, if the path which the stream must follow, from the time it percolates into the sand on the upstream side until it emerges on the downstream side, be made of a sufficiently great length, the resistance to flow will be great enough to so retard any movement of water through the sand that it can never attain an appreciable, or dangerous, velocity. Koenig states that: "No material, other than solid bed rock, offers a more secure support than sand which is properly confined; therefore, to make it serve as a safe foundation for a permanent dam is entirely a problem of making adequate provision against the only disturbing element—a current of flowing water."

The provision which is needed against a current of flowing water comprises one or more cut-off walls, either of wood, of steel, or of concrete. There should be not less than two of these walls, one at the upstream edge of the dam and one at a considerable distance downstream; their distance apart being approximately 4 times the height of the dam.

The following are empirical rules for the depth, d, of the cutoff walls below the surface of the stream bed:

$d = 2.5h$ for heads up to 8 ft.

$d = 2h$ for heads up to 15 ft.

$d = 1.6h$ as the minimum in any case.

If the dam has water flowing over its crest, there must be an apron below the dam, extending some distance downstream and made of masonry or concrete. This apron serves a double

[1] *Trans.* Am. Soc. Civ. Eng., January, 1911.

purpose. It prevents seepage water which passes through the sand, from rising into the stream until it has passed a considerable distance beyond the end of the apron. It also serves to protect the stream bed and the toe against the shock of falling water. The length of such an apron, therefore, is an important element in fixing the factor of safety.

Aprons for spillways, diversion weirs, and dams of the overfall type built on sand, should be submerged, so as to form a water cushion to absorb the shocks of falling water, ice, and debris. Such water cushion should have a depth equal to one-quarter, and preferably one-third, the height of the dam, and a length of not less than 1½ times the height of dam, with the downstream edge forming a deck, sloping upward. This deck forms a

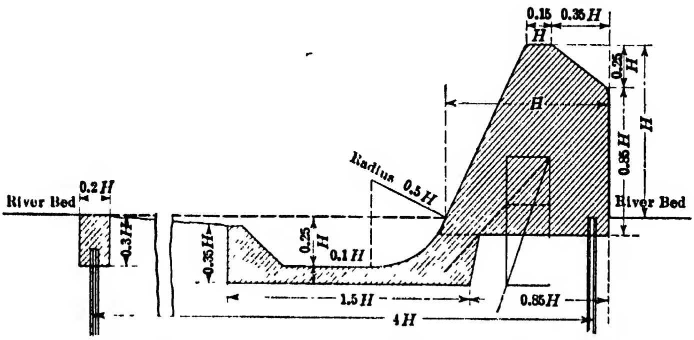


FIG. 106.—Proportions for protecting aprons.

covering for the stream bed and should extend downstream for a distance of not less than 1½ to 2 times the length of sheet piling used in the main curtain wall, ending with, and supported on, a secondary line of sheet piling of from one-third to one-half the depth of penetration of the curtain wall. Figure 106 shows the general proportions which should be observed in constructing a spillway dam on sand foundation.

It should be noted that the uplift pressure under dams constructed on sand, is not negligible, as it is in the case of rock or shale foundations, but may attain a value on the upstream side as high as 25 per cent. of the static head. In the exact equations for masonry dams, the value of ϕ should be taken as 0.25 for sand foundations. In cases where the dam is to be of considerable height, and the proportion for depth of the cutoff walls

as given in the foregoing empirical rules, would result in a wall or sheeting of impractical depth, it has been suggested that a low dam built a short distance downstream from the high or primary dam would allow the use of a cutoff wall of much less depth.

For instance, if a dam were to be erected 50 ft. high, the cutoff wall, according to the rule given, would have to be 80 ft. in depth. If this dam were built with a cutoff 30 ft. deep and then 500 or 600 ft. downstream, a second dam, 15 to 18 ft. high were constructed, with a cutoff 25 or 30 ft. in depth, and the second dam were provided with an apron on the downstream side to prevent erosion, this double system should prove safe and permanent for the impounding of a 50-ft. depth of water.

When reinforced, hollow concrete dams are built on sand foundations, it is necessary to cover the entire surface of the stream bed with a mat of concrete, which should have a small amount of steel reinforcement in it. The thickness of this mat and the reinforcement in it must be such as to spread the pressure of the footings over from one wall to the next one. This mat not only serves to limit the unit pressures on the sand but it prevents the water from emerging underneath the dam, and thereby shorten its path of travel. The downstream apron is a continuation of the mat and it is also joined to the spillway. In this way all the conditions of a solid foundation are maintained.

When confined between cut-off walls and protected against under-flow the safe bearing pressure of sand is from 5 to 8 tons per square foot.

Foundations on Clay.—Clay is a generic term which includes material of consistencies varying from hard shale or slate down to a mushy, water-bearing clay.

Hard shale of the slaty variety will bear any pressure that is likely to be imposed on it, certainly up to 10 tons, per square foot.

It can be treated as a limestone foundation, though it will seldom need grouting. A cutoff wall should, however, be constructed, its depth being from $0.1H$ to $0.2H$, depending on the conditions found in the cutoff wall trench. As a precaution, drill holes should be made at occasional intervals—say on 20-ft. centers—to make sure of the condition of the underlying strata.

With softer clays, the foundation should be treated as a sand foundation so far as the cutoff walls, apron and (for hollow buttressed dams) the mat, are concerned. The upstream wall,

however, does not need to be so deep as for sand. As a rough approximation, $0.3H$ to $0.5H$ is sufficiently deep for the upstream wall and $0.2H$ to $0.4H$ for the downstream cutoff, which latter wall is placed at the downstream end of the apron. The length of the apron may be equal to H. There need be no allowance for uplift pressure under the dam for any kind of clay.

The safe bearing power of soft clays ranges from 1½ to 5 tons, per square foot.

In certain kinds of soft clay containing a considerable proportion of water, practically the only safe dam is the hollow-buttress type. Piles are driven under each buttress wall, the size and number of piles depending on the bearing power of the soil and the weight imposed by the vertical component of all the

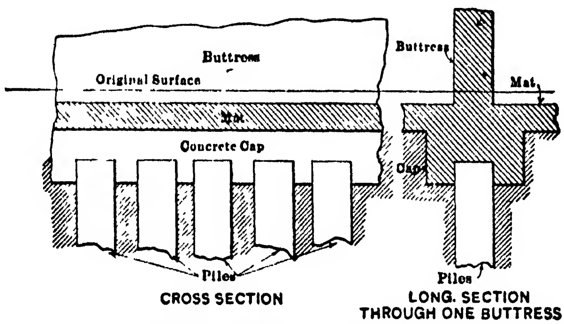


FIG. 107.—Construction details of pile-supported hollow dam.

forces acting on the dam, plus the weight of the triangular section of water lying on its deck. The unequal distribution of this pressure must be taken into account and the number of piles proportioned to the unit pressure at any point.

The piles may be of wood or of concrete. In case wood piles are used, they must be cut off at least 3 ft. below the water line, and a concrete capping cast on top of them, which is wider than the footing of the buttress walls. On top of the capping, a concrete mat must be cast, and the buttress walls placed on top of the mat. Fig. 107 shows the general arrangement.

Of course, variants of this method will suggest themselves. The author, however, is of the general opinion that when a dam can not be built without driving piles to support it, another site should be selected or the dam remain unbuilt.

Abutments and Retaining Walls.—In cases where a dam ends at an earth stream bank, or joins with an earth section, a masonry, or concrete, abutment is necessary to prevent erosion. The abutment should extend both upstream and downstream, as indicated by the sketch, Fig. 108. Preferably, the wall should be, in plan, as indicated in the sketch—that is, with the two ends further away from the stream bed than at the middle section, where a junction is made with the dam.

The top of the wall must be at least 3 ft. higher than the level of maximum high water.

A good foundation is an absolute necessity. The wall footings must be carried down to a point where;

1. Wash or erosion can not reach them.

2. The material underneath the bottom is incapable of sliding under any pressure that may come on the wall; that is, the whole mass of earth with the wall embedded in it may slide toward the stream, unless the wall penetrates below any stratum that is capable of sliding.

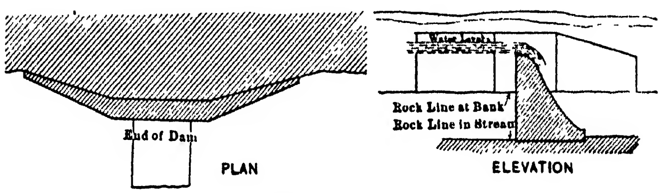


Fig. 108.—Abutment wall.

3. The pressure resisting qualities of the material on which the wall rests, will amply sustain the maximum pressures at the toe of the wall. It may, at times, be necessary to spread the footings in order to reduce the unit pressures on the foundation.

The same rules that apply to the design of dams are equally applicable to retaining walls. The resultant of all the forces acting should cut the base of any section, within the middle third.

The overturning moment, acting on a wall having a vertical, earth-bearing face, is

$$M = \tfrac{1}{6}wh^3 \tan^2 (45 - \tfrac{1}{2}\phi) \qquad (172)$$

In which w = weight of the earth per cubic foot.
h = height of wall in feet.
ϕ = angle of repose of the ear h.

The value of w is usually taken at 100 lb. per cubic foot.

A wall designed to resist overturning and of such dimensions that the resultant will cut the base at the end of the middle third, will have a thickness at the base equal to

$$l = \sqrt{1.25a + \frac{w}{w_1} h^2 \tan^2 \left(45 - \frac{\phi}{2}\right)} - \frac{a}{2}$$

w_1 = weight of masonry in wall per cubic foot.
l = thickness through any section, in feet.
a = thickness at top of wall, in feet.

For usual values of $w_1 = 140$, $w = 100$ and $\phi = 34°$.

$$l = \sqrt{1.25a + 0.202h^2} - \frac{a}{2} \qquad (175)$$

The usual values for a vary from 12 in. to 2.5 ft. according to the judgment of the designer. For abutments, the top width should be 1.5 to 2 ft.

Drains and "weep holes" should be built through the walls not only at the bottom, but at various heights. These relieve any excessive pressure that may be set up by an accumulation of water behind the cutoff wall. If water be caught and retained by a wall it will be absorbed in the earth behind the wall, giving it the quality of a semi-liquid, and thereby greatly decreasing the angle of repose, ϕ.

Expansion joints should be placed in retaining walls when their length exceeds 200 ft.

Unlike dams, it makes but little difference in the economy of the section whether the wall has its slope on the pressure side or the free side. This difference is due to the fact that the weight of earth is nearly equal to that of the masonry. The most economical section is that in which the pressure side is vertical and the free side sloping. A retaining wall, however, has a much better appearance if the visible, or free, side is vertical. Also, in the case of abutments for dams, a vertical side next to the stream gives a free fall of the water without striking an inclined face. This, of course, applies only to the case of a spillway section ending against a retaining wall. If the dam ends

in a bulkhead section, the magnitude of the retaining wall required will amount to little or nothing. Also, for terminating a bulkhead section it may have its most economical form, viz., with the slope on its free side.

No formula can be given for the length of an abutment wall. This is one of the factors in design which must be left to personal judgment. It may run from 30 to 100 ft. upstream and the same distance downstream, measured, respectively, from the face and toe of the dam, and, in certain special cases, these lengths may be exceeded.

Retaining walls may, also, be built of reinforced concrete, and usually a saving in cost can be effected by walls of this kind. The formulæ and methods of design of concrete walls are given in a succeeding discussion.

Reinforced Concrete Dams

Within the past few years. the use of reinforced concrete for the construction of dams has steadily increased, and at the present time (1916), most of the dams now constructed are hollow, reinforced structures. These differ, essentially, from the solid masonry dams in that they depend, almost entirely, on the weight of water which rests on the inclined upstream face, to hold them against overturning, and they are designed simply to resist crushing stresses.

In the opinion of the author, they possess many distinct advantages over solid masonry dams, the principal ones being:

1. The distribution of weight over the foundation is more uniform.

2. Due to the design and the angle of slope of the upstream face, sliding and overturning are practically eliminated.

3. The structure has a definite, known strength in every part.

4. As usually designed, they have a high factor of safety against crushing, and the factor of safety is considerably greater than that of a solid section.

5. When built on sand foundation there is practically no uplift pressure.

6. Owing to the inclined upstream deck, no allowance need be made for ice pressure as the expanding ice will slide up the deck instead of pressing against a vertical wall and producing an overturning moment.

7. It is suitable for any kind of foundation that will sustain 2 tons or more, per square foot.

8. Due to the fact that the whole structure is solidly joined together by the reinforcing steel, such dams have a girder-like action, and in case of foundation failure, or washout, under a comparatively short length, the dam does not fail, but spans the opening, being self-supporting.

9. Low cost. These dams, with a factor of safety of 4, cost from 18 to 35 per cent. less than solid dams having a factor of safety of 2.

10. Short time of construction. This last is particularly important. Time is of the essence of dam construction. The ability to get into the river and out of it during the same season of low water, escaping flood periods, means savings in many ways. Cofferdams are cheaper; the cost of handling the water is greatly reduced; the losses due to injury by floods practically eliminated, and in addition, savings are effected by reason of the plant being put into commission and becoming an earner of income without having to wait an additional year for the second period of low water and carrying the financial load of interest on bonds, staff of engineers, and the possibility of expiration of preliminary power contracts.

Types of Hollow Dams.—All hollow, reinforced-concrete dams consist, essentially, of an inclined, continuous slab forming the water barrier, which is curved at the top to form the crest, and a second slab sloping downstream, to make the spillway, which may be either complete or partial, and means for supporting these two slabs which may comprise simply transverse buttress walls, or a series of pillars, or a combination of both transverse and longitudinal interior walls which intersect at right angles giving both transverse and longitudinal support to the upstream deck and the spillway.

The best known type is the Ambursen dam, which is made up of a series of interior parallel buttress walls, set transverse to the axis of the dam and spaced from 14 to 20 ft. apart. On these, as supports, are cast the deck slab and the spillway. The deck spans from wall to wall, either as a simple beam or continuous girder, depending on conditions and the manner in which the reinforcement is placed. The angle which the upstream deck makes with the vertical, or θ, in the general formulæ, is taken at about 48°, this value being adopted because it is practically

the angle at which the friction produced by the weight of water on the deck of the dam, plus the weight of the structure itself, is approximately, equal to the thrust of the water tending to push the dam downstream. In other words, at this angle the danger of sliding is almost entirely removed, without the necessity of embedding the buttress wall footings in the earth, or rock, forming the foundation. Where the upstream deck comes down to the foundation, it is usually carried deep into a trench to form a cut-off wall and prevent percolation underneath the structure.

The spacing of the buttress walls is arbitrarily fixed by the designer. The closer together the walls, the greater the amount of concrete in them, and the greater is the amount of foundation work to be done in the cofferdam, while the quantity of concrete and reinforced-steel deck is less. As the walls are put farther apart, the quantity of concrete and reinforcing steel in the deck becomes greater, these varying, almost directly, as the distance apart of the buttress walls. It must be noted that the most economical adjustment of the materials between the quantity required in the deck and that necessary for the buttress walls, is seldom the most economical for the actual construction of the dam. A trench must be cut for each buttress wall, and the bottom of this trench should usually be grouted. This work is all done inside cofferdams, and one of the main objects in dam construction is to get the work brought up above the level of high water in the stream in the shortest possible time, after which the destruction of the cofferdam by floods is of no moment, as work can proceed as soon as the flood waters have receded. Therefore, the fewer the number of buttress walls, the more economical and rapid will be the progress of the work.

Another consideration is that one of the main elements of cost in building a dam, is for the construction and placing of the forms. The cost of the forms varies but little with changes in thickness of the concrete slab poured in them. Hence, the thickening of the deck slab to compensate for wide spacing of the buttress walls, means an increase in the cost of the deck of only the value of the additional raw concrete required to give this extra thickness. From these considerations, it follows that the spacing of the buttress walls should be from 2 to 3 ft. greater than theoretical economy of material would dictate.

In order to reduce the span of the deck slabs, dams have been made in which the walls were spread out at the upper edges into

corbels. This effects a considerable saving in concrete and steel, but the additional form work required, of a difficult character, retards the progress of the work and its cost is very nearly equal

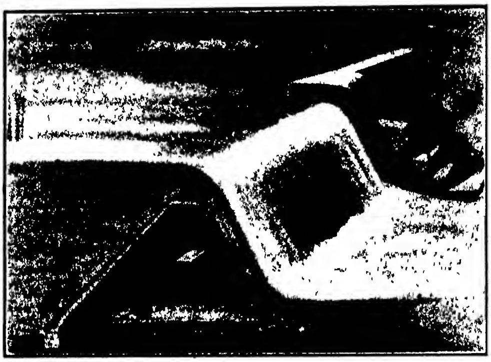


FIG. 109.—Cross-sectional perspective of hollow dam.

to the saving effected by the reduction in the quantity of materials used.

In designing hollow dams, special effort should be made to avoid all unnecessary angles and junctions. It is usually better to place extra concrete and steel rather than to save it in the

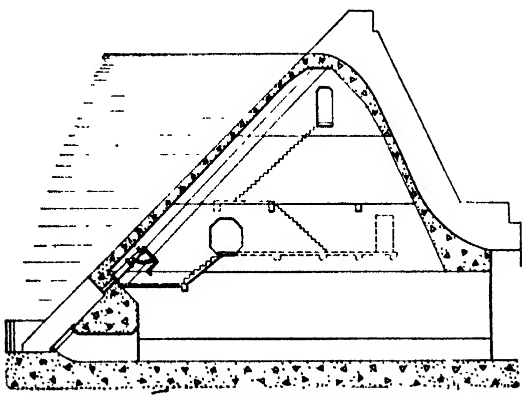


FIG. 110.—Section through hollow dam.

expense of considerable form work. It should be remembered that the construction of a hollow concrete dam is a carpenter job, and the speed of the work is determined, principally, by the rate at which carpentry can proceed.

Figure 109 shows a perspective cross-section of the Ambursen type of dam. Fig. 110 is a cross-section of the Cedar Falls dam, built in accordance with Ambursen designs. It will

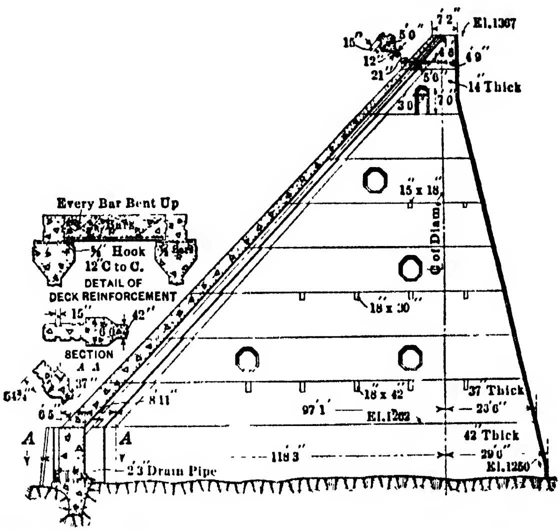


FIG. 111.—Section through bulkhead.

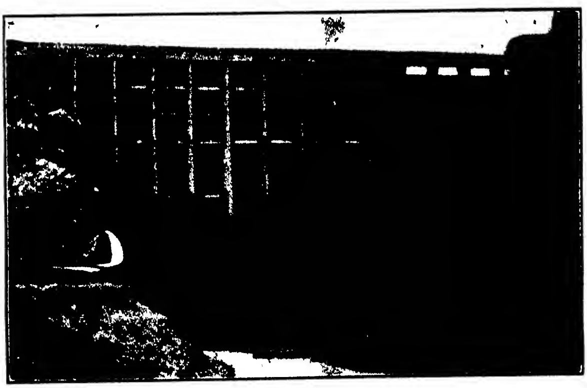


FIG. 112.—Downstream view of bulkhead.

be noted that there are openings made through the buttress walls, and runways may be placed inside the dam, passing from wall to wall through the openings, and making a continuous passageway inside the dam, which may be used for inspection,

or for reaching sluice gates when these are arranged to be opened from the inside.

These dams have been built up to 134 ft. in height. When

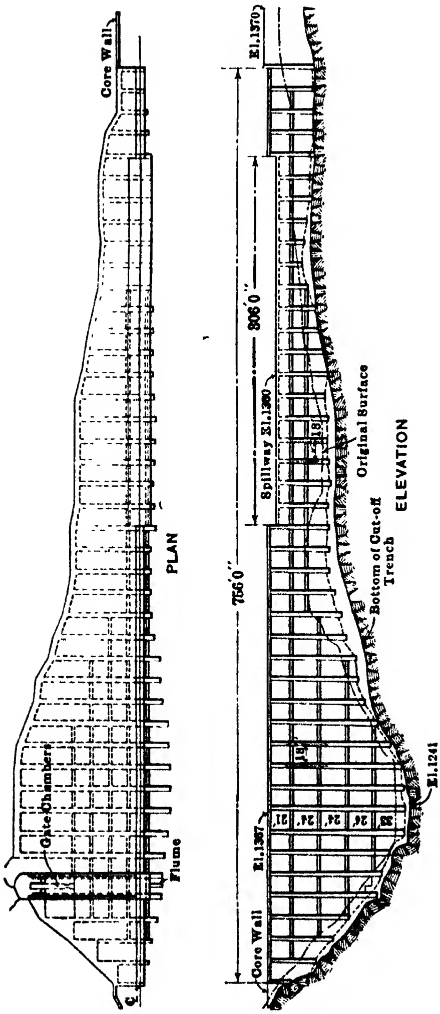


FIG. 113.—Plan and elevation of hollow dam.

built as bulkhead sections, the spillway is omitted, and the buttress walls have a smaller incline on the downstream side. Figs 111 and 112 show a bulkhead section of a dam of this type.

Fig. 111 is a cross-section and Fig. 112 is a picture of the downstream side. Fig. 113 shows the plan and elevation of this dam.

Where the stream bed is of stone, or where it is so soft that it is necessary to protect it with a mat (see discussion of "Foundations of Dams"), a saving in cost can be effected by making a partial, instead of a complete, spillway. This means that the usual form of curve is made of the crest of the spillway, which is carried down a short distance and then turned in a long sweeping curve to discharge the water horizontally, at a considerable height above the stream level. A partial spillway dam is shown in Fig. 114.

Figure 115 shows a diagram from which may be taken the quantities of concrete and steel, per foot length, required for

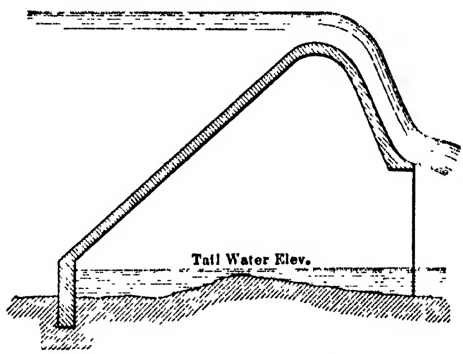


Fig. 114.—Hollow dam with partial spillway.

reinforced-concrete dams of various heights. It has been attempted to apply these curves to several types, so that they must be regarded as approximations only, for preliminary computation.

An example of the type of dam having both longitudinal and transverse walls, intersecting at right angles, is shown in Fig. 116, which is a section of the dam built on the Colorado River at Austin, Tex. The longitudinal walls are plainly shown in section. As indicated, it has an inclined upstream deck, making an angle of 42° to the horizontal. At a height of about 2½ ft. above low water level, this slope of the wall ends, intersecting a narrow horizontal bench, and from the upstream edge of this horizontal bench, a vertical wall goes straight down into the rock.

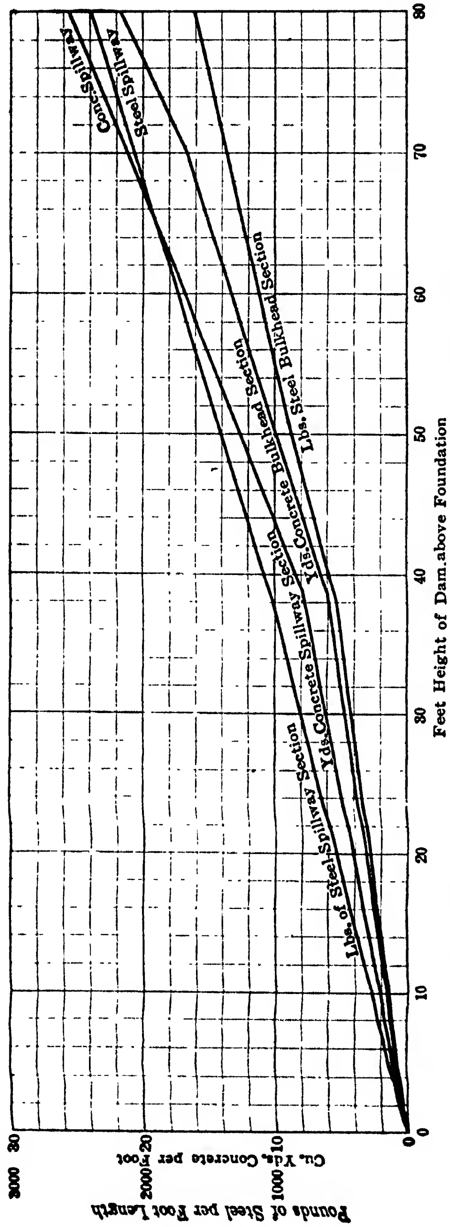


FIG. 115.—Quantities of material in hollow reinforced-concrete dams.

This vertical wall forms both the lower portion of the deck and the cutoff wall.

The object of the bench is to allow flow of water through any completed section of the dam during construction, the flat horizontal slab, which runs the entire length of the dam and which covers the upper part of the bench, being omitted until the completion of the work. Normally, the total flow of the river was

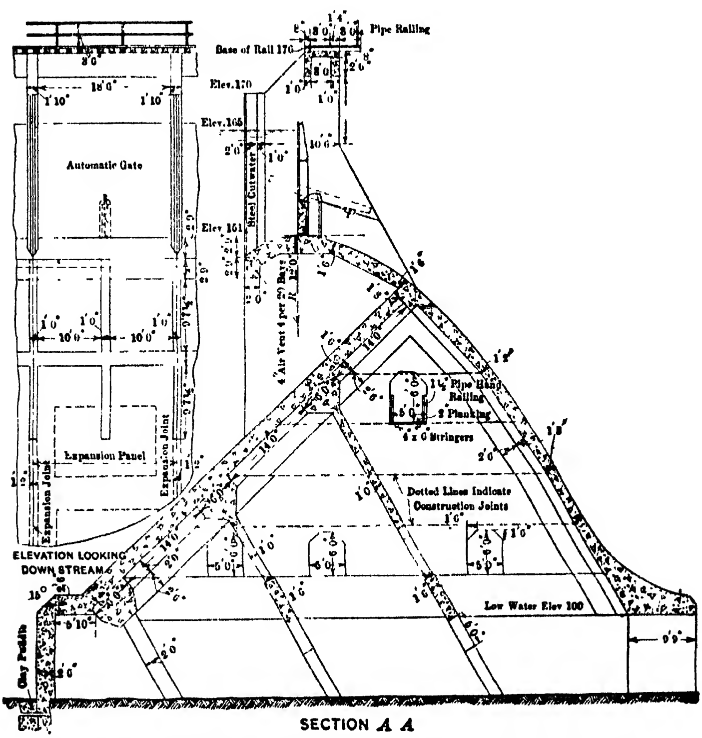


FIG. 116.—Cross-section and part elevation, Austin (Tex.) dam.

taken through the sluice gates in the bottom of the dam, but, in the case of floods, the openings through the horizontal bench, together with the area afforded by the sluice gates, would usually take care of the whole river flow at such times, without an elevation in the water level of more than 4 to 6 ft.

The transverse walls are set parallel to the flow of the stream and are 20 ft. apart, measured from center to center. The

longitudinal walls are also spaced 20 ft. apart. These walls are not vertical but are inclined, so that they make an angle of nearly 90° with the upstream deck. The slope of these walls is such that the resultant of the forces which they resist, made up of the water pressure acting against the surface of the deck and the gravity component of the deck and the walls themselves, has the same direction as that of the wall slope. The longitudinal and transverse supporting walls intersect at right angles and form a series of square openings when viewed in a direction normal to the deck. The deck panels are exact squares and, hence, may be reinforced in two directions. The reinforcing steel is, therefore, placed both longitudinally across the length of the dam and transversely up and down the deck. The spacing of the steel is logarithmic, the bars being laid closer together as they approach the middle of the panel. As is also shown in the section, these walls are surmounted by corbels, for the purpose of reducing the quantities of materials in the deck slab.

The contracted section was adopted for this design, which reduces the thickness through the base of the dam.

Owing to the difficult character of the foundation, it was desirable to make the distance through the base of the dam, measured from upstream to downstream side, as short as possible. In the usual form of design, the inclined deck goes up to the elevation of the crest of the dam, where it joins with the spillway. The form of the spillway crest is determined by the parabolic formula. The slopes of the deck and of the spillway and the width at the top, as fixed for the crest, all combine to produce a certain distance, measured horizontally, through the base of the dam. As shown in the typical section, the spillway was designed in accordance with the usual method, and the sloping deck made to intersect it at some 25 ft. further downstream and 6 ft. lower, vertically, than in the customary design. In this way, a proper length of theoretically designed spillway was made available, as well as an inclined deck of, substantially, the total height up to the crest, and, at the same time, the length through the dam from front to back was reduced about 25 ft., diminishing in a like proportion the length of the transverse walls, the length of each transverse wall foundation, and eliminating one longitudinal wall. While the saving in material and labor was greatly reduced by the adoption of this design, the

cost of the form work was increased so that the net saving effected, though considerable, was not so great as would, at first, appear.

As indicated in the figure, the vertical, transverse walls intersect the deck and the spillway, continuing on up above them in the form of piers, on the top of which are placed reinforced-concrete girders that carry a railway track. Between these piers are placed automatic crest gates.

The vertical buttress, or diaphragm, walls, have four large openings made in each, as indicated in the section. Through the topmost series of these openings is placed a walkway, with iron railings on either side, so that it is possible to walk through the new portion of the dam from end to end. Access is provided

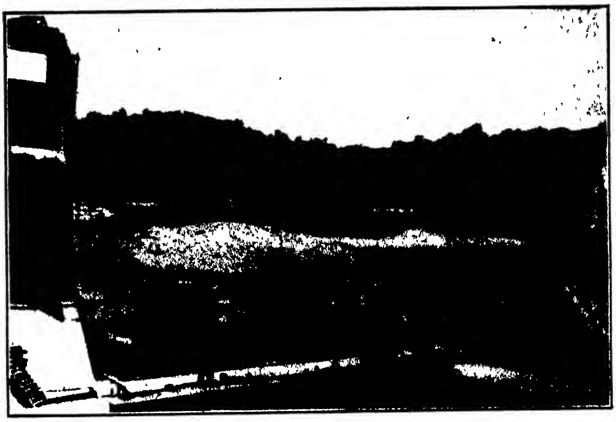


Fig. 117.—Austin dam during construction. View looking downstream.

to the interior by means of winding stairways, one placed at either end. These pass down through a stair-well, each well being formed in a pier, which is of sufficient width to accommodate it.

Figure 117 shows the Austin dam in process of construction, and the square openings formed by the transverse and longitudinal walls, are clearly shown near the right of the picture. To the left is shown a portion of the finished dam with the crest gate in place. Fig. 118 is also a picture of the dam during construction, in which the cross-section of the dam is shown.

A third type of hollow dam which has economic possibilities is the standard, vertical-walled dam, with longitudinal reinforced-concrete girders running along the upstream surface of

the walls underneath the deck slab, and spaced at such intervals as will make square openings over which the deck slabs are laid. The ability to reinforce in the two directions allows an appreciable saving in cost. This type is indicated in Fig. 119.

Formulæ for Reinforcement of Concrete.—The formulæ hereafter given are those which are in practical use. They are approximate, and dependent on the assumption that the neutral axis in a concrete beam, or slab, which is reinforced on the tension

FIG. 118.—Austin dam during construction. End view.

side only, is $\frac{5}{8}d$ from the center of the reinforcing steel, d being the thickness of the beam, in inches, less the depth that the steel is embedded in the concrete.

For tension in the steel:

If M = moment of load on beam, in inch-pounds.
S_s = steel stress inch-pounds, per square inch.
a = area of steel, in square inches.
d = distance of steel from compression side, in inches.

Then

$$M = \frac{7daS_s}{8} \tag{176}$$

$$a = \frac{8M}{7dS_s} \tag{177}$$

If b = breadth of beam, in inches, its cross-sectional area is bd sq. in., neglecting that portion lying below the steel. That portion is not to be regarded as adding to the effective area in flexure, but simply as a protecting covering for the steel.

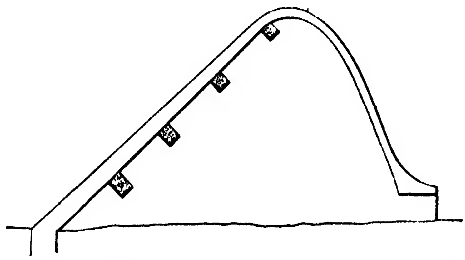


FIG. 119.—Hollow dam with longitudinal deck beams.

For compression in the concrete

$$M = \frac{bd^2S_c}{6} \tag{178}$$

S_c being the maximum compression in the extreme element of the beam, in pounds, per square inch.

Bending moments of uniformly loaded beams are;
Simple beams, supported but not fixed at ends,

$$M = \frac{Wl^2}{8} \text{ lb.-ft.} = \frac{3Wl^2}{2} \text{ lb.-in.}$$

W = weight per foot length, l = clear span, in feet.

Similarly, for beams fixed or cantilevered at both ends, *i.e.*, continuous over both supports,

$$M = \frac{Wl^2}{12} \text{ lb.-ft.} = Wl^2 \text{ lb.-in.}$$

For beams having one end cantilevered and one end simply supported,

$$M = \frac{Wl^2}{10} \text{ lb.-ft.} = 1.2\, Wl^2 \text{ lb.-in.}$$

The maximum safe stresses which are allowable, is largely a matter of opinion.

Some engineers consider 500 or even 600 lb., per square inch, safe compressive stresses on concrete, as doubtless they are. In the construction of reinforced-concrete dams, however, the sections are usually very thin, and a little more concrete costs only a small additional amount and adds stability to the thin walls. For these reasons, the author uses 400 lb. per square inch as the maximum compressive stress in concrete.

The depth of slab required for a given maximum compression in the concrete, in pounds per square inch, may be computed from the following formulæ:

For simple beams,

$$d = l\sqrt{\frac{0.75W}{S_c}} \text{ in.} \tag{179}$$

For continuous beams,

$$d = l\sqrt{\frac{0.5W}{S_c}} \text{ in.} \tag{180}$$

For beams supported at one end and cantilevered at the other,

$$d = l\sqrt{\frac{0.6W}{S_c}} \text{ in.} \tag{181}$$

d = depth of slab from compression side to steel, in inches.
l = length of clear span, in feet.
W = weight on slab, per square foot.
S_c = maximum stress in concrete per square inch.

The depth of slab as computed from formulæ (179), (180), and (181) are independent of the required area of steel and the unit stresses in it.

Most engineers limit the steel stress to 12,000 or 14,000 lb. per square inch, on the ground that any greater stress will allow such extension of the metal that the concrete will crack. If high elastic limit metal be used—such as standard twisted bars—this theory does not hold. For my own practice, 16,500 lb. per square inch has been adopted.

After fixing the maximum stresses allowable, there results a ratio of steel area to that of the concrete cross-section, neglecting always, the 1½ or 2 in. on the lower side of a beam or slab, which lies below the steel. This ratio is constant and, once fixed, a

computation of depth of slab may be made directly, and the steel area calculated without use of the bending moment formulæ.

Call p the *steel ratio*. This, by definition, is

$$p = \frac{a}{A} = \frac{a}{bd}, \text{ or } a = pA = pbd.$$

For $b = 12$ inches, $a = 12dp$.

a = area of reinforcing steel, in square inches.

$A = bd$ = area of concrete (exclusive of portion outside steel on tension side), in square inches.

b = breadth of beam or slab = 12 inches, for unit section.

d = depth of beam from compression side to steel, in inches.

S_s = stress allowed in steel, per square inch.

S_c = stress allowed in concrete, per square inch.

From equations (176) and (178),

$$\frac{a}{bd} = \frac{8S_c}{42S_s} = p \qquad (182)$$

For $S_c = 400$ lb. per square inch, and $S_s = 16{,}500$ lb. per square inch.

$$p = \frac{a}{bd} = \frac{8 \times 400}{42 \times 16{,}500} = \frac{1}{217} = 0.00462 \qquad (183)$$

Hence, $a = 0.00462bd = 0.0554d$, for 12-in. width of beam or slab.

Where a specific ratio of steel to concrete is desired, the following formulæ are useful to determine the depth of a uniformly loaded beam, or slab, required to resist a given applied load.

For simple beams

$$d = l\sqrt{\frac{0.1424W}{pS_s}} \text{ in.} \qquad (184)$$

For continuous beams

$$d = l\sqrt{\frac{0.0952W}{pS_s}} \text{ in.} \qquad (185)$$

For beams having one end simply supported, the other end cantilevered

$$d = l\sqrt{\frac{0.1143W}{pS_s}} \text{ in.} \qquad (186)$$

l = span of slab in feet.

W = load in pounds per square foot.

When the effective depth, d, of a slab is known, the spacing of reinforcing rods, to give a tension of 16,500 lb. per square inch in the steel, may be computed directly from the following formulæ.

TABLE 33.—FORMULÆ FOR SPACING OF REINFORCING RODS

Kind of beam	Size of reinforcing rod		
	1 in. sq.	¾ in. sq.	½ in. sq.
Simple, $M = \frac{2Wl^2}{3}$	$\frac{173{,}250d}{M}$	$\frac{97{,}450d}{M}$	$\frac{43{,}310d}{M}$
Simple continuous, $M = 1.2Wl^2$ ...	$\frac{208{,}900d}{M}$	$\frac{116{,}940d}{M}$	$\frac{51{,}970d}{M}$
Continuous, $M = Wl^2$	$\frac{259{,}900d}{M}$	$\frac{146{,}200d}{M}$	$\frac{64{,}970d}{M}$

d = depth of beam, in inches, from compression side to center of reinforcing rods.
M = moment, in pound-inches.

Obviously, for any value of the tension, other than 16,500 lb. per square inch, the spacing will be directly proportional to the allowable stress adopted.

The foregoing formulæ are for slabs reinforced in one direction only. When the construction admits of square slabs, supported around all four sides, the reinforcement should run in two directions, transverse to each other. This results in a considerable saving of concrete, for in computing the value of d, take $\frac{W}{2}$ for the value of W. This gives, of course, a slab having a thickness equal to 71 per cent. of the thickness it would have if reinforced in one direction and supported on two of its sides only.

Also the thickness of the slab may be reduced with respect to shear when it is the predominating stress, as is indicated later.

The steel is, likewise, computed as before, *i.e.*, $a = pd$, but twice this area of steel is required because a, as computed, is the amount required for reinforcement in one direction only, and based on a load of $\frac{W}{2}$. An equal amount is required for reinforcement in the other direction. This method is an approximation which errs on the side of safety.

The laws governing square, reinforced-concrete slabs, supported at all four sides, are not clearly developed. Tests show a far greater strength than the foregoing methods of computation would indicate, especially when the reinforcing bars are spaced logarithmically, *i.e.*, the distance apart decreasing as the middle of the slab is approached; the spacing of the bars being laid out from a logarithmic scale. This scale is conveniently made by proportionate measurements from the lower scale of a slide rule.

In order to supply the tensile strength at the upper side of the slab necessary for it to act as a continuous beam where it passes over a point of support, steel rods must be placed in the top of the slab, or the bottom rods bent up to the top of the slab.

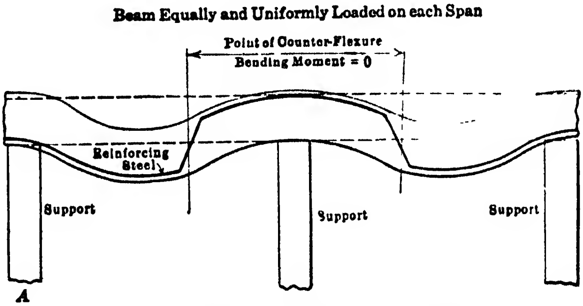


FIG. 120.—Showing flexure in beams.

Figure 120 indicates the stresses in a beam, uniformly loaded, and continuous over a support, the flexures being greatly exaggerated, for clearness. It is better practice to carry the bottom steel straight across the middle support at the lower level, and place additional rods in the top of the slab, which should be $\frac{l}{2} + t$ in length, t being the width of the support. This will give a proper length of cantilever rod projecting on either side of the support, namely, $\frac{l}{4}$. Unless this steel reinforcement is placed in the upper surface of the slab over the supports, it can not be considered as a continuous, but only as a simple beam.

The bending moments of the cantilevered portion and of that part in flexure, are equal when the upper reinforcement over the middle support extends a distance $= \frac{l}{4}$ on either side of the

support. Hence, the area of steel for this reinforcement should be just the same as that of the main lower bars.

Tension, Shear, Adhesion.—Good 1 : 2 :4 concrete, has a *tensile strength* varying from 175 to 300 lb. per square inch. The usual value adopted as having an ample margin of safety, is 50 lb. per square inch.

The safe value adopted for *shear* varies from 50 to 60 lb. per square inch.

After certain depths of water are reached, the thickness of the deck slabs is computed for resistance to shear instead of for flexure, as the thickness required to resist flexure is insufficient to resist the shear.

For plain buttress-supported dams, the depth at which the shearing stress equals the flexure stress is about 50 ft. For dams carried on both longitudinal and transverse supports, with square deck panels supported on four sides, the shear begins to predominate at a depth of about 75 ft. Hence, for depths under 50 ft. the thickness of the deck slabs is computed for flexure only, while for depths above 75 ft., the slab thickness is computed only for shear. Between 50 and 75 ft. depth, the slab thickness is computed both for flexure and for shear, and the greater thickness adopted.

The depth of water is, of course, $H + h$.

The formulæ for thickness, d_s, of slab to resist shear are, for dams on parallel buttresses,

$$d_s = \frac{Wl}{1440} \text{ in.} \tag{187}$$

For dams supported on walls running in both transverse and longitudinal drections,

$$d_s = \frac{Wl}{2160} \text{ in.} \tag{187a}$$

W = total load on deck, lb. per sq. ft.
l = length of clear span between supports, in feet.

These formulæ are based on a unit stress of 60 lb. per square inch.

The *adhesion* to steel reinforcing bars is taken at 50 to 60 lb. per square inch of steel surface embedded in the concrete for smooth rods, and 100 to 150 lb. per square inch for deformed rods. Adhesion to twisted bars may be taken as from 75 to 100 lb. per square inch. Actual tests show an ultimate bond strength

for smooth, undeformed bars of from 200 to 400 lb. per square inch.

Where the length of reinforcement required exceeds the length of a single standard steel bar, additional bars are used, and the connection from bar to bar is made by simply lapping them, one past the other, the concrete acting as a coupling for the two. The length of lap must not be less than twenty-four times the diameter of the bar.

Mixing Concrete.—The production of good concrete requires watchful care. The best results are obtained by proportioning the ingredients to fill the voids in the aggregate, (broken stone or gravel), rather than any specific and unchanging ratio of quantities.

The aggregate should be tested for voids about once each hour, and oftener, if it varies considerably in density. The void test simply comprises filling an iron pail, which contains a known volume, with aggregate, well shaken down and level full. An accurately graduated, measuring glass is filled with water, and the water poured into the vessel of aggregate, until the pail will hold no more. The amount of water poured into the vessel represents the volume of the voids in the known volume of aggregate. Then the quantity of sand required for good concrete is the amount necessary to fill the voids in the aggregate, plus 5 per cent. as a factor of safety, and the amount of cement required must be sufficient to fill the voids in the mixture of sand and aggregate, less 5 per cent. as allowance for water of crystallization.

The aggregate may be of broken stone, (granite limestone, sandstone or trap), or river gravel. It must be free from clay or soil of any kind. If stone, it must be crushed so that any piece of it will pass through a 1½-in. ring. Small pieces of stone are essential for reinforced-concrete work. If a piece should measure 2 in. in any direction it might abut at one end against a reinforcing bar, and the other end would strike against the form on the tension side of a slab, and thus two surfaces of the stone would not be covered by, or bonded with, the concrete.

Waterproofing.—The concrete for the deck slabs, and other portions of a dam under pressure, must be made waterproof, which is accomplished by mixing some substance with the concrete which causes it to become denser than it ordinarily does and closes up the pores so that no water can pass through it.

There are many excellent materials which fulfill this condi-

tion, most of them known under some trade name and having a specific brand. Hydrated lime is used extensively for this purpose and is thoroughly satisfactory.

The proper proportion of hydrated lime is from 7 to 10 per cent. of the amount of cement used, by weight. Eight per cent. of hydrated lime was used in constructing the Austin dam, and the inside surfaces of the deck slabs have remained dry for a year and a half, under a 65-ft. head on the outside of the slabs.

Reinforcing Steel.—While there is no need of a steel bar being notched, twisted, or otherwise deformed, in order for the concrete to take a firm hold on it, there is a feeling of security about the use of deformed bars and the surety that, under no conditions, can they slip, longitudinally, in the concrete. The principal value of twisting the bars is the reduction in ductility and increase in the elastic limit.

For the reinforcement of concrete, a bar having a great ultimate strength and a low elastic limit is of less value than one having a much less ultimate strength, but a greater elastic limit. The reason is that the bar can never be stressed to its ultimate strength, as the concrete structure would probably fail after the elastic limit is passed, because the elongation of the steel would be sufficiently great to shift the neutral axis toward the compression side, producing cracks in the tension side and crushing on the compression side. It is better, therefore, to take the elastic limit as the ultimate strength of the reinforcing steel and proportion the factor of safety to this value.

Some engineers hold different views and prefer metal having a high ductility, enough of it being used to make the concrete safe against cracking where under stress.

A short general specification for reinforcing steel is as follows:

"The steel rods must be of best quality, cold-twisted bars, of full dimensions. Weight, more than 3 per cent. in excess of normal not to be paid for. The bars when tested at random must show an elastic limit above 50,000 lb. per square inch, an ultimate strength above 85,000 lb. per square inch and an elongation of 8 per cent. or more.

Bars of 3/4-in. square, must bend through 180° around a radius equal to twice the thickness of bar without fracture.

Bars of 3/4-in. section and larger, must bend through 180° around a radius equal to three times the thickness of the bar, without fracture."

Of 22 tests made on hot-twisted bars, the minimum elastic limit was 55,200 lb. and the maximum 74,000 lb. per square inch.

The minimum ultimate strength was 90,700 lb. and the maximum 119,100 lb. per square inch, while the minimum elongation was 14.7 per cent. and the maximum was 20 per cent.

Calculation of a Reinforced-concrete Dam.—As an example, the following computations are given for a 60-ft. dam with a 10-ft. thickness of water over the crest.

Adopt spacing of buttresses 20-ft. apart, center to center.

For preliminary computations, assume angle of the deck with the vertical, or $\theta = 48°$, and the average thickness of the buttress walls $= 1.5$ ft.

Clear span between buttresses $= l = 18.5$ ft.

The outline of the section will be as shown in Fig. 121. The parabolic curve of the spillway is laid out down to a point, say 20 ft. below the crest, exactly in the same manner as before described in the discussion of the design of solid masonry dams. The spillway deck is given as small a slope as practicable and yet keep the overflowing water from springing clear of it. This slope is made at an angle of 24° with the vertical.

The toe is curved to a radius $= \frac{H}{2} = 30$ ft., as shown. It is usual to make the toe end at an elevation of from 1 to 2 ft. above normal low-water level.

Adopt the type of structure having no longitudinal walls, the panels simply acting as beams from buttress to buttress.

Unit stress in concrete 400 lb. sq. in. maximum.

Unit stress in steel 16,500 lb. sq. in. maximum.

Then by eq. 183, the steel ratio, p, becomes

$$p = 0.00462$$

Assume that expansion joints will be placed 40 ft. apart, that is, one joint for each two panels; this will make each longitudinal element of the upstream deck a beam that is partly a continuous girder and partly a simple beam, as previously shown in Fig. 120. Hence, the bending moment of the panels, per unit width, can be taken as $\frac{Wl^2}{10}$ lb.-ft., or $1.2Wl^2$ lb.-in. Pressure of water at bottom of deck slab $= W = (60 + 10)62.5 = 4375$ lb. per square foot.

From formula (186), and using values of l, p, and W, as given, the thickness of slab at the bottom to carry the water pressure only, and neglecting the weight of the slab itself, is

$$d_1 = 18.5\sqrt{\frac{0.1143 \times 4375}{0.00462 \times 16{,}500}} = 47.3$$

$$D_1 = 47.3 + 2 = 49.3 \text{ in.}$$

D, or D_1, is the total thickness of the slab which is equal to the distance from the compression side to the steel, plus the 2 in. protective covering for the steel, or

$$D = d + 2 \text{ in.}$$

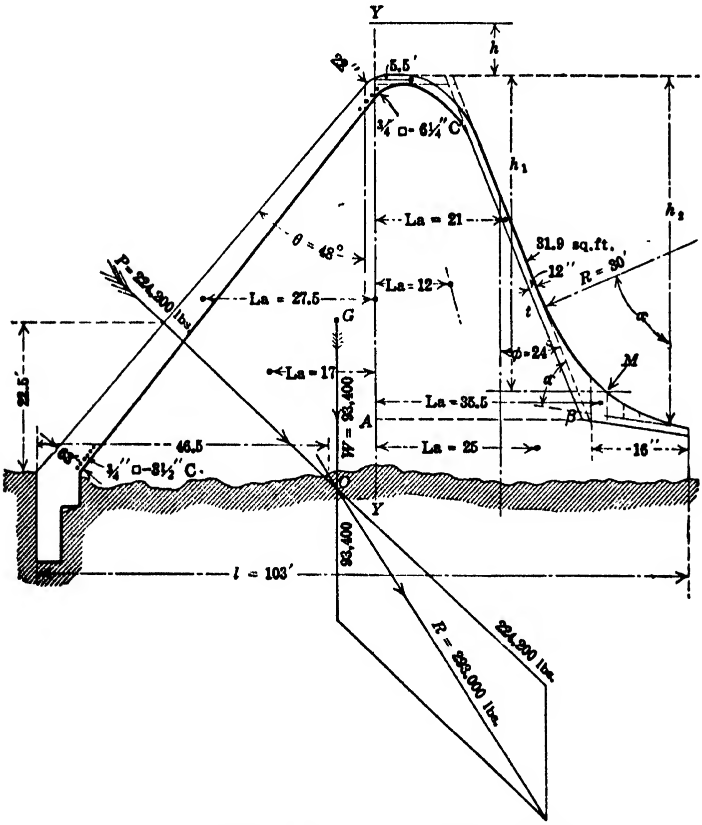


FIG. 121.—Design of a hollow reinforced-concrete dam.

To resist shear, the required thickness of the slab will be, from formula (187),

$$d_s = \frac{4372 \times 18.5}{1440} = 56.2 \text{ in.}$$

This, however, is the thickness required for the water pressure only. To this must be added the weight of the slab itself. Assume that this additional load will increase the slab thickness 2.8 inches, making the total thickness 59 in.

$$\text{Weight of slab} = \frac{59}{12} \times 140 = 689 \text{ lb. per sq. ft.}$$

This weight acts vertically downward. The component of this weight, normal to the deck, is $689 \times \sin\theta = 689 \times 0.743 = 512$ lb. per square foot.

Total load on slab $= 4375 + 512 = 4887$ lb. per square foot.

$$d_s = \frac{4887 \times 18.5}{1440} = 62.8 \text{ in.}$$

Which is the thickness of the deck slab at the bottom. Taking this at 63 in., the area of the steel for a 12 inch width of slab is, from formula (177).

$$a = \frac{8 \times 4887 \times (18.5)^2}{7\,(63 - 2)\,16{,}500} = 1.9 \text{ sq. in.}$$

If ¾ in. square bars are used, the area of a single bar will be 0.5625 sq. in. The spacing apart of bars is given by the formula

$$f = \frac{12a_0}{a} \text{ in.}$$

f = distance apart of bars, center to center, in inches.
a_0 = area of a single bar, in square inches.
a = total area required per foot width of slab, in square inches.

For this case,

$$f = \frac{12 \times 0.5625}{1.9} = 3.55, \text{ say, } 3.5 \text{ in.}$$

The load on the slab at the top of the dam is

$$10 \times 62.5 = 625 \text{ lb. per square foot.}$$

Required thickness of slab for load, neglecting weight of slab, is

$$d_{1t} = 18.5\sqrt{\frac{0.1143 \times 625}{0.00462 \times 16{,}500}} = 17.9 \text{ in.}$$

$$D_{1t} = 17.9 + 2 = 19.9 \text{ in.}$$

Increase this thickness slightly to compute weight of slab.

$$\text{Call } D_{1t} = 21.5 \text{ in.}$$

Then weight of slab $= \frac{21.5}{12} \times 140 = 251$ lb. per square foot.

Component of this weight normal to deck $= 251 \times \sin \theta$

$$= 251 \times 0.743 = 186 \text{ lb. per square foot.}$$

Total weight is 811 lb. per sq. ft.

Then

$$d_t = \sqrt{\frac{0.1143 \times 811}{0.00462 \times 16{,}500}} = 19.6 \text{ in.}$$

$$D_t = 19.6 + 2 = 21.6 \text{ in.} \quad \text{Adopt 22 in.}$$

Area of steel, per foot width of slab $= 12 \times 0.00462 \times 19.6 =$ 1.087 sq. in.

Spacing of ¾-in. square bars $= \frac{0.5625 \times 12}{1.087} = 6.21$ in.

Adopt 6.25 in.

Hence, the upstream deck diminishes in thickness uniformly, from 63 in. at the bottom to 22 in. at the top, and the spacing of the reinforcing rods gradually diminishes from 6¼ in. at the top to 3.5 in. at the bottom.

The thickness of the crest is arbitrarily fixed at 18 in., and that of the spillway deck at 12 in. The reinforcement of the crest should be of ¾-in. rods, spaced about 7½ in. apart, center to center.

The reinforcement of the spillway deck is arbitrarily fixed at a proper amount of steel to prevent cracking. The actual water pressure against the spillway deck is very small, and at high-water stages the water may actually leap beyond it without imposing any pressure on it at all.

A satisfactory reinforcement of the spillway deck is ½-in. rods spaced on 10-in. centers. This deck should be thickened considerably at the toe where the water is deflected from an inclined to a horizontal direction. There is a heavy pressure caused by the water at this point, which can be computed by the formula

$$T = \frac{222 \times FV^2}{r\alpha} \times \frac{\sin \alpha}{2} \tag{188}$$

in which T = pressure against curved portion of the toe in pounds per square foot, and normal to the surface at every point.

α = angle through which water is deflected = angle subtended between radii at ends of curve.

r = radius of curvature, usually $= \frac{H}{2}$.

V = velocity of water flowing over toe

$= 8.025\sqrt{h_v + \frac{h}{2}}$, in which h_v = height from crest of dam to middle point of curvature of toe, and h = depth over crest.

F = depth of water at toe $= \frac{3.33h^{3/2}}{V}$.

This formula may also be written

$$T = \frac{740h^{3/2}V}{r\alpha}\sin\frac{\alpha}{2}, \text{ lb. per square foot.} \qquad (188a)$$

In determining the quantity of reinforcing steel necessary, it is sufficiently accurate, and good practice, to take the whole toe and treat it as if it were of a uniform thickness, equal to its average thickness. In this instance its horizontal length = 16 ft., its average thickness 3.75 ft. = 45 in. This dimension is fixed arbitrarily by the designer, and in most dams is much greater than 4 ft., though for what reason the author is unable to say.

Average weight of toe per square foot = 3.75 × 140 = 525 lb.

Vertical component of water thrust is found from formula (188a)

$$h^{3/2} = 31.6, \quad h_v = 47.5, \quad \frac{h}{2} = 5$$

Then, $V = 8.025\sqrt{52.5} = 58.28$ ft. per second.

Also, $\alpha = 58°$, $\sin\frac{\alpha}{2} = 0.4848$.

$$r = \frac{60}{2} = 30 \text{ ft.}$$

Substituting the values found

$$T = \frac{740 \times 31.6 \times 58.28 \times 0.4848}{30 \times 58} = 378 \text{ lb. per square foot.}$$

Since the pressure is normal to the surface, this computed value is the pressure per square foot of horizontal section.

Adding this water thrust to the weight of the slab, the total stress is 903 lb. per square foot.

Area of reinforcing steel $= a = \dfrac{8 \times 1.2 \times 903 \times (18.5)^2}{7(45 - 2) \times 16{,}500} =$ 0.578 sq. in. per foot width of toe. (Equation (177).)

Spacing of steel, ¾ in. square, over bottom of toe is

$$\frac{12 \times 0.5625}{0.578} = 11.7 \text{ in.}$$

Adopt 11 in.

(0.5625 = area of a ¾ in. square bar)

To find the resultant of the forces acting on the dam and its position, the center of gravity is found by the method of moments. In this case, the axis Y-Y is a convenient reference line. All quantities to the left of Y-Y are negative; all to the right, positive.

For convenience, the figure is treated as a trapezoid having a flat top. This assumption is productive of only a negligible error. Dimensions and lever arms are scaled from the drawing.

Taking quantities on the right of Y-Y and remembering that for this condition, volumes instead of areas must be considered, and hence, each area must be multiplied by the longitudinal dimension of the section in order to find the volume,

Area of toe from point of tangency, t, to end = 99 sq. ft.

Longitudinal dimension = length from buttress to buttress = 20 ft. center to center measurements.

Volume = 99 × 20 =	1980 cu. ft.
Lever arm = 35.5 ft.	
Moment = 1980 × 35.5	70,290 vol. ft.

Area of spillway deck from crest to point of tangency, t = 31.90 sq. ft.

Volume 31.90 × 20 =	638 cu. ft.
Lever arm = 21 ft.	
Moment = 638 × 21	13,398 vol. ft.
Area of crest = 18.7 sq. ft.	
Volume = 18.7 × 20 =	374 cu. ft.
Lever arm = 5.5 ft.	
Moment = 374 × 5.5 =	2057 vol. ft.

Area of buttress wall from crest down to line AB = 1118 sq. ft. Thickness taken at 1.5 ft.

Volume = 1118 × 1.5 =	1677 cu. ft.
Lever arm = 12 ft.	
Moment = 1677 × 12 =	20,124 vol. ft.

Area of buttress wall below line AB = 400 sq. ft.

Volume = 400 × 1.5 =	600 cu. ft.
Lever arm = 25 ft.	
Moment = 600 × 25 =	15,000 vol. ft.

Totals on right of axis Y-Y

Volumes	5269 cu. ft.
Moments	120,869 vol. ft.

On the left of the axis Y-Y.

Area of upstream deck = 298 sq. ft.	
Length of section = 20 ft.	
Volume = 298 × 20 =	5960 cu. ft.
Lever arm = 27.5	
Moment = 5960 × 27.5 =	163,900 vol. ft

Area of buttress wall = 1340 sq. ft.
Thickness of buttress wall = 1.5

Volume of buttress wall = 1340 × 1.5 = 2010 cu. ft.

Lever arm = 17 ft.	
Moment = 2010 × 17 =	39,170 vol. ft.
Total volume on left of axis =	7970 cu. ft.
Total moment on left of axis =	203,070 vol. ft.

Taking the moments on the right as negative, the sum of the moments is 203,070 − 120,968 = 82,201 vol. ft.

The sum of the volumes = 5269 + 7970 = 13,328 cu. ft.

Then the center of gravity lies to the left of the axis, and is a distance away from it $= \dfrac{82{,}201}{13{,}328} = 6.17$ ft.

This locates the position of the center of gravity G, as shown.

The total volume of a 20-ft. length of dam, including one buttress wall, has been found = 13,328 cu. ft. or 666.4 cu. ft. per foot length of dam.

Weight per foot length of dam = W = 666.4 × 140 = 93,396 lb. or, practically, 93,400 lb. total.

The total water pressure acting on the deck is

$$P = \frac{H}{2 \cos \theta}(H + 2h) \times 62.5 = \frac{60 \times 80 \times 62.5}{2 \times 0.669} = 224{,}200$$

(approx.).

Height of center of pressure above base is $\frac{60}{3}\left(\frac{60 + 30}{60 + 20}\right) =$ 22.5 ft.

Hence, the line representing the water pressure will pass through the face of the deck at a point 22.5 ft. (vertically), above the base. It will be normal to the deck, and its length, to scale, will be such as to represent 224,200 lb.

Constructing the parallelogram of forces from the intersection of the water and gravity forces, it is found, graphically, that the resultant intersects the base 46.5 ft. from the upstream and 56.5 ft. from the downstream side. In this case, the length of base clear out to the end of the toe is included, because the mass of concrete is bound together with metal reinforcement. The resultant intersects the base somewhat too far toward the upstream side, and the slope of the deck could be safely made greater, shortening the thickness through the base of the dam and reducing the foundation work required. This would, however, reduce the resistance to sliding and make it necessary to sink the footings well into the rock of the stream bed.

The total pressure acting vertically on one buttress wall footing = $(P \sin \theta + W)20$, P and W being the water and structure weights, respectively, per foot length.

$P \sin \theta = P_v = 224{,}200 \times 0.743$	166,580
$W =$	93,400
Total weight per foot length =	259,980

Weight on one buttress wall = 259,980 × 20 = 5,199,600 lb.

If the wall footing were 1 ft. thick, the average weight, per square foot, would be

$$\frac{P_v + W}{l} = \frac{5{,}199{,}600}{103} = 50{,}481 \text{ lb.}$$

The greatest pressure would be at the upstream side and equal to

$$S = 50{,}481\left(1 + \frac{6\left(\frac{103}{2} - 46.5\right)}{103}\right) = 65{,}170 \text{ lb. per square foot.}$$

Since the assumed limiting pressure on footings is 200 lb. per square inch, or 28,000 lb. per square foot, the width of footings must be $\frac{65,170}{28,000} = 2.33$ ft., theoretically. As a matter of fact, the footings would be spread out to 3 ft. wide or, possibly, 3.5 ft. The height above the rock to which the footings are carried, must be equal at least to their width, after which they are stepped in until the thickness is reduced to that adopted for the buttress walls.

Thickness of buttress wall at the bottom $= \frac{65,000}{50,000} = 1.3$ ft. $=$ 15.6 in., if the limiting wall stress is 50,000 lb. per square foot. This wall should be slightly thicker for a 60 height of dam.

No vertical supporting wall should be less than 12 in. thick. The top section should be 12 in. thick down to about the 40-ft. level; then increased to 16 in. thick down to the 20-ft. level, and from that point should be 18 in. thick, down to the footing.

At expansion joints, the buttress wall may be 16 in. thick all the way up from the 20-ft. level to the crest. This added width is to give a full 8-in. bearing to each of the two abutting ends of the decks which rest on it. It is customary to make the walls at expansion joints, the same thickness as the standard walls, and place corbels on their upstream surfaces, 18 in. wide, so that the deck slabs will have ample bearing surface. This saves material but, unless the walls are 50 ft. or more in height, the extra cost of the added form work will be greater than the value of the material saved.

Expansion Joints.—Figures 122 and 123 shows two forms of expansion joints for making a waterproof, sliding connection between adjacent ends of concrete structures. Fig. 124 is a detail of the flashing in the joint shown in Fig. 123.

The joint shown in Fig. 122 is simply a section of some special iron which resists corrosion. One-half of it is embedded in the concrete of one of the abutting sections. The concrete is allowed to harden with one edge of the metal projecting beyond it. The edge of the concrete and the projecting metal are heavily painted with asphalt, then the other, adjacent section of concrete is cast. When finished, the metal key is firmly gripped by one section and is free to slide in and out of the other. This is a simple and thoroughly satisfactory joint.

The size of iron plate varies with the head of water acting,

and the thickness of the walls. Roughly, plates $\frac{1}{4}$-in. by 4-in. cross-section should be used up to 12-ft. head, $\frac{3}{8}$-in. by 6-in. up to 30-ft. head, and $\frac{1}{2}$-in. by 8-in. for any head above 40 ft.

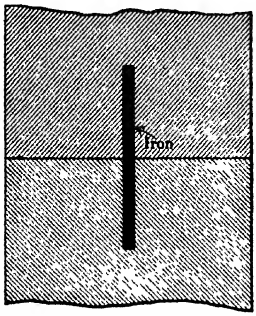


FIG. 122.—Tongue expansion joint.

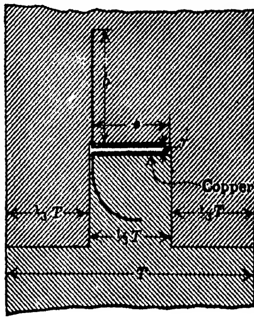


FIG. 123.—Dovetail expansion joint with flashing.

Having adopted a size of plate, it should be maintained in every part of the structure, for uniformity of construction.

The expansion joint shown in Fig. 123 is simply a dovetail made in the ends of the concrete sections, with a flashing of some soft metal, such as lead or copper.

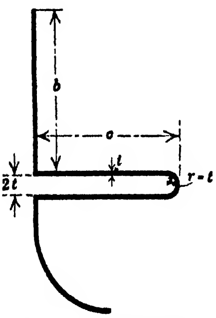


FIG. 124.—Flashing for dovetail expansion joint.

The end of one section is cast with one projecting leg of the flashing set in it. After this hardens, the end of the concrete section is painted, heavily, with asphalt, and the other section cast.

The proper proportions of the flashing are; $\frac{3}{16}$ in. thick up to 12-ft. head; $\frac{1}{4}$ in. thick up to 30-ft. head; and $\frac{3}{8}$ in. thick up to 100-ft. head. Separation of the leaves should be $2t$; radius $= r = t$, where t = thickness of metal, and $c = \frac{1}{3}T$, where T = thickness of wall.

$$b = c$$

(See Fig. 124 for b and c.)

Before pouring concrete where this joint is placed, always fill in between the leaves with a strip of wood that may afterward be withdrawn, to prevent spilled or leaking concrete from filling the space at any point.

Expansion joints should be spaced at intervals of $\frac{4080}{t_1 - t_2}$ ft., where the walls are free to move. Where they rest on foundations, or are thin and easily cracked, the spacing between consecutive joints should be $\frac{3000}{t_1 - t_2}$ ft.

t_1 = maximum temperature, in degrees F., to which structure will be subjected—this will, of course, be when the reservoir is empty.

t_2 = minimum temperature to which it will be subjected.

Thus, for a maximum temperature of 100°, and a minimum of 32°, distance apart of joints will be $\frac{4080}{68} = 60$ ft. In no case, however, should they be more than 100 ft. apart.

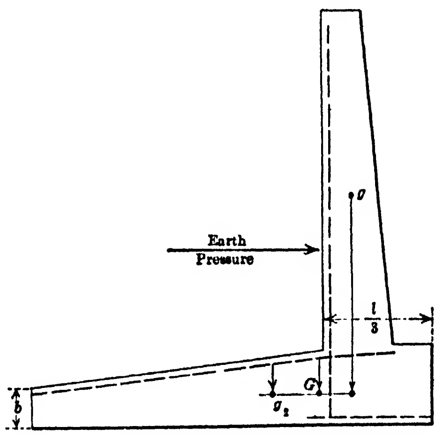


FIG. 125.—Reinforced-concrete retaining wall.

Reinforced-concrete Retaining Walls.—The general type of retaining wall, of reinforced concrete, is shown in Fig. 125.

The length through the base slab, for a factor of safety equal to F against overturning, is given by the formula

$$l = \sqrt{\frac{A + B}{C + G} + \left(\frac{E}{C + G}\right)^2} - \frac{E}{C + G} \qquad (189)$$

$A = 3Fwh^3 \tan^2\left(45 - \frac{\phi}{2}\right)$

$B = 9hD_1^2w_1$

$C = 9D_2w_1$

$E = 3hD_1w_1$
$G = 8wh$
D_1 = thickness of wall, in feet.
D_2 = thickness of slab, in feet.
w = weight of earth, per cubic foot.
w_1 = weight of concrete, per cubic foot.
h = height of wall, in feet.
ϕ = angle of repose of earth.

This formula is exact only for a section in which the thickness of the wall is constant from top to bottom, the thickness of the bottom slab constant over its whole length, and the distance from the earth face of the wall to the outer toe is $\frac{l}{3}$. The value of l will be too small for a wall having a batter, or a base slab which is thinner at the ends than at the point where the wall sets on it. In case these conditions exist, l should be increased from 5 to 10 per cent. above the computed value.

For the usual values of $w = 100$ lb., $w_1 = 140$ lb., $\tan^2\left(45 - \frac{d}{2}\right) = 0.286$, and factor of safety, F, $= 2$, the partial formulæ become

$A = 171.6h^3$
$B = 1260hD_1^2$
$C = 1260D_2$
$E = 420hD_1$
$G = 800h$

In designing a wall of this kind, the procedure is as follows:

First, the section of the wall, and the bottom slab, must be found. The wall acts partly as a gravity section but, principally, as a cantilever.

To find D_1

$$d_1 = \sqrt{\frac{24M}{w_1h + 48S_c}} \text{ in.} \tag{190}$$

and $D_1 = d_1 + 2$ in.

M = moment of earth tending to overturn the wall in

$$\text{pound-inches} = \frac{wh^3 \tan^2\left(45 - \frac{\phi}{2}\right)}{6} \times 12 \tag{191}$$

S_c = allowable compression stress in concrete, per square inch.

d_1 = thickness of wall, to center of reinforcement, in inches.
D_1 = total thickness of wall, in inches.
w_1 = weight of concrete, per cubic foot.
h = height of wall, in feet.

This formula is not exact, as certain negligible quantities have been omitted to simplify it.

The thickness of the base slab is now found, giving the values of d_2 and D_2.

$$d_2 = \sqrt{\frac{M}{2S_c}} \text{ in.} \qquad (192)$$

$$D_2 = d_2 + 2 \text{ in.}$$

Symbols having same meaning as for preceding equation.

As an example, take the design of a wall 21 ft. high, the proportions being such that the distance from the inner earth face of the wall to the toe $= \frac{l}{3}$, the usual values of w, w_1 and $\tan^2\left(45 - \frac{\phi}{2}\right)$, applying to this case. Limiting stress in concrete $= S_c =$ 500 lb. per square inch.

$$M = 100 \times 21^3 \times 0.286 \times \frac{12}{6} = 530{,}000 \text{ in.-lb.}$$

$$d_1 = \sqrt{\frac{24 \times 530{,}000}{140 \times 21 + 48 \times 500}} = 21.7 \text{ in.}$$

$$D_1 = 21.7 + 2 = 23.7 \text{ say, } 24 \text{ in.} = 2 \text{ ft.}$$

$$d_2 = \sqrt{\frac{530{,}000}{2 \times 500}} = 23 \text{ in.}$$

$$D_2 = 23 + 2 = 25 \text{ in.} = 2.08 \text{ ft.}$$

$$l = \sqrt{\frac{171.6 \times 21^3 + 1260 \times 21 \times 2^2}{1260 \times 2.08 + 800 \times 21} + \left(\frac{420 \times 21 \times 2}{19{,}420}\right)^2} - \frac{17{,}640}{19{,}420} = 8.46 \text{ ft.}$$

In the actual design, the thickness of the wall and slab might be tapered to advantage, so that the wall would have a thickness of, say, 1.5 ft. at the top and 2.5 ft. at the bottom, while the slab would be, say, 1.25 ft. thick at the inner end, 1.5 ft. thick at the toe and 2.7 ft. thick just under the wall. These dimensions will not, appreciably, change the quantities of concrete, and will greatly reduce the amount of steel reinforcement required. The steel reinforcement required in the wall and bottom slab, is determined by the formula

$$a = \frac{8M}{7dS_s} \text{ sq. in.} \qquad (177)$$

M = moment, in pound-inches, per foot length of wall, at whatever point in the height the moment is computed.

S_s = stress, per square inch, allowable in the steel.

d = depth of beam, from compression side to center of reinforcement.

a = area of steel, in square inches, per foot length of wall.

Obviously, the amount of steel required progressively diminishes from the bottom to the top of the wall, so that some of the rods will run from bottom to top, others only part of the way up to the top, and others a still less distance. The steel area for about each 5 ft. of wall height, should be computed and the amount of steel adjusted as nearly to these theoretical requirements as practical construction will permit. Too much refinement of construction may result in a greater cost than placing an excessive quantity of steel, and judgment must be exercised in fixing the actual design. Note that steel is required in the bottom of the base slab, extending from the line of the inner face of the vertical wall, out to the toe. The fact that this portion of the base slab is in tension is obvious.

Where walls are of considerable height—25 ft. and over—it is usually more economical to provide diagonal buttresses at intervals of from 8 to 15 ft., as shown in Fig. 126. The vertical walls and the base slabs are reinforced to act as uniformly loaded beams, from buttress to buttress, just like the slabs of reinforced-concrete hollow dams. The buttresses, with their diagonal tension rods, r, resist the overturning moment acting on the wall. The buttresses are anchored to the base slabs by the vertical rods, b, as indicated. The area of the steel to anchor the buttress to the slab is

$$a = \frac{16ML}{7NS_s} \text{ sq. in.} \tag{193}$$

M = overturning moment in pound-feet per foot length of wall.

L = distance between buttresses in feet.

N = width of bottom of buttress in feet.

S_s = stress in steel in pounds per square inch.

This assumes a uniform spacing of rods over the base of the buttress, from front to back.

The area of the tension steel, holding the wall against separating from the buttress, is fixed by the total earth pressure against the wall.

The area of the diagonal rods, a, is computed, just as for any cantilever,

$$a_t = \frac{8ML}{7dS_s} \text{ sq. in.} \tag{194}$$

d, in this case, is equal to the horizontal distance from the face of the wall to the center of the reinforcement, at whatever point in the height the computation is applied.

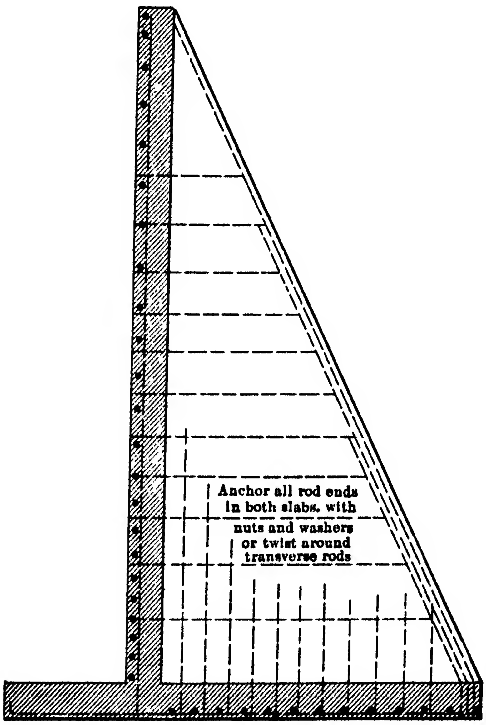


FIG. 126.—Reinforced-concrete retaining wall with buttresses.

The spacing of the buttresses should vary with the height of the wall. For walls up to 20 ft. in height, 5 to 6 ft. apart is an economical spacing. For 20 to 30 ft. height of wall, spacing should be between 7 and 8 ft., and for 30 to 40 ft. height, spacing should be 9 to 10 ft.

The buttresses, being only, partly, compression members, and principally carriers, separators and protectors of the steel rein-

forcement, may be made comparatively thin—from 10 in. thick for a 20-ft. wall to 16 in. for a 40-ft. wall.

After making designs of a retaining wall, the foundation pressure should be computed, and if it is found that the pressure at the toe is too great, or is negative (tension) at the inner end of the base slab, the base should be lengthened until the maximum pressure is within safe limits, and the negative pressure is brought to zero, or given a positive value. By adding an arbitrary 5 or 10 per cent. to the value of l, and then computing the moments of weight of section, plus weight of superimposed earth as the resisting moment, opposed to that of the sliding earth, the foundation pressures for the new length of base can easily be determined in the same way that they are for dams, and as indicated in that section of this work on "Foundations of Dams," wherein this subject is more fully discussed.

Earth Dams.—Earth dams are seldom used for water-power developments. They possess the fatal defect that no water can be allowed to pass over them, and, no matter how well they may be constructed, any flow over the crest will, inevitably, destroy them.

Occasionally, the topography of the region near a river is such that a dam will cause the water to flow around through a side depression, so that an earth dike must be constructed to prevent this. Also, at some dam sites, the rate of slope of one bank, upward from the stream bed, is very small, and the crest of a masonry dam running across the stream and far beyond, in order to reach a given contour, would be abnormally long. In such cases, the concrete, or masonry dam should be made long enough to afford an ample spillway for the heaviest known floods, and that portion of the dam, from the end of the spillway on out to the point where the desired contour is reached, may be an earth dike.

The usual method of earth dam construction is to build a masonry, or concrete, core wall, ranging from 1 to 3 ft. through at the top. It is battered in both sides, the usual batter being between 16:1 and 20:1. This wall must be sunk in a ditch which is excavated down to rock, or well into "hard-pan." The earth is then compacted on either side of the wall. The wall may be built up as the construction of the earth bank proceeds, being kept slightly higher than the earth.

The site must be cleared of all vegetation and top soil and a good bearing surface reached.

The proportions of the mixtures of different kinds of earth vary greatly with the character of the materials, the methods of handling and the personal views of the engineer. The thickness on top ranges from 6 to 30 ft. A formula which seems satisfactory is; top thickness $= 5 + 0.2H$, H being the height of the dam. The top of the core wall usually comes within 3 or 4 ft. of the top of the dam and 4 to 12 ft. above the water level.

The earth slope on either side of the wall ranges between 1 : 1½ and 1 :3.

The materials must be carefully selected, graded and compacted. The earth is always deposited in layers; then wet down and compacted by tamping and ramming.

The upstream face of an earth dam is usually covered with rip-rap or, in some cases, a layer of screened, coarse gravel on which is laid a pavement of concrete—5 to 8 in. in thickness.

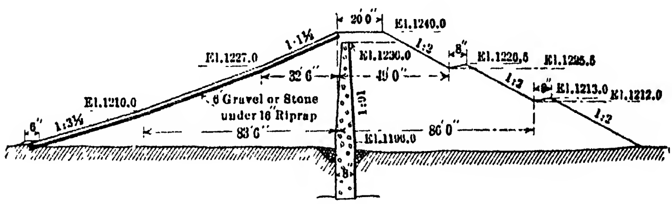


FIG. 127.—Section of the Hinckley earth dam.

One of the most important causes of failure of earth dams is the seepage of water along the side of a pipe or conduit laid through them. Any passage through an earth dam should be avoided, if possible. If necessary to lay a pipe through one, surround the pipe with a series of concrete rings cast around it, of not less than 2 ft. greater diameter than the pipe, and the outer diameter of the ring should be twice the diameter of the pipe when the latter is greater than 2 ft. in diameter. The thickness of these rings should range from 6 in. for a 3-ft. diameter, to 12 in. for one of 6 ft. Preferably, these rings should be reinforced with steel hoops. The number of rings should be from 3 to 5, beginning 2 ft. from the upstream face of the dam and spaced 4 to 6 ft. apart.

The earth must be compacted against the pipe and leakage rings with special care, and a mixture of clay with the run of

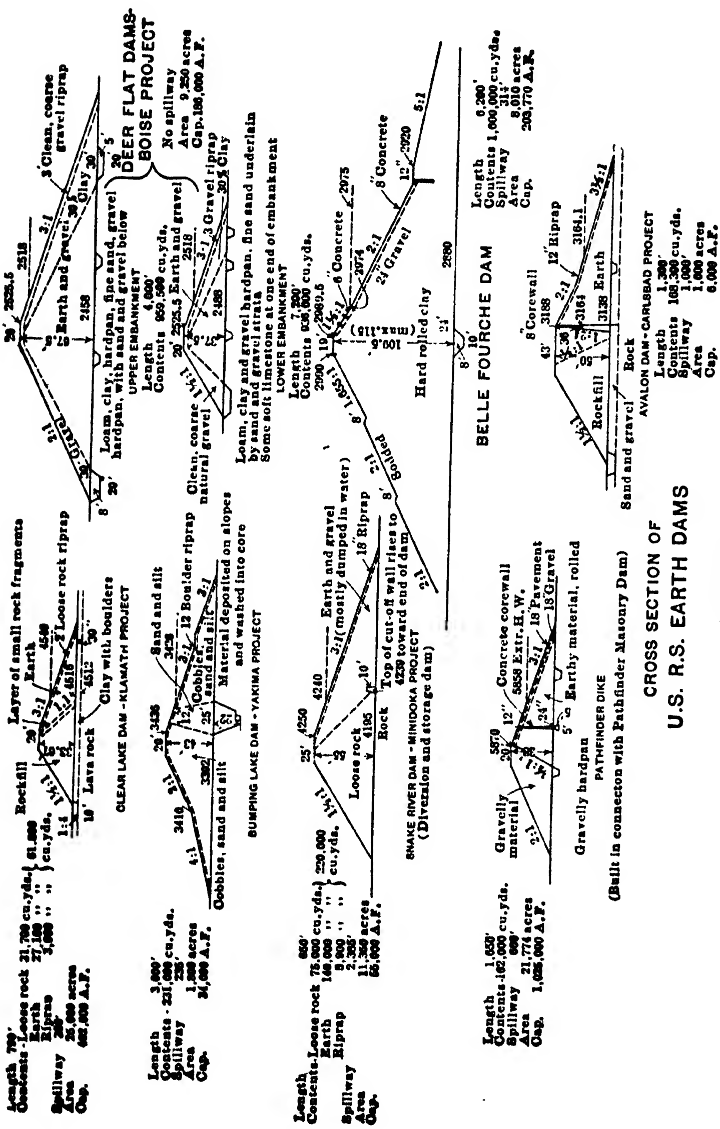

Length 700'
Contents - Loose rock 31,700 cu.yds.
Earth 27,100 " "
Riprap 3,000 " "
61,800 cu.yds.
Spillway 300'
Area 25,000 acres
Cap. 462,800 A.F.
Rockfill
Layer of small rock fragments
Earth
Loose rock riprap
Lava rock
Clay with boulders
CLEAR LAKE DAM - KLAMATH PROJECT
Length 3,000'
Contents - 231,000 cu.yds.
Spillway 235'
Area 1,300 acres
Cap. 34,000 A.F.
Sand and silt
12 Boulder riprap
Cobbles, sand and silt
Cobbles, sand and silt
Material deposited on slopes and washed into core
BUMPING LAKE DAM - YAKIMA PROJECT
Length 650'
Contents - Loose rock 75,000 cu.yds.
Earth 140,000 " "
Riprap 5,900 " "
220,000 cu.yds.
Spillway 2,365'
Area 11,350 acres
Cap. 55,000 A.F.
Earth and gravel
3:1 (mostly dumped in water)
18" Riprap
Loose rock
Rock
Top of cut-off wall rises to 4239 toward end of dam
SNAKE RIVER DAM - MINIDOKA PROJECT
(Diversion and storage dam)
Length 1,650'
Contents - 162,000 cu.yds.
Spillway 900'
Area 21,774 acres
Cap. 1,025,000 A.F.
Gravelly material
Concrete corewall
5858 Extr. H. W.
18" Pavement
18" Gravel
Earthy material, rolled
Gravelly hardpan
PATHFINDER DIKE
(Built in connecton with Pathfinder Masonry Dam)
Gravel
Earth and gravel
3' Clean, coarse gravel riprap
Clay
Loam, clay, hardpan, fine sand, gravel hardpan, with sand and gravel below
UPPER EMBANKMENT
Length 4,000'
Contents 959,500 cu.yds.
DEER FLAT DAMS - BOISE PROJECT
No spillway
Area 9,250 acres
Cap. 186,000 A.F.
Clean, coarse natural gravel
Earth and gravel
3 Gravel riprap
Clay
Loam, clay and gravel hardpan, fine sand underlain by sand and gravel strata
Some soft limestone at one end of embankment
LOWER EMBANKMENT
Length 7,200'
Contents 936,000 cu.yds.
6" Concrete
8" Concrete
24" Gravel
Bolded
Hard rolled clay
BELLE FOURCHE DAM
Length 6,200'
Contents 1,600,000 cu.yds.
Spillway 31½'
Area 8,010 acres
Cap. 203,770 A.F.
8" Corewall
12" Riprap
Rockfill
Earth
Sand and gravel
Rock
AVALON DAM - CARLSBAD PROJECT
Length 1,300'
Contents 168,300 cu.yds.
Spillway 1,000'
Area 1,000 acres
Cap. 6,000 A.F.
CROSS SECTION OF
U.S. R.S. EARTH DAMS

FIG. 128.

earth should be used to insure the prevention of the passage of water through the dam along the surface of the pipe.

The downstream side of an earth dam is sometimes stepped off with berms at vertical intervals of 10 to 15 ft. The berms are from 6 to 8 ft. wide. The whole of the exposed portion of the dam should be sodded, and a thick grass covering cultivated.

Drains should be provided, of tile pipe, from the bottom of the downstream side of the core wall, to the downstream face of the dam. Three-inch tile pipes spaced on 6 to 10-ft. centers and ending a foot from the core-wall face make a satisfactory draining system.

Figures 127 and 128 show examples of earth dams indicative of general practice.

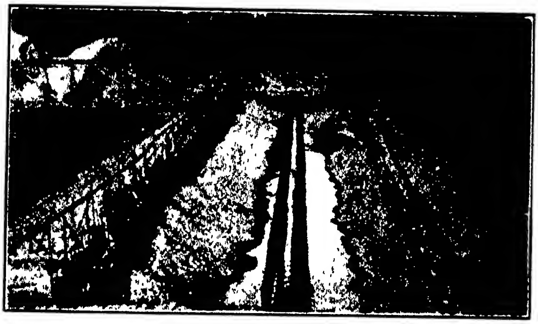


FIG. 129.—Second-stage flumes and artificial core, Conconully dam.

Hydraulic Fill Dams.—These are simply earth dams in which the material is moved by sluicing it down, then pumping the liquid, containing the earth in suspension, into troughs or chutes, which deliver it to the site of the dam. The water drains away, leaving behind the earth, and in this way the embankment is built up.

The initial portions are formed by placing planks or gravel around the boundary of the base to prevent the material from spreading too greatly. The embankment is usually brought up to near its full height in two parallel banks or ridges, one upstream and one downstream, with a depression in the middle which is filled with the water escaping from the liquid material.

Figure 129 shows the two banks with the pool of water between them, and also the core that is being placed along the middle of

the dam. The troughs, or sluiceways, which carry the liquid and distribute it over the banks are, also, clearly depicted.

After the dam has risen to near its ultimate height, a single flume discharges into the space lying between the two parallel banks, filling it and finally bringing the crest of the dam up to the desired elevation.

FIG. 130.—Third-stage flumes, Conconully dam.

Figure 130 shows a dam, nearly completed, with the single distributing flume over its crest. Both of these illustrations are of the Conconully dam of the United States Reclamation Service, and are taken from the *Proc.* Am. Soc. Civ. Eng. of April, 1911, in which a complete paper by Henny describes this dam.

CHAPTER VIII

MOVABLE CRESTS FOR DAMS

Due to the variable flow in streams, the elevation of the lake formed by a dam changes considerably from periods of high to low water. The minimum elevation is that to which the water is drawn down during periods of drouth, while the maximum elevation is that due to the height of dam, plus the thickness of water over the crest during floods. Obviously, the head acting on the water wheels, and the amount of water stored in the lake are both minimum during the time of low water. Since the critèrion of the value of a water power is the amount of power which may be continually developed during the period of low water, it is evident that any means whereby the lake level may be kept higher than the crest of the dam during such times will increase the value of the entire investment. It is equally clear that the level of the lake, during times of flood, must not be increased by such means as may be adopted to augment the power and storage volume for normal or low-water periods.

Many appliances have been devised to be placed on the crest of dams which, under normal conditions, increase the height of the dam, raising the water level above that of the crest, and during times of flood are in some manner moved off or away from the crest, so that the flood water pours over the normal crest of the dam. Some of these devices are automatic in their action, while others are moved at will, either manually or by power. A few of the most important of these are hereafter given.

Flash-boards.—Flash-boards are simply a series of planks, set vertically on top of the crest, forming a wooden barrier, and held in place by vertical iron rods which have their lower ends sunk into the masonry of the crest. Flash-boards vary in height from 18 in. to 4 ft. In flood periods, the excessive pressure of the water bends the supporting rods, which are designed to have a resistance to bending such that a flow of some predetermined height will bend them over. The boards are carried downstream in the flood and lost, and before the succeeding period of low water

arrives a new set of flash-boards must be installed. Obviously, this is a crude and expensive manner in which to accomplish the desired object. In spite of this fact, many dams are equipped with flash-boards.

In designing these, the rods are placed from 4 to 8 ft. apart, and the diameter of the rods is computed by calculating the water pressure in the usual manner and finding the center of pressure, which gives the moment of the force tending to overturn the

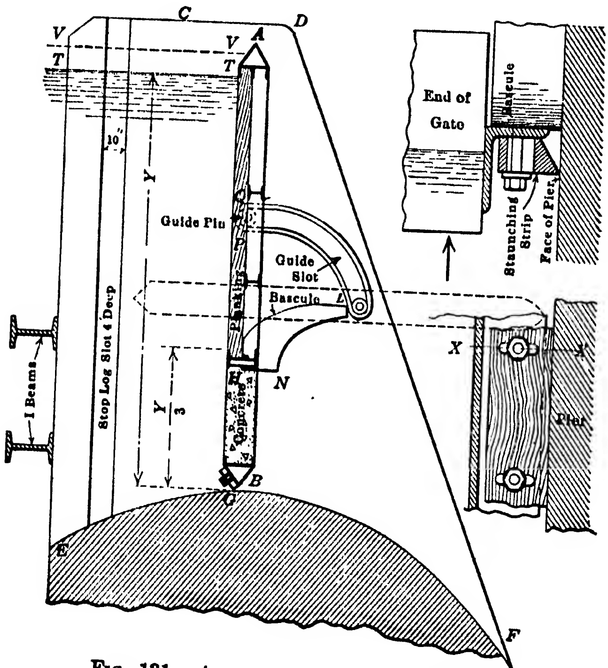


FIG. 131.—Automatic crest gate and details.

board. The iron rods are then selected of such size that when this moment has reached a certain value, they will bend. The thickness of the boards is likewise computed for the bending moment due to the load imposed by the water pressure with the span between successive rods. The stresses in the boards and their deflection are taken as considerably higher (practically double) than those assumed when permanent structures are designed.

Automatic Crest Gates.—There are several types of water barriers which are placed on the crest of dams, which move themselves automatically when the water level reaches a certain height, and restore themselves to normal position when the lake level has fallen a certain distance below the maximum allowable level. The simplest, most reliable and lowest cost device of this kind is the tilting crest gate.

An example of the use of these is on the Austin, Tex., dam, where the crest, 1100 ft. long, is provided with these gates, most of which are 18.5 ft. long by 15 ft. high—probably the largest crest gates that have ever been constructed. The construction of one of these gates is indicated in Fig. 131. It consists of a flat (or curved) slab, which revolves about a rocker or bascule. When the gate is vertical, and stops the flow of water over the crest of the dam, the point of contact between the bascule, against which the water pressure forces the gate, and the gate surface, is one-third the predetermined height of the water level above the crest of the dam When the water level has reached this fixed height, the moment of the pressure above the bascule is exactly equal to the moment of the pressure below it, as indicated by the full lines in Fig. 132, which shows the forces acting on the gate. When the water rises higher than this predetermined point, the moment of the forces above the bascule exceeds the moment of those below it, as indicated by the dotted lines in the figure. There being an unbalanced force tending to rotate the gate, it turns until it reaches a horizontal position, and the water pours over the spillway, the depth of water being equal to that at which the gates moved, while the gate is simply a vane in the current.

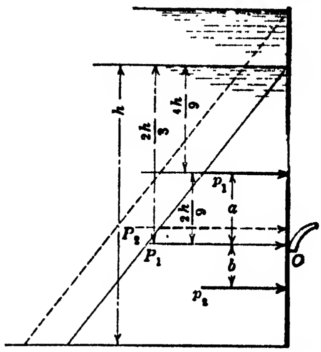


FIG. 132.—Diagram of forces acting on tilting gates.

The gate is so constructed that its center of gravity lies below the pivotal point when in its vertical position. As it revolves about the bascule, the lever arm of the center of gravity, tending to turn it back into its vertical position, increases, because the point of support of the gate rolling on the bascule, continually

travels away from the bottom and toward the top, as the gate turns toward its horizontal position. This is obvious from the figures.

When the gate is in a horizontal position, the weight of water over its upper surface is practically constant from end to end and, therefore, this mass of water has little or no tendency to return the gate to its vertical position. The water flowing underneath the gate tends to press against its under surface near its upstream edge, and falls away from it, down the spillway, at its downstream edge, and, therefore, the underflow of water tends to hold the gate against turning back to the vertical

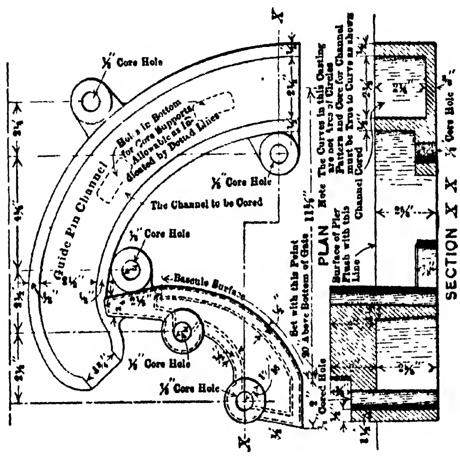


FIG. 133.—Bascule and guide pin channel for automatic crest gate.

position. Hence, in the design of these gates, the moment of the weight, acting through the center of gravity at a given distance from the point of support when the gate is in a horizontal position, must be substantially equal to the upward thrust of the water entering under the gate at its upstream edge, which thrust gradually diminishes to zero at the downstream edge. The actual resultant of these forces is practically impossible to compute, owing to secondary forces set up by eddy currents, and possibly, some air suction.

Theoretically, the bascule about which the gate turns has the form of an involute of a circle. In practice, this curve has to be somewhat flattened, and the amount of variation from the theo-

retical form is, in turn, dependent on the counterweighting of the gate, which fixes the position of its center of gravity and the amount of weight acting through it. Fig. 133 shows the outline of a bascule designed for a gate having a total height of 6 ft., and designed to begin turning when the height of water above its bottom edge is 5 ft. 3 in. This drawing also shows the guide slot in which works a heavy iron pin having a brass roller over it. In the usual design there is a bascule placed at each end of the gate, and in each end is fastened the iron pin which keeps the gate in position on the bascule. Without these holding pins, the gate, when turned in a horizontal position, would be carried downstream by the impact and friction of the rapidly moving mass of water.

The forces acting on the gate are as indicated in diagram, Fig. 132. The maximum bending moment for the vertical members of the gate occurs just before it tilts, and is at the point of contact with the bascule. The upper portion of the gate above the bascule is treated as a cantilever which has a water pressure against it equal to

$$P_t = \frac{62.5}{2} \times \left(\frac{2h}{3}\right)^2 = 13.9h^2 \text{ lb. per foot length of gate} \qquad (195)$$

The center of pressure at which this force acts is at a height $= \frac{1}{3}\frac{2h}{3}$, or $\frac{2h}{9}$ feet above the point of rotation, which distance is marked a in Fig. 132. Hence, the moment tending to bend the upper portion of the gate downstream is

$$M_1 = 13.9h^2 \times \frac{2h}{9} = 3.1h^3 \text{ lb. ft.}$$

in which h = height of water from crest of dam to elevation at which gate is designed to begin turning.

From this, the most economical distance between vertical members, and their dimensions, can be computed.

The bending moment tending to buckle the gate horizontally downstream is

$$M_2 = \frac{62.5h^2}{2} \times \frac{l^2}{8} = 3.9h^2l^2 \text{ lb. ft.} \qquad (196)$$

l being the clear span between supporting bascules.

M_2 should be computed for several different values of h, because it is, obviously, much greater at the bottom of the gate than

near the top, and in construction, the longitudinal members *are made heavier and stronger,* and placed closer together near the bottom than at the top.

In practice, these gates are made up of structural-steel frames, which are of sufficient strength to take all of the stresses imposed on the entire gate. The upper portion of the gate, or about 66 to 70 per cent. of the total gate height, is covered over with heavy wood planking, the adjacent edges of the planks being fitted together with wooden tongues. The thickness of this wood varies with the height, being 2 in. for a gate 6 ft. high and 4 in. for a gate 12 ft. high. The disposition of the longitudinal members against which the planking rests, and which fixes the distance apart of the planking supports, influences, considerably, the thickness of planking adopted.

The lower portion of the gate is generally filled with concrete, properly reinforced to withstand the stresses, and this acts not only as a filler for the lower part, but also as a counterbalance. Figs. 134 and 135 show the framework and details for a gate having a height of 6 ft. and length of 18.5 ft. The location of the guide pin is shown in both figures. The diagonal rods, extending from near the middle of the gate to each of the upper corners, are used in adjusting the gate to make it absolutely rectangular. Iron frames of this character will vary from the rectangular form, and these means must be used to bring them into adjustment.

The ends of the framework have angles rivetted on them, which project beyond the edges of the gate, and these angles are backed up with heavy reinforcement over that portion of their length which rests against the bascule. This reinforcement is for the purpose of strengthening the angles, so that they will not be bent in by the water pressure tending to force the gates downstream. The wood planking is held, as shown, by rivetting an angle section along the transverse I-beam, which is just above the concrete filling, forming a channel between the inside of the I-beam flange and the upwardly projecting leg of the angle. Into this channel the lower ends of the planks are driven, the upper end being held by a similar channel in the top of the upper cross-beam, which latter is removable by unscrewing the top nuts which hold it down. After the wood planking is in place, this top member is driven down onto the upper ends of the wood planking and firmly bolted in place by the end bolts. To pre-

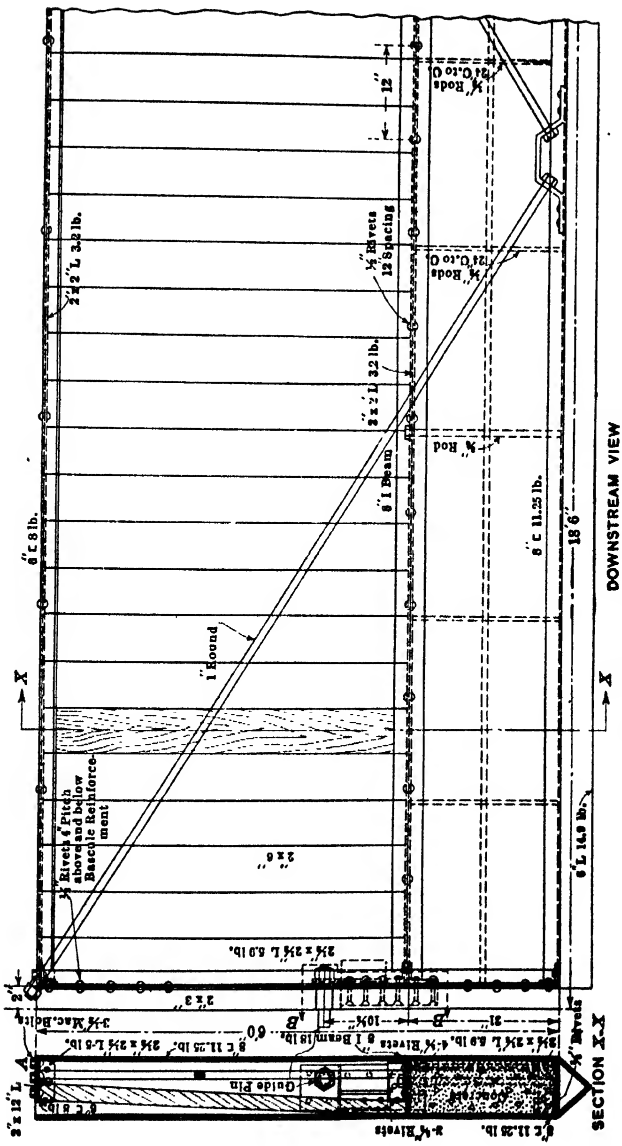


FIG. 134.—Automatic tilting gate.

vent upward deflection of the top member, any upward movement of which would free the upper ends of the planks, a vertical tension rod is placed in the middle of the gate, as shown. This rod passes up through the top channel, and a nut, screwed down on its upper end, prevents the bending upward of this member.

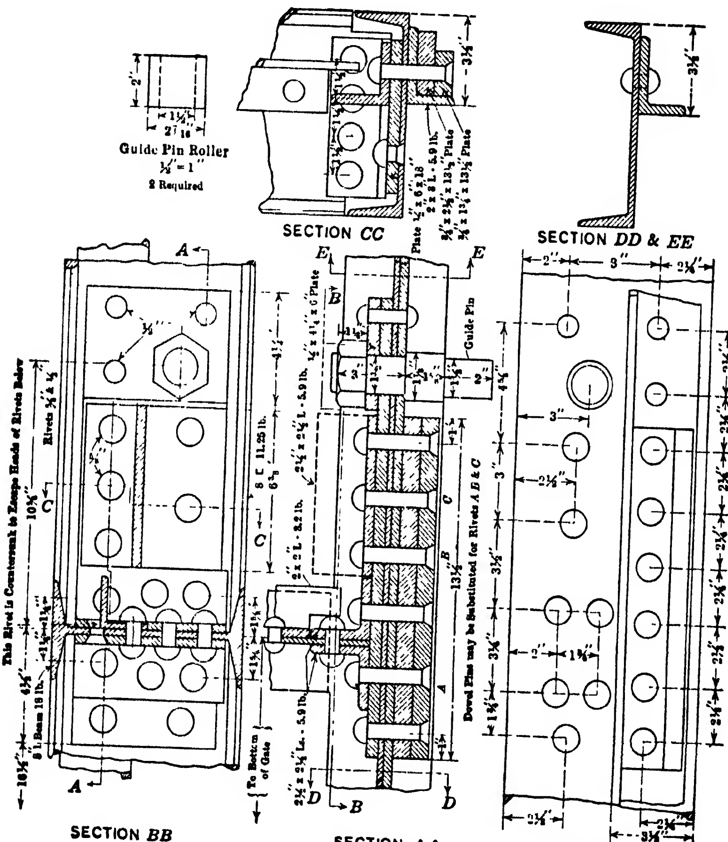


FIG. 135.—Details of automatic crest gate.

The downstream face of the gate framework should be covered with light sheet iron, say, about 3/16 in. thick. This is for the purpose of forming a smooth surface for the water to slide under when the gate is turned in its vertical position. If the framework is left open, the projecting steel members will cause eddies in the water, and set up heavy pressures tending to carry the gate

downstream, which may break the holding pins, or twist the side of the gate so that the pins can come out of the guide grooves, and the gate carried downstream. Where the guide pins pass through the ends of the gates, the steel must be heavily reinforced with flat plate and angles to strengthen the end members against twisting from the guide pin stresses, and, also, to provide a sufficient bearing surface for the pins.

The top and bottom edges of the gates should be brought to sharp angles, either by a special form of steel shape, or the use of wide standard angles. If the surface of the gate bottom is flat, the pressure set up by impact of the flowing water is very great, and, in addition, the quantity of water which may be discharged through the opening will be considerably reduced. Theoretically, it would be better if a vertical cross-section of the gates were parabolic in form instead of a straight line, so that when the gates turn into a horizontal position they would follow in contour the stream line of the nappe. The advantage to be derived from this construction is not sufficiently great to warrant the increased cost.

These gates are staunched by means of staunching strips, such as are indicated in Fig. 131. As shown, wooden timbers are fastened to the end angles, which timbers are bevelled on one edge. These timbers are placed so that the bevelled edges contact against the supporting piers, the sharp edge of the bevel being upstream. They are fastened to these angles by means of bolts which pass through slotted holes in the timber, the slots running transversely across the staunching timbers, as shown. The bolts are not set down tight against the surface of the strips, but a small amount of play is allowed. The water pressure on the upstream side forces the staunching strips against the pier surface, thereby making a tight joint. The pressure which moves the staunching strips outward against the piers is equal to the unit water pressure, multiplied by the area of the inner edge of the strips. Since they are bevelled on the outside edge, any water leaking past them can not set up a counter-pressure to move them away from the piers, because there is no surface against which it can act.

Gates constructed in this manner have been found satisfactory and reliable, and they can be so designed as to completely open and close within a range of 14 in. change in lake level.

Protection for Crest Gates.—Where flood waters of streams bring down quantities of driftwood and heavy débris, the floating

logs and trees are so interlocked and piled up on each other, that the depth of driftwood below the surface of the water may, at times, be many feet, and when the mass of drift attempts to flow over the crest of the dam and through the gate openings, the upper portion will pass freely over the upper surface of the gate, but the lower portion will often wedge itself between the crest of the dam and the under-surface of the gate. The accumulation of additional driftwood, urged downstream by the flood waters, will produce a heavy thrust against the mass wedged on the underside of the gate, and will buckle the gates upwardly. It, therefore, is necessary to place a protecting screen in front of the lower portion of the gate opening which will effectively prevent an interlocking mass of driftwood from wedging underneath the gate. This protective screen may be made up of two or more horizontally placed I-beams, which are fastened to the upstream faces of the piers, between which the gates operate. Even a single beam placed across the lower portion of the opening, at a point halfway between the crest of the dam and the bascule surface, will protect the gates against the driftwood. These I-beams have to be of considerable strength, owing to the heavy pressure set up by the driftwood. It is impossible to fix the size by computation, owing to the fact that the force acting can only be surmised. With a span of 12 ft. and a total height of gate equal to 6 ft., the beam should be a 6-in. I, weighing approximately 14 lb. per foot. The allowable bending moment on this beam is 11,000 lb.-ft. = 132,000 lb.-in. With this as a basis, the size of beam for any other span and height of gate may be computed, the bending moment increasing as the square of the span and, also, as the square of the height. The I-beams should not be closer together than 2 ft. Fig. 131 indicates this method of protection.

The piers must extend sufficiently far upstream to allow the formation of stop-log channels in the sides, which channels must be far enough away from the bascule for the gate to turn and clear the stop logs when in its horizontal position. This is likewise indicated in Fig. 131.

Stoney Roller Gates.—These gates are well adapted for placing on the crest of a dam to regulate the water level and discharge over the dam. They are regular sliding sluices comprising a panel, or slab, which moves vertically upward to open, and downward to close. In the sliding gate, the enormous thrust of the

water against its surface produces a corresponding friction between the gate and the slide in which it moves. The Stoney gate was devised to eliminate this friction and thereby enable large and heavy gates to be moved with a comparatively small exertion of power. The principle of the Stoney gate is indicated in Fig. 136 herewith.

As shown, a series of rollers, which have their centers along the same vertical line, are imposed between the gate and the supporting pier. When the gate moves, the rollers travel in the same direction as that in which the gate does, but at half the speed of movement of the gate. There is no pressure carried to, or through, the axles of the rollers. The pressure passes directly

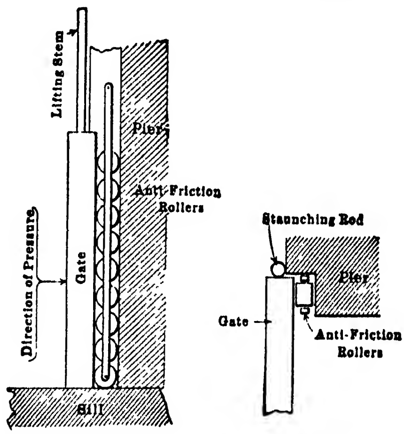


FIG. 136.—Diagram of the Stoney sluice gate.

across a diameter of the roller from one side of the periphery to the other.

The rollers are all mounted in a vertical frame, which moves with the movement of the gate, its distance of motion being one-half that travelled by the gate.

Obviously, it is impossible to make the gate seal itself against leakage at its sides which connect with the supporting pier, and, in order to prevent leakage, staunching rods must be used. These consist of wooden, or rubber-covered steel rods, circular in section, which are loosely hung at their top and bottom ends and extend from top to bottom of the gate at either side, lying in the corner between the gate and the pier.

These gates are moved by means of a hoisting apparatus placed on a bridge which is built above the crest of the dam and rests on the piers. They have been built in sizes larger than it is possible to make any other form of moving water gate.

Some of the gates for the Assuan dam are 58 ft. in height by 21 ft. in width. One of these is shown in Fig. 137. Others have been made 28 ft. in height by 32 ft. in width; gates which it would be impossible to move, unless provided with anti-friction rollers. Fig. 138 shows details of practical construction of gates of this type.

The objection to the use of these as crest gates lies in the excessive cost of the installation. The height of the hoisting bridge above the crest of the dam must be more than twice the height

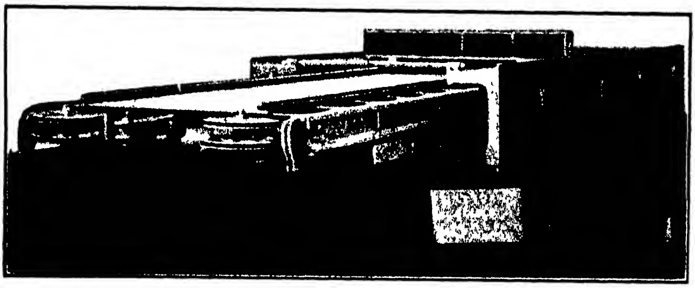


FIG. 137.—Stoney gate for Assuan dam. Sluice frame, with sluice entering, showing rollers, pulleys, etc.

of the gate. This requires either heavy piers with considerable masses of masonry or expensive structural-steel frameworks.

The hoisting apparatus may be operated by hand or power, as desired, and is simply a special form of winding drum. A hoist is not required for each gate. A railway track laid across the bridge on which the hoist can travel, and one hoist to move any gate is usually a satisfactory arrangement. Two hoists are always required so that, in case of one becoming inoperative for any reason, the other can be used to move the gates.

In the design of these gates the steel framework is computed with the vertical members of sufficient strength to withstand the vertical bending moment due to the water thrust. Means for computing these moments have been before given. The framework may be covered with planking, steel plate or a thin slab of reinforced concrete. The latter, however, adds considerably to the weight of the gate and, therefore, is not to be recommended

except under certain special conditions. The gates are usually counterweighted by means of concrete weights attached to wire cables or chains, which pass over sheave wheels located on the hoisting bridge.

Theoretically, the amount of power required to move these gates is very small, but the presence of any silt, leaves, twigs or

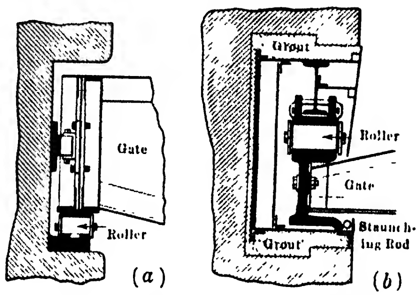


FIG. 138.—Ends of Stoney gates.

other substances which may lodge against the roller surfaces and over which the rollers may have to travel, increases this theoretical power by several hundred per cent., and in the design of hoisting apparatus this possibility should be taken into account.

The Stickney Gate.—This gate and its method of operation are indicated in Fig. 139. As shown, it comprises two

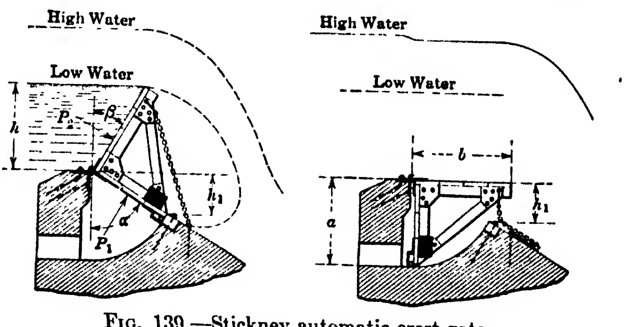


FIG. 139.—Stickney automatic crest gate.

leaves, set approximately at right angles to each other, hinged, at the point where they join, to the crest of the dam, which latter is provided with a long recess into which one of the leaves may sink, taking a vertical position, the other leaf then taking a practically horizontal position. Openings pass through the dam

from the recess to the upstream side, so that there is always a water pressure inside the recess proportional to the height of the water above it. A counterweight is attached to one of the leaves, and indicated by the black rectangle shown in the figures. It produces a moment tending to turn the gate to open, (downward), position.

The water pressure against the leaf which forms the water barrier, also produces a moment tending to open, or lower, the gate. Opposed to these two moments is that of the water pressure in the recess acting on the second leaf. Therefore, in order to cause the gate to rise and stand upward, when the water level is within certain limits, and to lower it automatically when the height of the water exceeds these limits, there is a necessary relationship between the lengths of the two leaves and the moment of the counterweight. The equations for these relationships are as follows:

Referring to the figure, the values of h_1, α and β are fixed by the design of the crest of the dam.

$$b = \frac{h_1}{\cos\beta} = \text{length of upper leaf.} \tag{197}$$

h_1 = height of hinge above downstream edge of recess in crest.
β = angle leaf b makes with the vertical when raised.
h = height of water above hinge.

$$a = \sqrt{\frac{3b^2h + h_1^3}{3(h + h_1)}} = \text{length of lower leaf} \tag{198}$$

Moment of counterweighting is

$$Wl = \frac{10.4}{\sin\ \alpha}[3h(a^2 - b^2) + 3a^2h - 4h_1^3 + 2b^3 \cos\beta] \tag{199}$$

Find the weights and locations of the centers of gravity of a and b. The sum of their two moments about the hinge is deducted from Wl, and the remainder is the moment which the counterweight must produce. The weight of the counterweight is then found by dividing the net moment by the distance from the hinge to the center of the weight, the amount of weight required being dependent on its distance from the hinge.

This gate has the advantages that no overhead bridge is required; it is automatic in operation and gives a clear passageway for driftwood or other débris.

It has the disadvantages that the crest of the dam must be cut

away to make the longitudinal recess, which becomes a serious matter when the height of the gate exceeds 3 or 4 ft., and that the path of the water pouring over the spillway is not the proper parabolic curve. This, also, becomes important under certain conditions which have previously been explained in the chapters on "Dams." A third disadvantage is the necessity of staunching the edges of both of the leaves and also the bottom edge of the lower leaf.

Automatic Gates with Rolling Counterweight.—This form of gate is indicated in the sectional drawing, Fig. 140. The methods

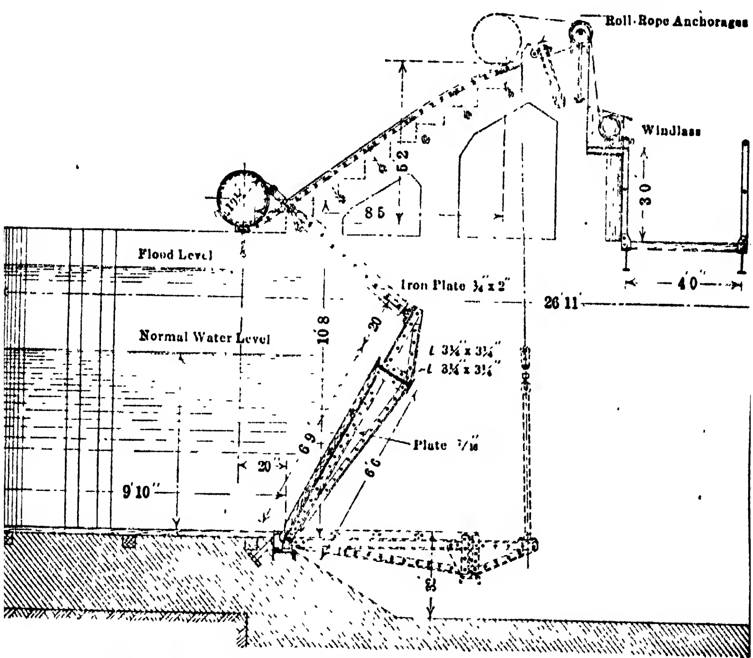


FIG. 140.—Automatic gate with rolling counterweight.

of construction and operation are obvious from the figure. The gate, which turns about the knife edge at its lower end, is held against the water thrust by flat iron links attached to its upper end, and these links, in turn, are fastened to chains, or wire ropes, which wind around a heavy steel drum. The drum rolls on an upwardly inclined path and, as the gate tends to fall, by reason of the increase in pressure due to raising of the water level, the chain

or cable must unwind from the drum, and this, in turn, causes the drum to roll along its inclined path. Hence, opening of the gate causes an elevation in the position of the drum.

The weight of the drum, the diameter of that portion of it over which the cable or chain is wound, and the inclination of the path over which the drum rolls, are all adjusted to coördinate with the water pressure acting to overturn the gate. Referring to Fig. 141, which represents the end of the drum and the forces acting on it, the vertical line OW represents the weight of the drum. T is the point of contact where the drum rests on the path. BP represents the pull on the chain or cable, set up by the water pressure acting against the gate. If a_1 and a_2 represent, respectively, the lever arms of these two forces with respect to the tangent point T, then, for equilibrium, Wa_1 equals Pa_2.

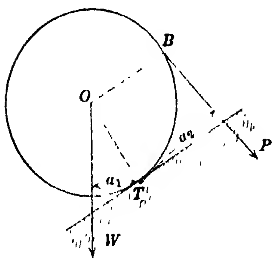


FIG. 141.—Diagram of forces acting on rolling counterweight.

The form of the path of the roller was determined experimentally.

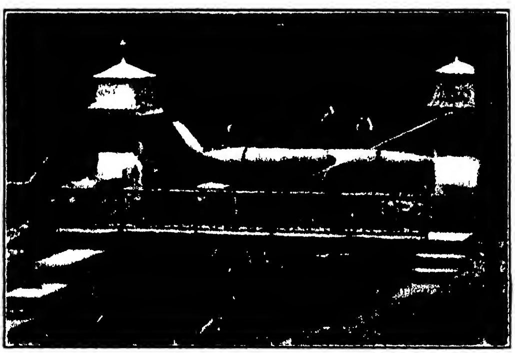


FIG. 142.—Counterweight and piers.

The roller is filled with concrete in order to add to its weight. To prevent it from slipping on the track and constrain it to roll evenly as it moves upward, roll ropes are provided. These are wire ropes which are anchored at the upper end of the track, pass down underneath the roller and are wrapped around it with one

turn and fastened into the roller surface. The roller, therefore, rolls up and down, winding and unwinding itself on these roll ropes. Rope sheaves are formed on the ends of the roller to receive these ropes as well as the counterweight ropes.

Obviously, this gate must be placed between piers, there being one at either end, and a roller track on each pier, the roller spanning from pier to pier, as shown in Fig. 142.

An ingenious method of staunching has been devised for these gates and is indicated, in section, in Fig. 143. There is a clearance of, approximately, ½ in. between the pier and the end of the gate. Thin sheet metal is bent into the form of a closed trough which has a length equal to the length of the gate, and is placed in the clearance space between the end of the gate and the pier. The trough is not entirely closed, but there is a small opening between the upstream surfaces, so that water may enter past the edge *b*, filling the space *a*. The water pressure tends to spread open the trough and thereby push its outer surface against the surface of the pier.

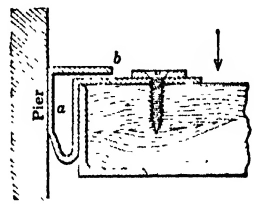


Fig. 143.—Method of staunching gates.

These gates operate satisfactorily, but, as is obvious from their construction, the cost of equipping a dam with this type of automatic gate would be very great as compared with some other kind.

Taintor Gates.—A form of crest gate which is used to considerable extent in America is the Taintor gate. Its construction is as indicated in Fig. 144. As shown, these gates are circular arcs in cross section, well supported on a framework of steel or wood, the convex surfaces being turned upstream, the concave surface downstream. They are pivoted on trunions, which are placed downstream with reference to the gates, their position coinciding with the center of the circle of which the gates are arcs. To open them, they are rotated upwardly around the trunions, movement being effected in several ways. One method is by means of a pinion which engages in a segmental rack, there being one of these racks and a coöperating pinion at either end of the gate. These gates work between the end surfaces of piers, and a shaft is provided running across from pier to pier on which the two operating pinions are carried. This shaft may be moved by

hand or power, as desired. Fig. 145 shows the general arrangement of the upstream side of a gate of this type, the operating pinions and racks in which they engage being clearly indicated.

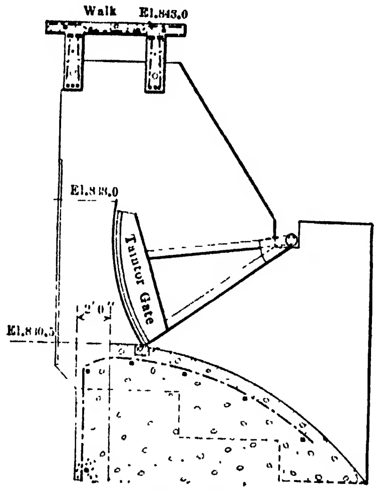


FIG. 144.

Where a number of these gates is placed on top of the crest of the dam, it is usual to substitute chain, or cable, lifts for the rack and pinion. Sheave grooves are formed in the gate, there being

FIG. 145.—Gear-operated Taintor gate.

one at either end; a chain or cable laid in each groove, is fastened at the bottom and passes upward to a bridge or walkway above the gates, on which a winding hoist is placed. By means of this

hoist the chains or cables may be wound around drums, and in this way, the gate pulled upward until it is wide open. Two or three hoists running on rails placed on the bridge are sufficient to move a number of gates in succession.

Figure 146 shows a movable hoist for Taintor gates. The lifting ropes, or chains, wind on the helically grooved windlasses of the machine. The hoist may be operated by either hand or power. The one shown in the figure is worked by an electric motor. The amount of power required to move Taintor gates is comparatively small, as the friction around the trunion bearing is the only resistance to be overcome, and the lever arm of the operating racks, or chains, about the trunion, is considerable. The only objectionable feature of these gates is the extent of the piers downstream to support the trunions, or the necessity of

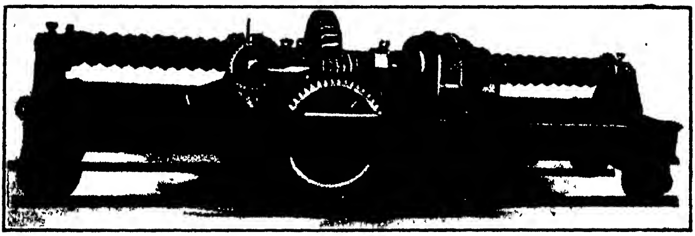


FIG. 146.—Portable hoist with motor, for Taintor gates.

building up some form of support to carry them and, in some cases, the cost of the hoisting bridge above.

From equations and data already given concerning other forms of crest gates, the forces acting can be readily computed and the necessary mechanical structure designed to resist them.

Rolling Dams.—Rolling dams have been used to a considerable extent in Germany, and a few installations have been made in America. Primarily, these consist of a cylinder placed between piers, the diameter of the cylinder being equal to the height to which it is desired to elevate the water. This cylinder, or drum, is built up of boiler plate and braced to withstand the stresses to which it is subjected. Inclined paths, one at each end of the drum, are provided, up which the drum may be rolled by hand or machine power. On each end it has a gear which engages with a rack laid on the inclined abutments and, by means of a sprocket chain wrapped around one end of the cylinder and connecting with the operating mechanism, the dam is rolled up or down the abutments

as desired. When rolled up from the crest of the dam and to a considerable height above it, a clear opening is provided for the passage of water and débris.

In its later development, the drum is made much smaller than the height to which the water is to be raised and a segmental sheath is attached to it, this sheath having a height equal to the desired elevation. Fig. 147 shows a roller dam of this type, as installed on a division dam of the United States Reclamation Service near Boise, Idaho. As shown in the figure, the sheath

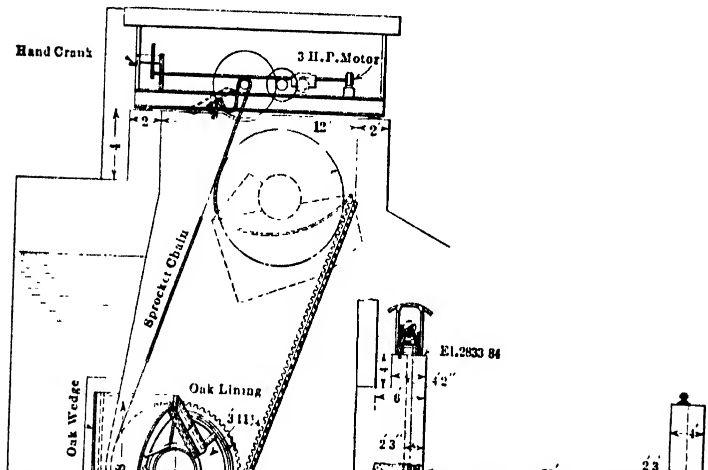


FIG. 147.—Roller dam.

is the arc of a circle of 6-ft. radius, its chord length and, therefore, its height, being 8 ft. This arc is not concentric with the main cylinder or shaft, but its center of curvature is located 4 ft. downstream from the center of the shaft and about 9 in. below it in elevation when the dam is closed. The dam is 30 ft. long. The main cylinder or shaft is about 2½ ft. in diameter, and the gears at each end of it are 7½ ft. in diameter. The racks are laid at an angle of 21½° with the vertical.

The inside faces of the concrete piers forming the abutments converge slightly upstream, and at each end of the sheath is a flex-

ible plate bound with oak timbers, which spring against the abutments as the dam is being closed, thus making it practically water-tight at the ends. When the dam is entirely open, it clears the crest by about 18 ft., and, as the maximum height of the water above the crest of the dam is only 10 ft., there is plenty of clearance for logs or débris to pass at the maximum high-water stage. As the dam opens, it recedes slightly from the position it occupies when closed, so that any drift of sediment that has accumulated on the crest in front of the dam offers no hindrance to its opening.

The controlling mechanism consists of a sprocket wheel, a shaft and the necessary gears, and is arranged so that it may be operated by a direct-connected motor or hand power. The hand-operated mechanism gives a very slow motion, however, and is intended for use only in case of trouble with the motor.

This dam is moved by a 3-hp., 500-r.p.m. motor. This motor will open the roller in about 15 to 20 min.

This form of gate is satisfactory in every respect except that of first cost. The height and size of the pier necessary to roll it up from the crest and give sufficient clearance, make an expensive construction. It has, however, the advantage that the distance between piers may be made practically any desired length up to 100 ft. For a mathematical discussion of the forces acting, refer to article, by Hillberg, in the *Engineering Record,* December, 1913.

There are many forms of movable water barriers and no attempt is made to describe them all here. The principal ones only, and their methods of operation are set forth, and it is probable that many variants of those described, as well as original methods of handling the same problem, will occur to the minds of designing engineers who have the question of making a variable height of dam crest, with all the attendant conditions, before them.

CHAPTER IX

HEADWORKS

Forebay.—The forebay is the name usually given to the portion of the lake which lies immediately upstream above the headworks. Generally, this is a small bay in the lake in which the water is comparatively still, except for such motion as is produced by the inflow passing through the headgates. A log boom is usually placed across the forebay, making an angle with the direction of flow of the stream. The boom is to prevent floating materials and débris from entering the forebay and, as they are kept within the influence of stream flow and carried off by it, any accumulation in front of the boom is prevented.

When the water carries sand in suspension, it is necessary to have a forebay of considerable area, so that the velocity of the water moving in it, toward the headgates, will fall to less than 0.75 ft. per second, in order that the sand may all be deposited before the water enters the penstock. If sand is present in the water, and it reaches the water wheels in even an infinitesimal quantity, the wheel runners and vanes are soon cut away.

Stop Logs.—Stop logs are temporary water barriers which are placed in front of headgates or crest gates when it is necessary to repair them. They are simply timbers, ranging from 4 by 8 to 10 by 12 in. in cross-section. These logs are dropped into stop log slots, which are recesses, or grooves, made in piers or abutments to receive them. In Figs. 132 and 148, these slots are indicated in the piers which project upstream from the headgates. It is better to place these slots upstream, above the racks as well as the headgates. When it is necessary to reach the racks, or gates, the logs are dropped into the slots, one by one, and the water pressure holds them tight against the rear of the slot. The logs have to be piled up higher than the level of the water, otherwise the whole mass will float and not make contact with the bottom.

These logs are removed by lifting them, one at a time, with the usual log-lifting devices, having sharp spiked projections which

catch into the surfaces of the wood. In some cases, special metal castings are sunk into the ends of the log, one being placed near each end, and a lifting rod, with an iron shoe fastened on one end which fits into these castings, is used to lift them out.

Where the span between piers is small, say under 5 ft., and the depth not more than 8 ft., wooden gates in sections of 2 to 3 ft. in height, may be slid into the slots instead of individual logs. The thickness of planking for the gate, or the thickness of the logs used, is determined by the usual formula for flexure in simple beams. A large factor of safety should be adopted, not less than 5, because, whenever the water is shut off by these means it is always for the purpose of allowing workmen to get down behind them for repairs or inspection.

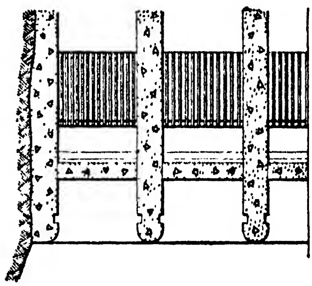


FIG. 148.—Piers and racks showing stop log slots.

Log Booms.—Booms are sometimes placed upstream, above the headworks, to catch heavy driftwood, and prevent it from injuring the headworks. A boom for this purpose is conveniently made up of large timbers, either round or square, which are flexibly connected at their adjoining ends by short sections of chain spiked on to the timbers, and the two ends of the boom are fastened in position by a chain at either end, which is anchored to the stream bank. The forces which will act against this boom are great, as shown by the following equations, and the strength of the timbers, chains, spikes and anchorages must be dimensioned to resist the stresses. The following analysis is due to Prof. I. P. Church.

Consider the conditions, as indicated in Fig. 149, which shows a boom with driftwood backed up behind it. The forces tending to move the boom downstream comprise; (*a*) the pressure due to the impact of the flowing water against the upstream surface of the mass, and which is proportional to the vertical projection of the submerged area, and (*b*) the force due to the friction of the water flowing underneath the mass and rubbing against its irregular under-surface.

The chains must hold the entire mass against these two forces. The force due to impact, or P_1, is

$$P_1 = kF\gamma\frac{V^2}{2g} = 0.972kFV^2 \qquad (200)$$

$F = LD$, where L = length of boom, D = depth of submersion in feet.

The force due to fluid friction under all the logs, or P_2, is

$$P_2 = fS\gamma\frac{V^2}{2g} = 0.972fSV^2 \qquad (201)$$

In these two equations, $\gamma = 62.5$, and k and f are empirical coefficients. k may be taken at 1.9, while f would be, approximately, 0.03.

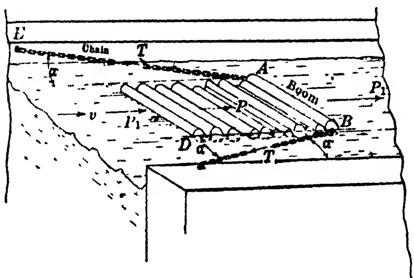


Fig. 149.—Forces acting on log boom.

S = number of square feet area of the mass of driftwood.

The tension in each chain, or T, is

$$T = \frac{P_1 + P_2}{2\cos\alpha} \qquad (202)$$

α = angle the chain makes with the direction of stream flow. As an example, if the length of boom were 200 ft.; its depth of submersion 1 ft.; the length of the mass of drift, measured along the stream, (*DB* in the figure), 300 ft., and the velocity of stream flow 4 ft. per second;

$$P_1 + P_2 = 0.972 \times 4^2[(1.9 \times 200 \times 1) + (200 \times 300 \times 0.030)]$$
$$P_1 + P_2 = 33{,}900 \text{ lb.}$$

If the anchor chains make an angle of 30° with the direction of stream flow, the stress on either chain will be

$T = \dfrac{33{,}900}{2\times 0.866} = 19{,}573$ lb., requiring a chain with links of ¾-in. iron.

Trash Racks.—Trash racks are, almost universally, made up of rods, or bars, spaced at varying distances apart, set at an angle to the vertical, and placed in front of the headgates. Wooden planking set with the edges turned upstream, round iron pipe and other forms of bars have been used, but the present-day standard practice has fixed on flat, rolled-steel bars, placed with the flat side parallel with the flow of water and spaced from 1 to 2 in. apart in the clear, as the most satisfactory form of rack. The proper sizes of bars for different sizes of rack are given in Table 34.

TABLE 34.—TRASH-RACK DIMENSIONS

Length of bars, ft.	Size of bars, in.	Diameter of connecting rods, in.	Number of bars per section
10 to 15	$\frac{1}{4} \times 2$	$\frac{1}{2}$	20
15 to 20	$\frac{1}{4} \times 2\frac{1}{2}$	$\frac{5}{8}$	15
20 to 25	$\frac{1}{4} \times 3$	$\frac{3}{4}$	10

From this table it is seen that the size of rack bar varies from $\frac{1}{4}$ by 2 to $\frac{1}{4}$ by 3 in. The table also shows the size of the round rods which pass through the bars to hold them together. The construction of the rack sections is clear from Fig. 150. Figs. 151 and 152 are vertical cross-sections which show the rack bars placed at an angle of approximately 20° to the vertical and supported by the lower ends resting in a channel beam, with I-beams behind the bars to prevent flexure due to water pressure when the racks become clogged with débris.

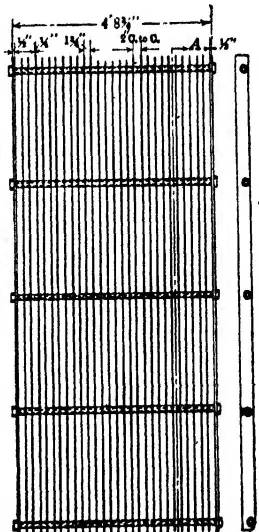


FIG. 150.—Rack section.

It is to be noted that the connecting rods which bolt the individual bars together to form sections, do not have their center lines pass through the center line of the bars, but the holes for these rods are drilled nearer to one edge of the bar than to the other, so that these connecting bolts, with the spacers which they carry on them, lie near the lower, or downstream, edge

of the surface of the rack, as shown in the section, Fig. 150. The object of this is to allow a clear space from top to bottom between each of the bars, so that the tooth of a rake can pass all the way up the rack without encountering any obstruction other than leaves, brush or other substances which are caught on the surface of the rack.

Racks must be designed so that they may be lifted away from the penstock or canal opening for thorough cleaning between the bars at intervals. For this reason, they are seldom made in complete units which span from one side to the other of the penstock opening. It is usual to bolt every 10 or 20 bars together, with spacers between them, and each set constitutes a

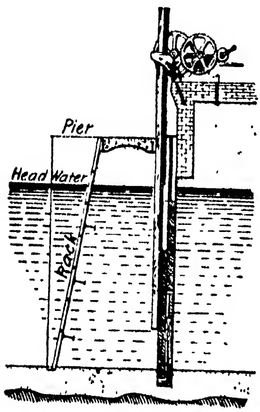


FIG. 151.—Rack supported by I-beams held by pier walls.

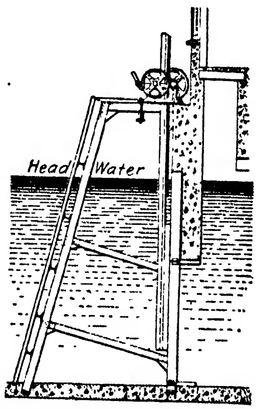


FIG. 152.—Rack supported by structured steel framework.

section which may be lowered into place, either by hand, or by means of a special hoisting apparatus, depending on the size and length of the bars and the number of bars per section. In the table giving the number of bars per section, the basis has been to keep the weight of each section within the limit of easy handling. This should not exceed 500 lb. and, preferably, be kept within 350 lb.

The I-beams placed behind the racks are designed on the assumption of the racks being fully clogged, and having exerted on them a hydrostatic pressure equal to that produced by the same depth of water and same width of section. The spacing apart of the supporting beams, and their sections, are fixed on the basis of these

stresses, and the spacing of the supporting beam is such that the rack bars are not stressed beyond safe, limiting, unit stresses. The possibility of a rack being entirely clogged, so that full hydrostatic pressure is set up against it, exists but is remote. Therefore, the rack bars and the supporting structure should be designed to withstand these stresses, but not with any such factor of safety as would be employed if this condition of full pressure existed continuously. The unit stresses in the steel which may be adopted as safe, should range between 28,000 and 30,000 lb. per square inch, assuming full hydrostatic pressure acting on the rack.

The spacers used between the bars may be either small castings, or they may be cut from extra-heavy, wrought-iron pipe. The castings have the advantage that they can be obtained of exactly the dimensions wanted, but cuttings from wrought-iron pipe have usually a better bearing surface for the sides of the bars to rest against.

The angle at which the rack bars are set is arbitrarily fixed by the designer. The slope which they make with the vertical may be more or less steep, as the engineer may decide. The general practice makes this slope from 15° to 20°. The object of having the bars slope is to enable the station attendants to rake them clear of any floating materials which they screen from the intake. In order that this raking may be done, a walkway, not less than 3 ft. in width, must be provided at the top of the racks so that the men may easily reach the entire surface. Figs. 151 and 152 show racks with walkways above them. The racks in Fig. 151 are supported on I-beams which span from pier to pier, the racks being set in bays between piers. Fig. 152 shows a continuous rack supported by a structural-steel framework.

The bars should be placed not less than 1 in. apart in the clear, nor more than 2 in. A clear opening of greater width than 2 in. might permit the passage of floating chips or branches of trees heavy enough to injure the turbine if they should pass into the wheels. If the spacing is less than 1 in., the racks will be particularly subject to clogging with small twigs and leaves, which could easily pass through the water wheels without injury. The actual spacing adopted is dictated largely by the character of country. The bars should be wider apart where the stream bears a large quantity of leaves on its surface during certain seasons of the year, and the less the liability

to clogging from leaves, the less distance apart the bars should be, within the before mentioned limits. In certain sections of the United States, during the late autumn, continuous raking of the racks, night and day, is required to prevent stoppage of the water wheels due to the clogging of the racks with leaves. Inexperienced engineers show a tendency to make the distance apart of rack bars very small. This leads to continual difficulties with clogging and the water wheels are no better protected than they would be if the bars were spaced a proper distance apart. Care must be taken, however, to prevent any opening or space around the rack, under the bottom or past the ends. Whenever water wheels have been injured by the passage of floating pieces of wood through them, it will be found, in practically every instance, that the racks were not properly placed and sealed in at the ends and bottom. The bars should extend not less than 1 ft. above the highest water level and should be not much greater than 2 ft. above it, because if the bars are longer than necessary the racks not only cost more, but are more difficult to handle and to keep clear by raking. Individual racks should be provided for penstock openings, a rack being placed in a bay between two piers which project upstream. This arrangement is better than that of one long, continuous rack for all the penstocks. The reason is that if any rack is injured or any section of it has to be removed for cleaning, the independent racks allow continuity of operation of the power plant, while if there be one long continuous rack for all the penstocks, removing any one section would admit débris into any or all of the water wheels which might then be running.

Loss of Head Through Racks.—When water passes through racks or gratings, there is a loss of head from two causes; one being due to the entry into the racks, the other being due to the sudden increase in cross-section of the water after it passes through the racks.

The loss due to entry is $\frac{V_1^2}{2g}\left(\frac{1}{C^2} - 1\right)$, in which V_1 is the velocity through the racks at the section of smallest area, and C is a constant, having an average value of 0.80 for flat-bar racks and 0.98 for racks made of round bars or pipes. The loss due to sudden expansion behind the racks is $\frac{(V_1 - V_2)^2}{2g}$ for flat-bar racks, in which V_2 is the velocity of the water after passing through the racks. This loss is zero for racks made of round bars or pipes.

Therefore, the losses due to racks are:

For flat-bar racks, $h_{r} = \frac{0.56\,V_1^2 + (V_1 - V_2)^2}{2g}$ ft. (203)

For round-bar racks, $h_{r} = \frac{0.04\,V_1^2}{2g}$ ft. (204)

There is a further drop in head through racks due to the increase in the velocity of the water from the initial velocity, or "velocity of approach," to the velocity V_1. The value of this additional drop in head is $h_1 = \frac{V_1^2 - V_0^2}{2g}$, in which V_1 = velocity through the racks, and V_0 = velocity of approach.

This drop in head, however, is not a loss chargeable to the presence of the racks, because, even if they were not interposed in the path of the water, a velocity head would be abstracted from the potential energy of the water when it enters the penstock or canal, which velocity head would be $h = \frac{V_3^2}{2g}$, in which V_3 = velocity in the penstock. The velocity given the water in passing through the rack assists in producing the required velocity in the canal or penstock, so that the head used in producing the required velocity through the rack, is not a loss chargeable to the rack, but is part of the velocity head needed for moving the water through the conduit.

Rack booms are, usually, heavy floating timbers placed end to end, the adjacent ends being flexibly connected together by short sections of chain spiked to the timbers. This construction makes a floating barrier to obstruct the passage of heavy floating objects when the two ends are anchored to confine it to some point in the stream above the headworks. Usually, this rack boom is anchored across the forebay so that the latter is kept clear of floating materials, and light objects, such as twigs and leaves, are prevented from entering the forebay and clogging the racks. In some developments it has been found advantageous to make a floating structure of a heavy pair of booms, the parallel timbers of which are spiked together by transverse timbers, and a vertical wooden rack is built on this floating structure as a support. The bars of the outer rack are made up of 2 by 4 in. timbers, which are nailed vertically to the floating longitudinal timbers. The rack-bar timbers are cut of sufficient length to reach at least 4 ft. below the water surface and 2 ft. above it. The spacing of these bars is from 3 to 3½ in. apart, in the clear.

Headgates.—Headgates, which admit water to, or shut it off from, penstocks or canals, are of many kinds and types. They form an element of the development which is considerably more important than is usually supposed. The idea seems to prevail, even among engineers of some experience, that almost any kind of water gate is satisfactory as a headgate.

The usual headgate is a sliding leaf, made of timber. It is arranged to be raised and lowered by a geared, hand-wheel mechanism, placed on the bulkhead above the headgate. The connection between the gate and the mechanism is made by means

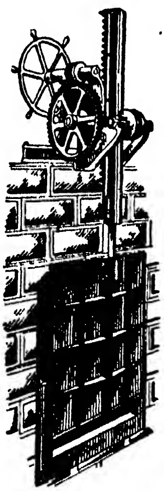


FIG. 153.—Headgate and hoist.

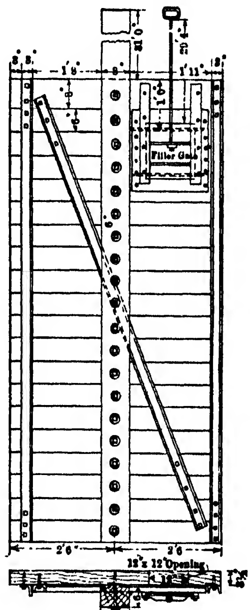


FIG. 154.—Wooden headgate.

of a long wooden stem, to which latter is fixed a section of toothed rack, into which meshes a pinion on the hoisting device. The leaf, or gate, slides in grooves prepared to receive it or slides against the surface of the flanged end of the penstock. Fig. 153 shows a headgate in which the moving slide is of cast iron, working against a cast-iron frame set in the face of the dam, with which frame the penstock connects. The vertical stem with the toothed section at the top and the worm-geared, hand-operated hoisting device, are all clearly indicated.

Figure 154 shows a wooden headgate of heavy oak planking $3\frac{1}{2}$-in. in thickness, with a vertical stem made up of timber 6 by 8 in. bolted to it. This gate has the individual planks connected together by structural-steel braces to which the planks are bolted. A filler gate is also attached to this headgate.

Filler gates are small, sliding, sluice gates, placed at a convenient point on the main gate. The filler gate is opened by hand, which is easily done, owing to its small area. Water passes through the filler gate and fills the penstock, it being prevented from passing on out through the penstock by the turbine gates, which are closed. When the penstock is full of water, the pressures on the upstream and downstream faces of the headgate are equalized, and the only force required to lift or to close the gate, is that due to the weight of the gate itself, and a comparatively small amount of friction. Obviously, if no filler gate were used, the total hydrostatic pressure, due to the depth of water over the headgate and its area, would act to press the gate against its guides, or the penstock flange, producing such a great amount of friction that the ordinary headgate hoist could not move it. It should be noted that the pressures on a headgate are very great when the penstock is empty, and it must be designed to withstand the hydrostatic pressure with an ample factor of safety, certainly not less than 5 or 6. Frequently, men are at work in the water-wheel chambers, and the only protection they have against drowning is the ability of the headgate to resist the pressure acting on it. In order to fix the filler gate on the headgate, some of the cross-members of the latter must be cut in two, and the transverse flexure of these cut portions must be transmitted across the opening by the filler-gate frame. Hence, to provide uniform strength from side to side of the gate, the filler-gate frames must be very heavy and strong.

Figure 155 shows the design of a filler gate. The heavy framework, and the bolt holes provided for fastening the ends of the cut timbers thereto, are clearly shown. Filler gates should have their sliding surfaces covered with bronze strips, so that the parts may not rust together and make it difficult, or impossible, to move them.

In some cases, the headgate is made up of two sections, one of which extends from the bottom of the gate frame up to near the top, the other section extending from this point to the upper limit of the gate, the arrangement being the same as if the gate

were cut through, horizontally, from edge to edge near the top. Vertical rods, having nuts on their upper ends, are screwed into the lower section and pass through the upper section, so that the

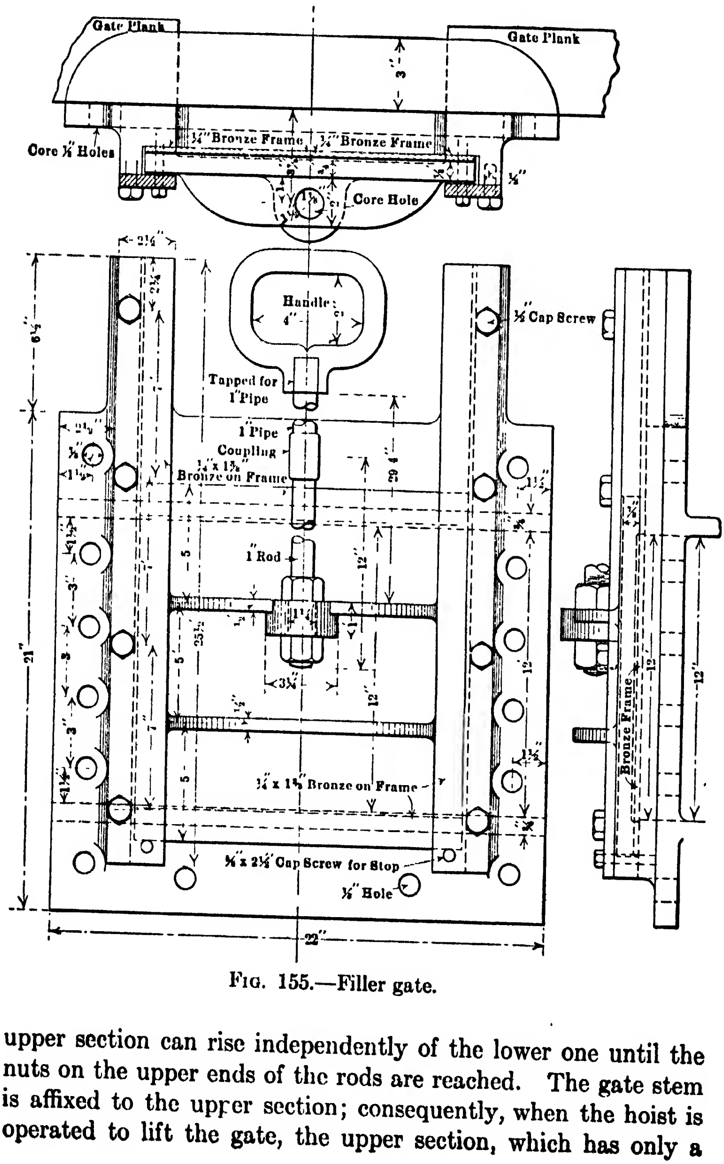


FIG. 155.—Filler gate.

upper section can rise independently of the lower one until the nuts on the upper ends of the rods are reached. The gate stem is affixed to the upper section; consequently, when the hoist is operated to lift the gate, the upper section, which has only a

small area, rises first, making an opening into the penstock, which is filled with water through this opening. In other words, it acts as a filler gate. Further motion of the gate hoist will then lift both the upper and lower parts, the total upward pull of the lower portion being taken by the rods and nuts. In closing the gate, the reverse operation takes place. Fig. 156 shows this type of gate. Though used on several developments in America, it has the objection that it is expensive and increases the distance of lifting for a given gate opening. With well-constructed filler gates, there is little or no difficulty in lifting them by hand and accomplishing the same object at a less cost.

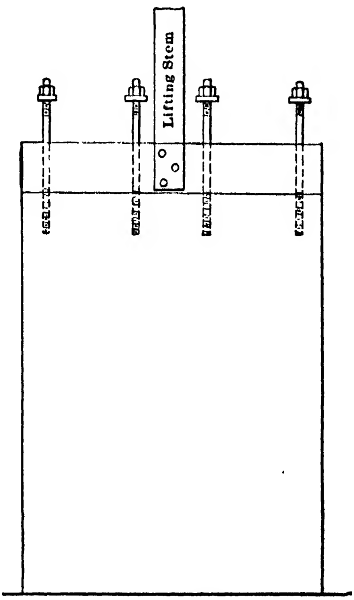


FIG. 156.—Divided headgate.

Where headgates are made of metal, the sliding surfaces on these also should be bronze-lined. It is better, in every case, to construct headgates of steel or cast iron. The gates are usually made in one casting, the leaf and the supporting ribs being all formed in the mould, as indicated in Fig. 153.

The width of a headgate should not exceed 4 ft., although its length may be as great as desired. Some have been made as long as 31 ft. If a greater width than 4 ft. is required for any single penstock, the opening should be divided into two or more sections and a headgate placed over each. The reason for this limitation in width of the gate lies in the fact that when it is nearly closed, and in the position where the lower edge is just making contact with the lower edge of the frame, there is practically the full hydrostatic pressure, due to its depth and area, acting against it. This sets up a transverse flexure tending to bend the gate and cause a downstream deflection. If this deflection is appreciable, the lower gate edge can not seat against the lower edge of the gate frame, because one is slightly

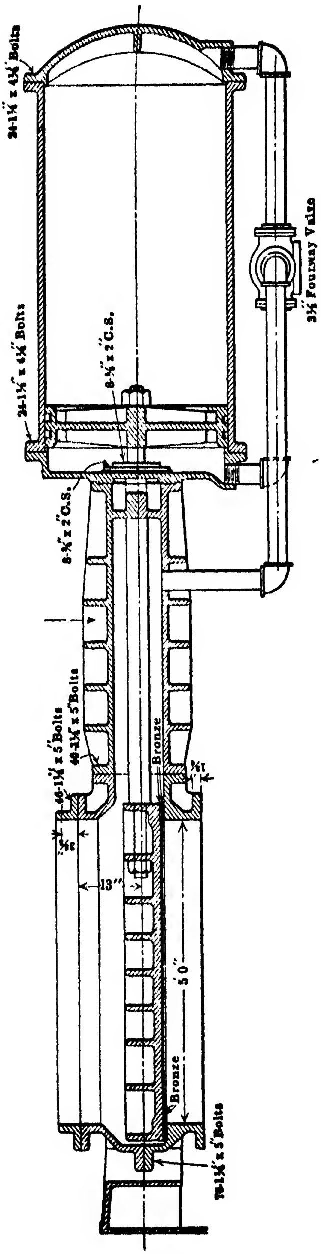


Fig. 157.—Gate operated by hydraulic cylinder.

curved while the other is straight, and the curved edge of the gate will not slide over the straight edge of the frame, but the gate edge and frame edge will overlap, and the gate can not be seated except by cutting its way into the bronze lining of the frame. It is difficult to get a reasonably water-tight headgate even under the best conditions, and if it is wide enough to allow a considerable deflection, with the result of cutting of the contact surface of the lower edge of the frame, it will leak badly, and the leak will increase with the number of times the gate may be raised and lowered.

Instead of the use of headgate hoists, these gates are sometimes operated by pressure cylinders placed above the gate and fastened to the frame, as depicted in Fig. 157. The piston rod passes through the lower cylinder head and is fastened to the gate. By means of the pipe connection and a four-way valve, pressure can be admitted to either side of the piston, and the gate, thereby, made to ascend or descend.

Usually, oil is used as the working fluid, though water is also sometimes employed. The necessary pressure is produced by means of a motor-driven pressure pump.

It is to be noted that when the valve is near the point of closure the vertical thrust is very great, and the piston rod is in compression. The size of the rod must be fixed by these conditions, it being computed for strength as a long column. The maximum lifting force is generally taken at 60 per cent. of the total hydrostatic pressure against the face of the gate.

Installations in which the headgates are equipped with these operating cylinders should have one gate arranged to be moved by hand. Otherwise, the condition might arise of the whole station being shut down and no power available to operate the pressure pump supplying oil or water to the lifting cylinders.

Wherever all the gates are arranged to raise and lower with pressure cylinders, it is necessary to provide a hand pump by means of which working fluid may be pumped to lift one of the gates high enough to supply power to one of the water wheels, from which energy can be taken to work the motor-driven pressure pump.

Wherever the depth of water over the penstock is not too great, the best form of head gate is the Stoney roller gate, which has been described in the preceding chapter. The bottom edge of this gate, which is made of structural steel, abuts against the heavy timber and does not slide into a seat. The staunching rods at the sides of the gate, make it more nearly water-tight than the usual sliding headgate; also, the amount of power required to move these gates is so much less than that necessary for sliding gates that filler gates can be omitted.

The Stoney gate does not have to be subdivided in narrow vertical sections of 4 ft. or less, but may be made as wide as desired. Its use as a headgate, however, is limited to conditions where the distance from the bottom of the penstock to maximum high-water elevation is 15 ft., or less.

The Taintor gate, which turns about trunions, is suitable for a headgate where the water passes into a canal and this type of gate has been used for this purpose. This gate is described in the preceding chapter.

Headgate Hoists.—The mechanism depicted in Fig. 153, for raising and lowering a headgate, is applicable to comparatively small gates only. Where gates are of a considerable size, the mechanism must be heavier, the gearing ratio between gate pinion and operating handle increased, and the gate must be provided with two lifting stems instead of one, to better distribute the stresses.

A hoist of this latter kind is shown in Fig. 158.

The more satisfactory type of hoist is the screw hoist, which is made in a number of different forms, one of which is shown in Fig. 159.

This hoist is operated by rotation of the horizontally placed wheel on top, which is bevel-geared and meshes with a pinion on a horizontal shaft, the latter being operated by the hand wheel. The hub of the horizontally placed wheel is threaded and within it works a threaded vertical shaft, which forms the upper portion of the gate stem. Therefore rotation of this wheel screws the shaft up or down and, with it, the gate. In order to reduce

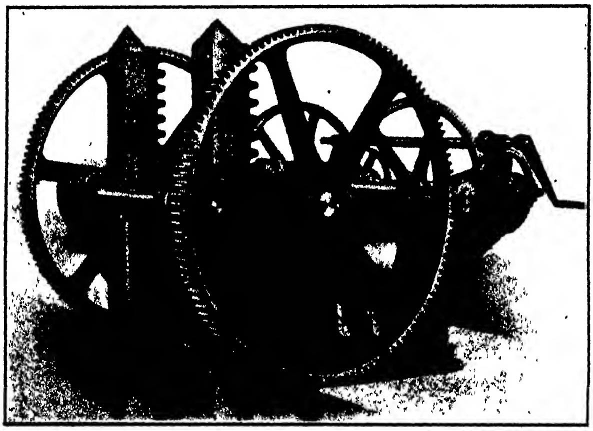


FIG. 158.—Double stem, geared hoist.

friction, the thrust of this main gear wheel is taken up against ball or roller bearings.

The hoist is also provided with a back gear, by means of which the gearing ratio between the hand wheel and the main operating wheel can be greatly increased. This is to start the motion of the gate in opening, or to finally seat the gate in closing. At such times, the resistance to motion is very much greater than after starting the gate from its seat, or before seating it in closing. This back gearing, therefore, gives the ability to exert a great starting or closing force, after which it may be thrown out of gear and the gate made to travel over the rest of its path at a comparatively high speed. Obviously, these hoists can be arranged to be driven by electric motors and frequently are.

Headgate hoists are sold on the basis of the ability to lift a net load. The distance of lifting for any hoist can be made as great as desired by making the rack or the threaded stem, whichever may be used, as long as required. The hoists should always have ample capacity and be strong enough to close or open a gate under full hydrostatic pressure against it. In case of breakage of the turbine gates, it is impossible to fill the penstock through the filler gate, and, unless the hoist has a capacity to close the head-gate with the full pressure against it, the resulting conditions may prove serious.

The cost of hoists is too small a proportion of the total equipment cost to attempt mistaken economies in getting them too small. Their lifting capacity must not be based on a mere

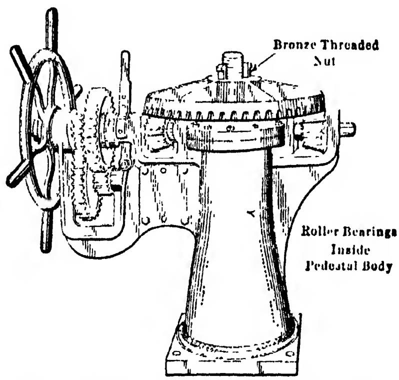


FIG. 159.—Screw hoist with change gears.

coefficient of friction and the resulting resistance to motion produced by a given pressure, because the gate or the gate frame may become slightly distorted and the force required to start it from its seat may be considerably greater than that which might be, normally, expected.

Sluice Gates.—Sluice gates differ from headgates, only in name, the term being applied to any large valve having a flat leaf which slides into, or out of, position to close or open. In hydraulic-power developments this term is usually applied to gates which are placed at the bottom of the dam, in order to drain the lake. These drain gates are an essential feature of every dam, not so much because they are necessary after the construction is completed and the plant is in operation, but, by their use, the cost

of constructing the dam is considerably reduced, and, in case of leaks under or around the dam, it may become necessary to drain the lake. For lowering the lake below the level of the crest, in order to work on the crest gates or flash-boards, drain gates are indispensable. Many dams have been built with drain gates of such small dimensions that it would be impossible to lower the lake level, because the minimum stream flow exceeds the quantity of water which the gates can discharge. The area of the

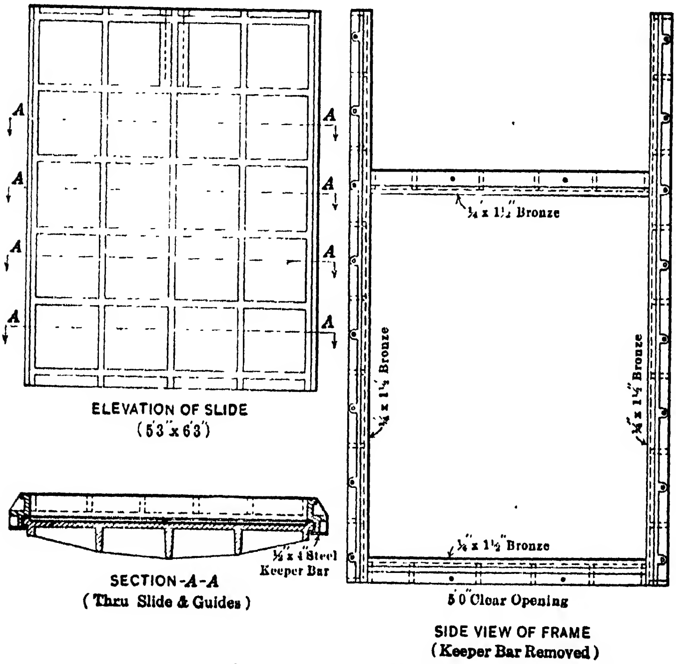


FIG. 160.—Sluice gate.

gates should always be sufficiently great to pass the normal stream flow, and an additional amount of water such that within from 3 to 7 days the lake could be emptied.

Sluice gates are nearly always made up of heavy cast-iron slabs, provided with strengthening and supporting ribs, and arranged to slide in a strong iron frame, the sliding surfaces being faced with bronze strips. Fig. 160 shows the details of design for a sluice gate and frame to give a clear opening 5 by 6 ft. and to work under a head of 65 ft.

The location of these gates and methods of placing in dams vary, as do also the means used to operate them.

In solid dams, where the height of the dam is not too great, say 30 ft. or under, the gate frame is set directly in the masonry of

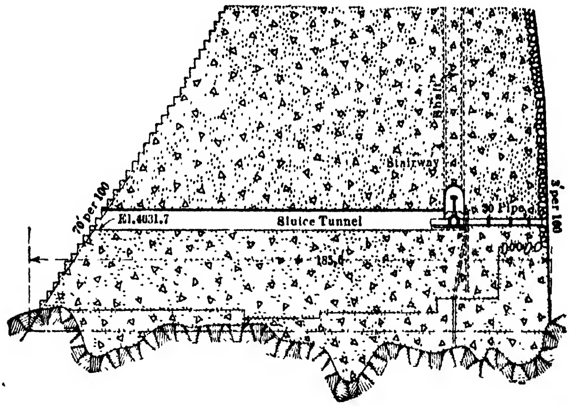


FIG. 161.—Section through solid dam showing sluice way, longitudinal tunnels and entrance shaft.

the dam, at the upstream face, and the operating rod or stem carried up to the top of the dam, on which is placed a mechanism for raising and lowering. In this case, the gates are set in bulkhead sections and not in spillway sections. In other instances,

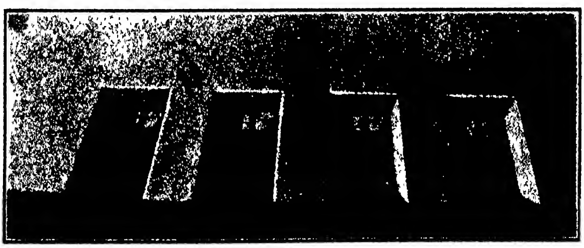


FIG. 162.—Sluice gates, inset in dam.

the gate is placed in a tunnel running longitudinally through the dam, and the tunnel is reached by means of a winding stairway descending through a vertical shaft, which shaft is made from the top of the dam down to the tunnel in the bulkhead section. This latter method of installation is indicated in Fig. 161.

In the case of hollow reinforced-concrete dams, it is customary to place each gate frame in an inset made in the face of the dam, the operating rod passing into the interior of the dam where the

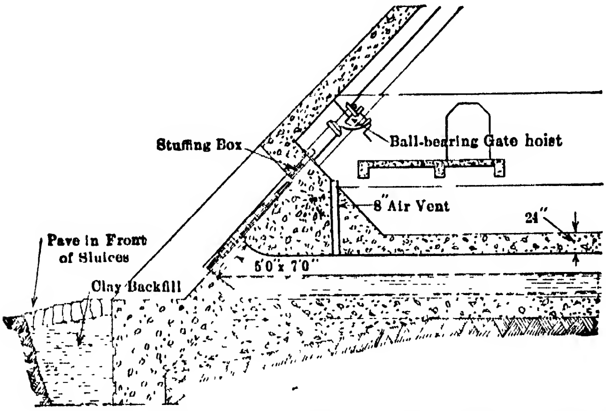


FIG. 163.—Section through dam showing interior hoist for sluice gates.

hoisting machine is placed. Fig. 162 shows a group of four gates located in this manner, while Fig. 163 shows a section through the dam, and in it may be seen the hand-operated

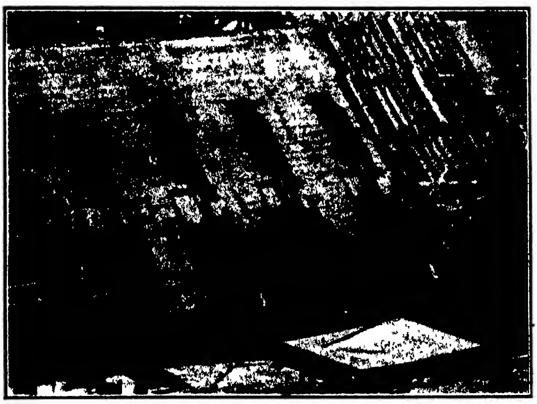


FIG. 164.—Sluice gates and operating cylinders on face of dam.

gate hoist, which is reached from a concrete platform on the interior. In the Austin, Tex., dam, the gate frames were set directly on the face of dam and the hoisting device, which in this case is a piston worked in a cylinder and actuated by oil

pressure, was also placed on the face of the dam, the whole apparatus being submerged. This arrangement is shown in Figs. 164 and 165.

The hoisting mechanism may comprise any one of the devices which have been described under the caption "Headgates," provided that they be arranged to shove the gate downward to close it, as well as to lift it up in opening it.

Under the depths where these gates operate, the pressures required to open and close are very great, and under certain

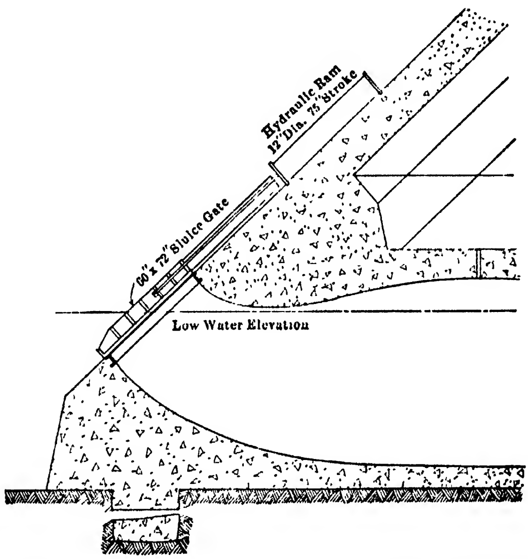


FIG. 165.—Section through sluice way showing cylinder and sluice gate, mounted on face of hollow dam.

conditions it would be better to use some type of balanced valve, such as the Johnson valve, later described. Wherever a flat sliding valve is used, the author's preference is for cylinder-operated valves, the valve and cylinder being both placed on the deck of the dam, without any insets being made in the structure. This arrangement has been criticised on the ground that the entire apparatus is submerged, and that it is impossible to reach it for inspection or repairs. The only part of the device which may need inspection or repairs is the sliding gate itself, and this is, of necessity, submerged and inaccessible, regardless of how it may be set, or the means used to operate it.

The cylinders for moving the gates of the Austin dam are of heavy cast iron and the pistons are of bronze. The stuffing boxes, through which the piston rod pass, are made tight with metallic packing. There are no valves or other operating parts, and, therefore, no possibility of any disarrangement of the device. Oil is supplied from a pressure pump in the power station, at 750 lb. per square inch, and two pipe lines, one a pressure feeder, the other the exhaust line, run from the power house into, and through, the dam. The four-way valves, by which the pressure may be applied to either side of the piston, are located inside the dam, and are accessible from the runway which passes through it.

The indicator, to show the operator the position of the gate, at any point in its travel, is made as follows: A 3/8-in. stranded copper cable is attached to the upper surface of the gate, and carried through a 1 in. pipe fixed to the face of the dam, up to a point near the crest, where it passes over a 4 in. sheave wheel of brass, then turns down vertically into the interior of the dam, through a second piece of 1-in. pipe which is sealed into the concrete, and has a stuffing box on its lower end, through which the cable passes. A 50-lb. weight made of concrete, is attached to the free end of the cable and behind this weight is placed a scale, the measurements on which, referred to the lower edge of the weight, show the distance of opening the gate. These gates have been opened and closed many times, and have never given any trouble from the date of installation.

The Johnson Valve.—This valve consists essentially of a circular body forming an enlargement of the pipe line or penstock, and having an internal cylindrical chamber containing a sliding plunger. The closed end of the internal chamber and the nose of the plunger are of conical form. They are designed to guide the water smoothly as it enters and leaves the valve.

No external source of power is required for operation. When the plunger is withdrawn into the internal or operating chamber, the valve is open and presents an unobstructed passage for the water. When the plunger protrudes from the operating chamber, it seats against a ground ring in the neck of the valve body, forming a water-tight joint. The standard control mechanism provides for only the open and closed positions of the plunger, but it may be specially arranged to hold the plunger at intermediate positions if desired.

Figure 166 shows the essential parts of this valve and the manner in which it works, while Fig. 167 shows a complete valve. At the rear end of the latter figure may be seen an indicator which shows the position of the plunger at any point in its movement.

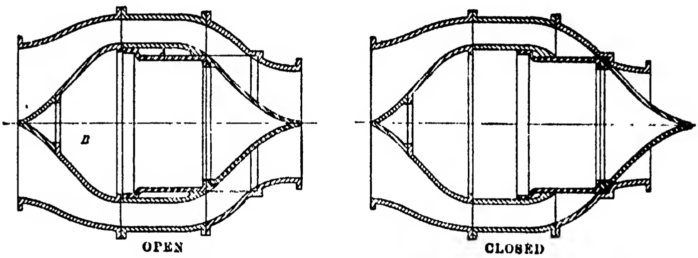


FIG. 166.—Johnson balanced valve. Longitudinal sections.

The valve plunger is differential, and forms an annular chamber *A* within the operating cylinder, in addition to the central chamber *B*. By means of a suitable external control valve and

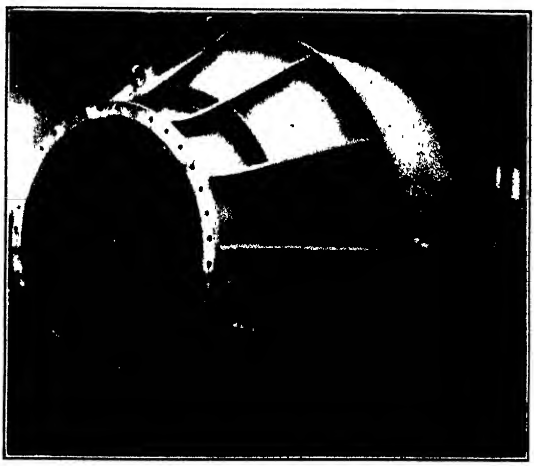


FIG. 167.—Johnson valve.

piping, either pipe-line or atmospheric pressure may be alternately applied to the chambers *A* and *B*. Admitting pipe-line pressure to *A* and exhausting it from *B* opens the valve; reversing the operation closes it.

The valve works equally well with the water flowing through it in either direction, and it may be operated readily in either still or flowing water. There is no danger of water hammer, because it is impossible for the valve to close any faster than at the usual rate. This is adjustable and may be made as slow as desired.

The external control valve is of the balanced-piston type in large sizes, and the ordinary four-way plug type in small sizes. Both types may be operated by hand or electricity. The control valve and its operating mechanism may be located at a considerable distance from the valve, if desirable. Also, the control mechanism may be electrically operated from a distance. Thus, the valves may be placed at the forebay or intake of a water-power plant and operated from the switchboard in the power house.

The valve may be placed in any position, horizontal, vertical or at any angle.

Another application of this valve is for automatic pressure relief. The valve plunger is held closed by air pressure so arranged that it is automatically released, permitting the valve plunger to open when the pipe-line pressure exceeds normal by some predetermined margin. The advantage of using air, rather than water, lies in the rapidity with which the air may be discharged, and the consequent rapid opening of the relief valve.

All standard valves may be arranged to close automatically in case of a break in the pipe line on either side of the valve, provided the break is sufficient to reduce materially the pressure at one end of the valve

The obvious objection to this valve is its high cost as compared with other forms of water gates. In spite of the greater expense of their installation, the advantages of these valves have induced their adoption in a number of recent developments.

CHAPTER X

WATER WHEELS

There are many types of water wheels, varying from the old overshot to the modern improved wheel, but the two kinds which are used, practically, to the exclusion of all others at the present time, are the mixed-flow reaction turbines and the curved-bucket impulse wheel, generally known in the art as the "Pelton" wheel. Therefore, this discussion of water wheels will be limited to these two types.

Reaction Turbine.—The essential elements of a reaction turbine comprise: a series of vanes set at equal intervals around the circumference of a circle, which vanes are stationary and guide the water to the wheel, and a wheel, free to rotate and having around its periphery a number of vanes into which water is directed from the stationary, or guide, vanes. The pressure set up against the wheel vanes by the action of the water produces a torque and causes the wheel to rotate and deliver power.

In the original development of turbines, the relative arrangement of the guide vanes and the wheel vanes varied according to the views of designers. In some cases the guide vanes were placed inside the wheel and were encircled by it. The flow was then radial, from inside to outside, and this, therefore, is called an outward-flow turbine. In other instances this relationship was reversed, the guide vanes being outside and encircling the wheel, in which case the water would flow from the periphery toward the center, and this is termed an inward-flow turbine. A third arrangement was that of an equal diameter of the wheel and of the guide vanes, the guide vanes being placed above the wheel, so that the water flows axially, from the guide vanes to the wheel vanes, and this is called a parallel-flow turbine.

In most of the turbines which are now being installed, the guide vanes are placed outside of, and around, the wheel, and the water enters the wheel at its periphery, flowing toward the center, so that the action begins as an inward-flow turbine. The

water, after entering the wheel, turns from a radial direction to one parallel with the shaft, and is discharged when flowing in a direction parallel to the shaft. This, therefore, is a mixed-flow wheel. This type of wheel is also called the "Francis" turbine, after its designer, J. B. Francis.

FIG. 168.—Reaction, or "Francis," runners.

Figure 168 shows wheels of this type, without the guide vanes and the large runners not mounted on shafts. From the picture, the manner in which the water enters and is discharged is clear. Fig. 169 shows, diagrammatically, the arrangement of the guide vanes and the wheel vanes relative to each other.

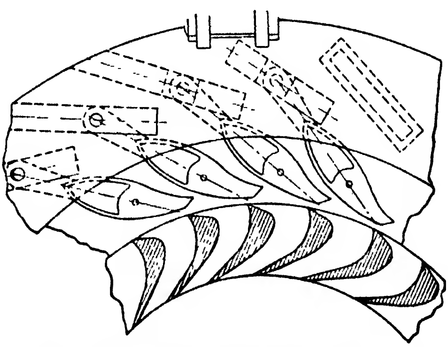


FIG. 169.—Section of turbine and gates.

The wheel and gates must be placed in a closed chamber, in which the full water pressure, due to the head, exists. The water discharged from the wheel, passes through an opening in this chamber, which opening connects with the wheel casing, and

no water can pass from the chamber except by going through the wheel.

If a draft tube is provided, the wheel may be set at any height above the level of the water on the downstream side of the dam or tail water, up to a limit of 25 or 26 ft. The draft tube is simply a water- and air-tight tube extending from the discharge opening of the turbine case down to tail water and carried a sufficient depth below the water surface to seal the bottom of the tube against entry of air. The action of this column of water in producing power at the water wheel is the same as if the wheel itself were set down at the tail-water level. Physically, this action may be described as similar to that of a syphon, the water exerting a suction proportional to the height of the column. The theoretical height at which a column of water may be maintained by air pressure, at sea level, is approximately 34 ft., and this would appear to be the height above tail water at which a turbine might be set. For reasons which will later appear, this full height can never be used. Also, at elevations above sea level the available draft head diminishes with the altitude.

In order to vary admission of water to the turbine wheel, to accord with variations in load on it and the power which it is required to deliver, there are three general methods of varying the area of the opening to the wheel vanes. One of these is by the use of a cylinder gate, which is simply a cylinder of thin sheet metal that surrounds the wheel, arranged to move in an axial direction. When the cylinder is moved so that it completely surrounds the wheel, no water can enter the vanes and the power is shut off. If it be moved to uncover more and more of the length of the vanes, the area of opening to the wheel is increased by the distance through which it moves, and, in this manner, the quantity of water delivered to the wheel, and its output, may be varied in accordance with the load. This gate, while simple, cheap and reliable, has the defect of reducing the efficiency of the wheel at partial loads, and it, therefore, is not used in hydroelectric developments.

The register gate is made of a metal cylinder surrounding the guide vanes and having in its periphery a number of openings which correspond in position and length to the entrance openings of the guide vanes. When the openings in the cylinder correspond with the guide-vane openings, the full flow of water passes to the

wheels. If, however, the cylinder be rotated through a small angular distance, the holes through it will not register with the guide-vane openings, and the solid portions will intercept more and more of the entrance openings of the guide vanes, until, in its extreme position, the guide-vane openings are completely shut off from the water by the solid portions of the cylinder. This gate has the difficulty that it is likely to bind and is almost sure to do so if any grit or sand works between it and the periphery of the guide-vane structure, in which case it is difficult to move the gate, and it, therefore, is not suitable for automatic governing.

The wicket gate is the one which is now used to the practical exclusion of all other types.

As a matter of fact, this is not a gate at all, but is a method of arranging the guide vanes so that they are movable, each being arranged to turn about a pair of trunnions. Movement in one direction will separate them further apart, while with movement in the other direction they are brought closer together, the extreme positions being the maximum width of opening in one direction, and in the other direction, contact between the adjacent vanes so that the water is completely shut off from the wheels.

The details and arrangement of this form of gate are given more fully hereafter.

General Theory.—(1) *Force of Jet Against a Vane.*—If a flat vane be pushed against the end of a nozzle, closing it, and having against it the pressure of the water which tends to emerge from the nozzle, the pressure against the vane, in pounds, will be

$$P = 62.5HF \tag{205}$$

in which H = head in feet and F = area of nozzle, in square feet. This condition is shown in (*a*) Fig. 170, at the left.

If the vane be allowed to move away from the nozzle a short distance, as shown on the right in (*b*) Fig. 170, the pressure exerted against the vane by the jet which issues from the nozzle will be

$$P = 2 \times 62.5HF \tag{206}$$

or twice the hydrostatic pressure.

If, instead of a flat vane, the jet strikes against a curved one of semicircular cross-section, entering at one side and departing on the other, as in (*c*) Fig. 170, the pressure against the vane will be

$$P = 4 \times 62.5HF \tag{207}$$

The apparent doubling or quadrupling of the pressure against a vane when the head remains constant seems, at first, paradoxical. The explanation lies in the fact that in the cases of the jets striking against the vanes, the kinetic energy in the continually flowing stream of water, acts on the vanes.

The general energy equation of mechanics is

Force = *Mass* × *Acceleration*, or

$$P = MA \tag{208}$$

The acceleration of a mass which changes its velocity from V_1 to V_2 is equal to the rate of change in velocity, or the total change in velocity divided by the time, T, in which the change occurs.

$$A = \text{acc.} = \frac{V_2 - V_1}{T} \tag{209}$$

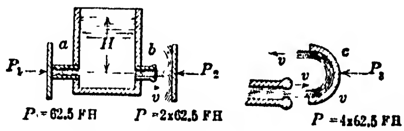


Fig. 170.—Water pressures on vanes.

If the time be taken as 1 sec.,

$$T = 1, \text{ and } A = V_2 - V_1$$

The quantity of water, Q, flowing through a nozzle in the unit time of 1 sec., is the velocity of efflux through the nozzle, multiplied by the area of the jet, or $Q = VF$, F being the area of the jet.

The mass is

$$M = \frac{62.5Q}{g} = \frac{62.5VF}{g}$$

when F is in square feet and V in feet, per second.

When water emerges from a nozzle and strikes against a flat vane, its velocity in the direction of the vane is totally destroyed, so that the change in velocity is equal to the initial velocity of the jet, or

$$A = \text{acc.} = V_2 - V_1 = 0 - V_1 = -V_1$$

Substituting the values for M and A in equation (208)

$$P = \frac{62.5V_1F}{g} \times (-V_1) = -\frac{62.5V_1^2F}{g} \tag{209}$$

But, $V_1 = \sqrt{2gH}$

Hence

$$P = -\frac{62.5 \times 2 \times gHF}{g} = -2 \times 62.5HF$$

The negative sign shows that the reaction of the vane is opposite in direction to the pressure of the jet.

From this computation, the reason now becomes clear for the production of the pressure double that produced by the hydrostatic pressure, when the water is allowed to flow from a nozzle and strike a flat, fixed vane.

When the vane is curved, and the jet striking it is completely reversed, the pressure produced by the jet against the vane, up to the point where the path of the water is at right angles to its initial direction (*i.e.* after travelling through 90° of arc and reaching the midpoint of the curve), is

$P = 2 \times 62.5HF$, just as for the preceding case.

The absolute velocity of the water at this point is the same as that with which it entered the vane, but the velocity in the direction of its initial path is zero. Hence, the change in velocity with reference to the vane is $0 - V_1 = -V_1$ as in the preceding case.

The water passes from the middle point of the arc out through the opposite side of the vane, leaving it with a velocity $= V_1$. Therefore, the acceleration, or change in velocity, with respect to the direction of pressure against the vane, is $0 - V_1$. Hence, the pressure against the vane to produce this acceleration of the jet must be

$$P = 2 \times 62.5HF.$$

That is, the reaction of the jet is equal to the pressure which it produces against a vane. The total pressure, or P, is the sum of the two pressures produced by the stream entering and departing. Hence, the pressure produced against a curved vane by a jet, when its direction is completely reversed, is

$$P = 4 \times 62.5HF.$$

If a jet impinges on a cone, or any surface of revolution, the axes of the jet and of the surface lying in the same straight line, as shown in Fig. 171, the velocity of flow when the water leaves the surface is V, as shown. Its velocity in the direction of the axis of the jet, on leaving the surface, is $V \cos \beta$, β being the

angle through which the water is deflected, or the angle between the axis of the jet and the path of the water on leaving the surface.

The change in velocity, then, is from V to $V \cos \beta$, and, therefore,

$$A = \text{acc.} = V - V \cos \beta = V(1 - \cos \beta) \qquad (210)$$

Hence, the pressure against the surface, in an axial direction, is

$$P = 2 \times 62.5HF(1 - \cos \beta) \qquad (211)$$

From this, the previously given formulæ may also be deduced.

For a flat vane, $\beta = 90°$, $\cos \beta = 0$, and $P = 2 \times 62.5HF$.

For complete reversal of jet, $\beta = 180°$ and $\cos \beta = -1$.

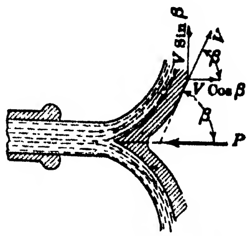


FIG. 171.—Action of a jet on a surface of revolution.

Hence, $P = 2 \times 62.5HF\ (1 - (-1)) = 2 \times 62.5HF \times 2$.

In the previous examples of a jet striking a vane, the forces other than those parallel to the axis of the jet were all neutralized by equal and opposite forces.

In the case shown in Fig. 172, the jet exerts both an axial pressure and one perpendicular to its axis.

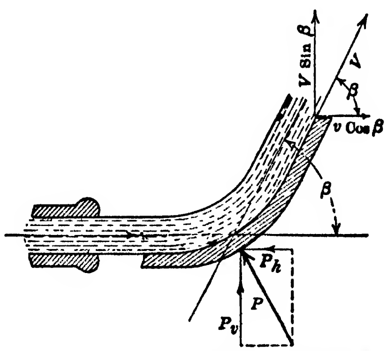


FIG. 172.—Action of a jet on a curved vane.

As in the foregoing example, the axial component of the pressure is $P_h = 2 \times 62.5HF\ (1 - \cos \beta)$, the angle β being indicated in the figure and representing the angle of change in direction of the path of the water; H = head on jet and F = area of jet, in square feet.

Also, the pressure perpendicular to the axis of the jet is

$$P_v = 2 \times 62.5HF \sin \beta \tag{212}$$

The total pressure $P = \sqrt{P_v^2 + P_h^2}$ (213)

Substituting the values of P_v and P_h and reducing,

$$P = 125HF\sqrt{2(1 - \cos \beta)} \tag{214}$$

If the vane shown in Fig. 172 is moving away from the jet and following a path in the line of the axis of the jet, the velocity of the water striking the vane is reduced by an amount equal to the velocity of the vane itself. Obviously, if the vane moved faster than the water, there would be no pressure produced at all, nor could there be any pressure if the vane moved as fast as the water.

The pressure must always be due to the velocity with which the jet strikes the vane, and this is the difference between the velocity of the jet and that of the vane, that is, the relative velocity of the two.

Also, the quantity of water which impinges on the vane is not equal to the quantity discharged from the nozzle. If the vane should move as fast as the jet velocity, or faster, none of the water would strike it; the amount of water discharged by the nozzle which will reach the vane is proportional to the difference in the velocities of jet and vane. The velocity of the water with respect to the vane being $V - U$, the value of the head, H, in the formula $P = 125HF$, therefore, is $\frac{(V - U)^2}{2g}$. Hence, the pressure, P, on a moving vane, is proportional to $(V - U)^2$. Obviously, this pressure is proportional to V^2 when the vane is stationary, $(U = 0)$. Hence, if P_1 denote the pressure on a moving vane, and P the pressure when the vane is stationary,

$$P_1 : P : (V - U)^2 : V^2 \tag{215}$$

V = velocity of the jet.
U = velocity of the vane.

This applies only to a single moving vane, however.

If there is a continuous succession of vanes passing in front of the jet, then the whole quantity of water discharged reaches the vanes, and the pressure P_2, for the action of a nozzle on a series

of moving vanes, is related to the pressure P of a nozzle against a fixed vane, thus:

$$(V_1 - U) : V_1$$

and the pressures against vanes moving axially away from the jet are:

$$P_1 = \frac{62.5FV_1(V_1 - U)}{g}, \text{ for a flat vane} \tag{216}$$

and

$$P_3 = \frac{62.5FV_1(V_1 - U)(1 - \cos\beta)}{g}, \text{ for a curved vane} \tag{217}$$

Composition of Velocities.—The foregoing discussion of moving vanes included motion of the vane in the same direction as that of the jet only. Obviously, in this case, the velocity of the jet with reference to the vane—or relative velocity—is numerically equal to the difference of the velocities of the two, and the direction of action is in the same path as that of the jet and also that of the vane.

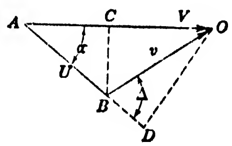


FIG. 173.—Compositions of velocities.

When the direction of motion of the jet and the vane are not parallel, or coincident, but make an angle with each other, there is a change in the conditions before set forth.

Assume that the jet acts in the direction AO, moving at a velocity V, and the vane is moving in the direction AB at a velocity U, and that the vane travels through the distance AB while the jet moves through distance AO (see Fig. 173). The vane moves away from the jet a distance equal to AC, measured axially. Hence, the difference in axial velocity is $AO - AC = CO$. In the same length of time the jet has had to move at right angles to its axis, a distance equal to CB. The components of the relative velocity are, therefore, CO and CB. The diagonal constructed on these is BO, which represents in both direction and magnitude the relative velocity between the jet and the vane.

If α = angle between the two directions of motion, and Δ = angle between the direction of the resultant velocity and that of the vane, then,

$$V \cos\alpha = U + v \cos\Delta$$

and

$$V \sin\alpha = v \sin\Delta$$

The tangential component of the velocity V, is, evidently,

$$AB + BD = U + V_1 \cos \Delta = V \cos \alpha \tag{218}$$

THEORY OF THE REACTION TURBINE

It is somewhat difficult to apply the equations for a single vane, moving in the path of the jet, to a series of vanes turning about an axis. The simpler method of analysis is as follows:

Turning Moment on Wheel.—Conceive a series of guide vanes which are fixed, and through which the water discharges to vanes on the periphery of a rotating wheel, as indicated in Fig. 174. Consider one of the nozzles, or openings between the guide vanes,

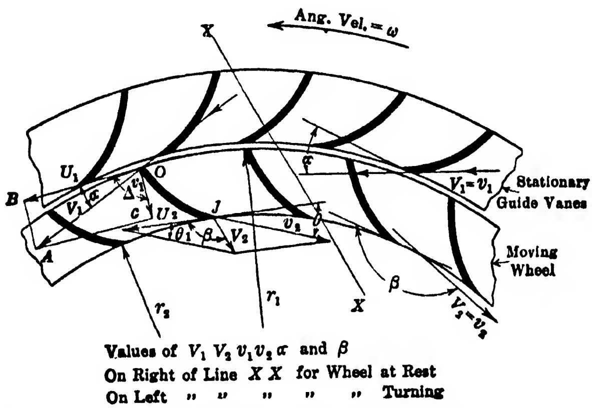


FIG. 174.—Diagram showing velocity relationships in a turbine.

which is delivering water to the wheel at a velocity V_1, in the direction shown by the arrow. This water strikes against a wheel vane which is turning in a counter-clockwise direction, the motion of the vane being, for the instant, in the direction of the line OB which is tangent to the wheel periphery at the outer end of the vane. This line also represents the velocity of the vane, or U_1. The water is discharged from the wheel vane at a velocity V_2. The velocities V_1, U_1 and V_2 are absolute—not relative or resultant velocities.

The tangential component of V_1 is $V_1 \cos \alpha$, and the tangential component of V_2 is $V_2 \cos \beta$, α being the angle between V_1 and the tangent at O, and β the angle between V_2 and the tangent to the inner circle of the wheel at J. Δ = angle between U_1 and v_1.

The pressure set up by the water against the vane in a tangential direction, at entry, is:

$$P_1 = MV_1 \cos \alpha \tag{219}$$

in which M = mass of water per second through the guide vanes, or $M = \frac{62.5\,Q}{g}$, Q being the quantity discharged against the vanes, in cubic feet, per second.

In the same way, the pressure set up by the water discharged from the wheel vane is

$$P_2 = MV_2 \cos \beta \tag{220}$$

The pressure due to V_1 acts through a lever arm r_1 about the center of rotation, while the lever arm of the pressure due to V_2 acts through the lever arm r_2.

Hence, the two turning moments of these forces are:

$$MV_1r_1 \cos \alpha, \quad \text{and} \quad MV_2r_2 \cos \beta \tag{221}$$

The net turning moment or the torque, T, acting on the wheel is the difference between the moment due to entry of the water to the wheel vanes and that due to the discharge of the water from the vanes, is

$$T = M(V_1r_1 \cos \alpha - V_2r_2 \cos \beta) \text{ lb.-ft.} \tag{222}$$

If the wheel turns at a velocity equal to $2\pi n$, or ω radians per second, the work done, R, is $T\omega$ ft.-lb. per second, or

$$R = \omega M (V_1 \cos \alpha r_1 - V_2 \cos \beta r_2) \text{ ft.-lb. per second} \tag{223}$$

But ωr_1 is the linear velocity of the outer rim of the wheel at the point of entry, and ωr_2 is the linear velocity of the inner rim at the point of discharge.

$\omega r_1 = U_1$ and $\omega r_2 = U_2$. Then

$$R = M (U_1V_1 \cos \alpha - U_2V_2 \cos \beta) \text{ ft.-lb. per second} \tag{224}$$

Substituting the value of M in the equation (224).

$$R = \frac{62.5Q}{g} (U_1V_1 \cos \alpha - U_2V_2 \cos \beta) \text{ ft.-lb.} \tag{225}$$

which is the total power delivered to a wheel in 1 sec.

Since one horsepower is 550 ft.-lb. per second

$$HP = \frac{62.5Q}{550g} (U_1V_1 \cos \alpha - U_2V_2 \cos \beta) \tag{226}$$

Obviously, the greater the velocity of the wheel, the less will be the torque for a given value of entering velocity, V_1. If the speed of the wheel is greater than the velocity of the water, it is clear that the water can not impart power or turning moment to the wheel. In equation (222) for the value of the torque, it is seen that all the quantities are given in terms of the absolute velocities of the water entering and discharging from the wheel vanes. This seems, at first glance, to give a value for the torque that is independent of the speed of the wheel.

Equation (222) is not, however, independent of the wheel speed, because the absolute value of V_2 and of the angle β are both dependent on the wheel velocity.

Referring to Fig. 174, the comparative values of V_1 and V_2, when the wheel is held against turning, are indicated to the right of the line $X - X$. On the left of this line are indicated the corresponding values when the wheel is in motion. The direction of discharge for a blocked wheel is, obviously, tangent to the curve of the vanes at the point of discharge. When the wheel is in motion, however, a velocity is imparted to the water within the wheel vanes, and its absolute velocity is changed, both in magnitude and direction.

In the same manner as has before been set forth, the velocities of v_1 and v_2 relative to the wheel are determined. If a parallelogram be constructed on $V_1 = OA$ as a diagonal, $OB = U_1$ as one side, with the angle α between them, the line Oc will represent the velocity of the entering water relative to the wheel at the point of entry. This relative velocity is denoted by v_1.

In order that the water may enter the turbine wheel and impinge against the vanes without shock, the shape of the vanes must be such that at the point of entry their surfaces are parallel to the direction of the relative velocity v_1, and if the line denoting v_1 be drawn through the periphery of the wheel and touching the end of the vane, it must be tangent to the curve of the vane at that point. Notice that this relation obtains in Fig. 174.

V_2 must be as small as possible, because the velocity in the discharged water is lost energy and equivalent to a loss in head equal to $\frac{V_2^2}{g}\cdot$ The general formula for V_2 is

$$V_2 = \frac{v_2 \sin \theta}{\sin \beta} \tag{227}$$

For the condition of best efficiency, the wheel speed is such that v_2 equals, approximately, U_2, and for this condition

$$V_2 = 2U_2 \sin\frac{\theta}{2} = 2U_2 \cos\beta \tag{228}$$

θ being the angle between the direction of the relative velocity v_2 and the tangent, or line of wheel velocity U_2, as indicated.

The direction of v_2 is fixed by the condition that the water must emerge from the wheel in a direction tangent to the curvation of the vanes at the point of discharge, so that the values of θ and $\sin\frac{\theta}{2}$ are fixed, and V_2 can be computed from the constants of the wheel.

V_2 is then found, either analytically from formulæ (227), (228), or graphically. Graphically, V_2 is the diagonal of a parallelogram constructed on U_2 and v_2 as two adjacent sides, as indicated in Fig. 174.

The velocity through the gates, or guides, is not necessarily equal to the spouting velocity due to the total head, but it is influenced by any internal pressures in the wheel.

The velocity, actually is

$$V_1 = C\sqrt{2g(H - h_1)} \text{ ft. per sec.} \tag{229}$$

in which, $h_1 = p/62.5$, p being the internal pressure at the periphery of the wheel, and C a constant of discharge, usually taken at 0.96. The value of p differs for different wheels and, also, with change in condition for the same wheel. In general, $V_1 = 0.6$ to 0.85 of $\sqrt{2gH}$. An average trial value for preliminary computation is 0.7. The actual value of V_1, for any turbine, can be approximated from formula (231).

The effective areas of the water passages are not the actual areas measured normal to radii, or their widest cross-sections, but are those areas normal to the direction of flow of the water. The water from the guides, entering the wheel, has a direction nearly tangential to the wheel, which makes a small angle α with the tangent. The effective area of entry, therefore, is $A_0 \sin\alpha$, in which A_0 is the area of the passages taken on a cross-section normal to a wheel radius. In the same way, $a_2 = A_2 \sin\theta$, a_2 being the effective area of the wheel exit passages, and A_2 their area normal to a radius from the wheel center.

Obviously, the quantity of water, Q, delivered to the wheel is the product of the *absolute* velocity of entry from the guide vanes, by the effective area of the guide vanes, or

$$Q = V_1 a_0 \text{ cu. ft. per sec.}$$

The velocity of the water through the wheel passages is the *relative* velocity of the water to that of the wheel. For instance, $v_2 = \frac{Q}{a_2}$, v_2 being the relative, not the absolute, velocity of the water. Hence, $Q = V_1 A_0 \sin \alpha = v_1 A_1 \sin \Delta = v_2 A_2 \sin \theta$, and

$$V_1 = \frac{Q}{A_0 \sin \alpha}, \quad v_1 = \frac{Q}{A_1 \sin \Delta}, \text{ and } v_2 = \frac{Q}{A_2 \sin \theta}.$$

(See Fig. 174 for α, θ, Δ, V_1, v_1 and v_2.)

An approximate formula for the best wheel speed is

$$U_1 = \frac{gH}{V_1 \cos \alpha} \tag{230}$$

H, being the head on the wheel and V_1 the absolute velocity through the guide vanes.

Formula (228) shows that the best wheel speed should be such that $U_2 = v_2$. Take $U_1 = \frac{U_2 r_1}{r_2}$, $v_2 = \frac{Q}{a_2}$, $Q = V_1 a_0$, $a_0 = 2\pi r_1 B \sin \alpha$, and $a_2 = 2\pi r_2 B \sin \theta$, which relationships have been established or are obvious from the conditions.

r_1 and r_2 are radii of wheel at entry and discharge respectively.

B = height of vanes.

Combining the foregoing quantities and substituting in formula (230),

$$V_1 = \sqrt{\frac{gH a_2 r_2}{a_0 r_1 \cos \alpha}} \tag{231}$$

Referring to Fig. 174, which shows the parallelograms of velocities at entry and at discharge.

$$v_1 = \sqrt{V_1^2 + U_1^2 - 2 V_1 U_1 \cos \alpha} \tag{232}$$

and

$$\operatorname{Tan} \beta = \frac{v_2 \sin \theta}{U_2 - v_2 \cos \theta} \tag{233}$$

It is to be noted in equation (233) that if $v_2 \cos \theta = U_2$, $\tan \beta$ = infinity, showing that $\beta = 90°$.

If $U_2 - v_2 \cos \theta$ is negative, the angle β is greater than 90° and the values involving β as a factor become negative.

All these quantities may be found graphically, if desired, by constructing the parallelograms of velocities and scaling off the values.

It is often more convenient to express the value of the power in terms of V_1 and v_2, thereby avoiding the necessity of computing the values of V_2 and β.

In this case formula (226) becomes

$$Hp. = \frac{62.5Q}{550g}[U_1V_1 \cos \alpha - U_2(U_2 - v_2 \cos \theta)] \qquad (234)$$

Obviously, the power imparted to the wheel is zero when the velocity of the rim, U_1, exceeds a certain value relative to the spouting velocity of the water due to head, or V_1. This condition is reached when

$$U_1 = \frac{U_2(U_2 - v_2 \cos \theta)}{V_1 \cos \alpha} \qquad (235)$$

or when

$$V_1 = \frac{U_2(U_2 - v_2 \cos \theta)}{U_1 \cos \alpha} \qquad (236)$$

The condition of high peripheral velocity of the water wheel, with a low spouting velocity of the water, occurs when turbine gates are opened suddenly, causing a drop in head on the wheel, but the energy stored in the rotating parts of the unit keeps the speed nearly constant over short intervals of time during governor action.

The wheel vanes need not receive and discharge the water in one plane for the foregoing equations to apply, but, as in the mixed-flow turbine, the water may enter radially and be discharged axially in a different plane from that of entry. Fig. 175 illustrates the mixed-flow wheel and certain of the factors used in the formulæ.

It is to be observed that the form or curvature of the wheel blades do not influence the torque or power of a wheel. The only elements that determine these are the angles of entry and discharge, and their respective radii. The efficiency of the wheel may be greatly affected by improperly designed vanes, but not the power.

In standard American wheels α varies from 10° to 40°; r_1, from $0.75\sqrt{F_0}$ to $2.5\sqrt{F_0}$; r_2 from 65 to 85 per cent. of r_1; θ from 16° to 24°; pitch of wheel vanes (circumferential) = 4.5 to 12 in. F_0 = area of guide passages at periphery of wheel.

Example of Use of Formulæ.—From the measurements of a turbine, the proper speed, horsepower, quantity of water dis-

charged and efficiency under any head can be, approximately, computed from the foregoing formulæ.

As an example, consider a wheel having the form and dimensions shown in Fig. 175. The difficulty of fixing exactly the average distance of the point of discharge is obvious from the figure, and this factor, more than any other, prevents the results of the computations from being exact. Determine speed, power and quantity of water used under 1 ft. head.

Outside diameter of wheel = 3 ft.

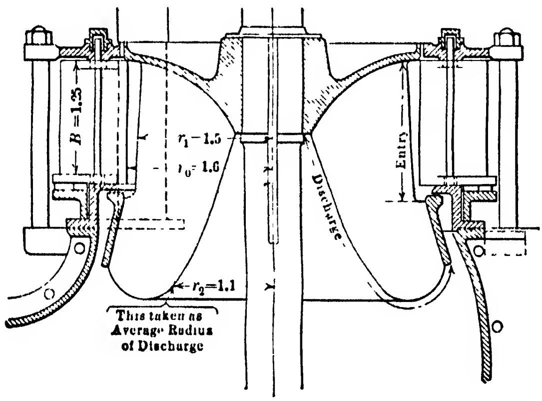


FIG. 175.

B = height of vanes = 1.25 ft.
r_0 = inner radius of guide vanes = 1.6 ft.
r_1 = outer radius of wheel = 1.5 ft.
r_2 = inner radius of wheel = 1.1 ft.
n_0 = number of guide vanes = 18.
n_1 = number of wheel vanes = 20.
t_0 = thickness of guide vanes = 0.5 in. = 0.04166 ft.
t_1 = thickness of wheel vanes = 0.4 in. = 0.0333 ft.
α, from measurement, = 22°, $\sin \alpha = 0.3746$, $\cos \alpha = 0.9272$.
a_0 = net affective area of guide vanes is
$(2\pi r_0 \sin \alpha - n_0 t)B = (6.28 \times 1.6 \times 0.3746 - 18 \times 0.04166)1.25 = 3.767$ sq. ft.
θ, by measurement, = 24°. $\sin \theta = 0.4067$. $\cos \theta = 0.9135$.
A_2 or area of discharge openings, by measurement = 12.4 sq. ft.
a_2 = net effective area of discharge = $A_2 \sin \theta = 10.5 \times 0.4067 = 4.27$ sq. ft.

Following quantities all computed for head on wheel, H, = 1 ft.

V_1 = velocity of water through guide vanes for best efficiency $= \sqrt{\frac{a_2 g H r_2}{a_0 r_1 \cos\alpha}} = \sqrt{\frac{4.27 \times 32.2 \times 1 \times 1.1}{3.767 \times 1.5 \times 0.9272}} = 5.37$ ft. per second.

Q, when wheel is working at highest efficiency $= V_1 a_0 = 5.37 \times 3.767 = 20.23$ cu. ft. water per second.

U_1 = linear velocity of periphery at entry $= \frac{gH}{V_1 \cos\alpha} = \frac{32.2 \times 1}{5.37 \times 0.9272} = 6.467$ ft. per second.

U_2, for best efficiency $= v_2 = \frac{Q}{a_2} = \frac{20.23}{4.27} = 4.74$ ft. per second.

For $U_1 = 6.467$ ft. per second, $U_2 = \frac{6.467 \times 1.1}{1.5} = 4.74$ ft. per second. These values for wheel speed check.

Rev. per sec. $= \frac{6.467}{2\pi r_1} = \frac{6.467}{6.28 \times 1.5} = 0.6865$ rev.

Rev. per min. $= 0.6865 \times 60 = 41.2$ rev.

$$\text{Tan}\,\beta = \frac{v_2 \sin\theta}{U_2 - v_2 \cos\theta} = \frac{4.74 \times 0.4067}{4.74 - 4.74 \times 0.9135} = 4.7.$$

Hence, $\beta = 78°$.

Sin β = 0.978. Cos β = 0.208.

V_2 = absolute velocity of discharge $= \frac{v_2 \sin\theta}{\sin\beta} = \frac{4.74 \times 0.4067}{0.978} = 1.97$ ft. per second.

Horsepower $= \frac{62.5 \times 20.23}{550 \times 32.2}[6.467 \times 5.37 \times 0.9272 - 4.74 \times 1.97 \times 0.208] = 0.07139[32.2 - 1.94] = 2.16$ hp.

Gross horsepower in 20.23 cu. ft. per second, under 1-ft. head $= \frac{20.23}{8.8} = 2.3$ hp.

Hydraulic efficiency $= \frac{2.16}{2.3} = 93.9$, say 94 per cent.

ϕ = ratio of theoretical spouting velocity of water under given head, to peripheral velocity of wheel $= \frac{6.467}{8.025} = 80.7$ per cent.

The power could have been computed with less labor by using formula (234) which is simpler than the one used here.

Having the discharge, speed, and horsepower for 1-ft. head, the corresponding values may be quickly computed for any other head by a simple relationship, as is shown later.

The hydraulic efficiency is simply theoretical, that is, the ratio of the work done on the wheel to the gross energy delivered to the wheel.

There will be hydraulic losses due to friction of the water in the passages, leakage and eddy whirls in the vanes and the wheel chamber, which will amount to from 5 to 10 per cent., or about 6 per cent. average, leaving the hydraulic efficiency $94 - 6 = 88$ per cent.

The mechanical losses due to friction range from 2 to 3 per cent. Taking the value as 2 per cent. in this case, the mechanical efficiency is 98 per cent.

Total efficiency, water to turbine shaft $= 88 \times 98 = 86.2$ per cent.

Owing to uncertainties as to the radius of the point of complete entry of water into the vanes; the radius of complete discharge; friction, leakage, eddy whirl and mechanical losses, it is practically impossible to predict the exact performance of a given wheel. If, however, a wheel of a certain design has been tested, the result will apply proportionately to a similar wheel of any size, as will be shown later.

Unit Quantities.—The horsepower, discharge and speed of a wheel under a 1-ft. head are herein called the "*unit*" power, discharge and speed, respectively. There is some difference among authorities concerning the nomenclature of the units, however. Many engineers term the speed of a runner under 1-ft. head and reduced to such a diameter that it will give 1 hp., the "specific speed." Others call it the "characteristic speed." This latter name is the more logical one and is, therefore, adopted in this work.

Relationship between Speed, Power, Discharge and Diameter.—The relationships between diameter, discharge, power and speed, for variation in head H on any given wheel, are as follows:

(*a*) The speed varies as the square root of the head.

(*b*) The discharge varies as the square root of the head.

(*c*) The power varies as the head raised to the three halves power, or as $H^{3/2}$.

For a given type of wheel under a head H:

(*d*) The power and discharge vary as the square of the diameter of the wheel.

(e) The speed varies inversely as the diameter

(f) The diameter varies as the square root of the power.

It is to be noted that the power varies as $H^{3/2}$, only for the conditions of fixed gate opening and a variation in speed proportional to $\sqrt{H}$. It is not true for a wheel operating at constant speed under variable head.

The foregoing relationships are expressed algebraically, as follows:

Having the horsepower, discharge, speed and diameter, under 1-ft. head, the values at any other head are:

$$P = P_1 H^{3/2} \tag{237}$$

P_1 = horsepower at 1-ft. head = unit power.
P = horsepower at H ft. head.

Also

$$S = S_1 \sqrt{H} \tag{238}$$

S_1 = speed at 1-ft. head = unit speed.
S = speed at H-ft. head.

If D_1 = diameter to give P_1 hp. under 1-ft. head, diameter to give P hp. under 1-ft. head is

$$D = D_1 \sqrt{\frac{P}{P_1}} \tag{239}$$

If Q_1 = discharge through runner under 1-ft. head = unit discharge, the discharge Q under H-ft. head will be

$$Q = Q_1 \sqrt{H}$$

Taking the previous example, determine the speed, discharge and power under 60-ft. head for the wheel which under 1-ft. head gives 2.16 hp., uses 20.23 cu. ft. of water per second, and rotates at 41.2 r.p.m.

Speed = $41.2 \sqrt{60} = 41.2 \times 7.746 = 319$ r.p.m.
Discharge = $20.23 \times \sqrt{60} = 156.7$ cu. ft. sec.
Horsepower = $2.16 \times (60)^{3/2} = 1003.5$ hp.

If the values, found for a 1-ft. head, were derived from a test of the wheel, and were the exact values, these just computed would represent exactly the performance of the wheel under the head of 60 ft.

Turbine Characteristics.[1]—"Characteristic speed" is a term used to designate the type of a turbine runner or wheel. It is

[1] See Discussion by CHESTER W. LARNER (Wellman-Seaver-Morgan Co.).

the speed at which the wheel would run if it were so reduced in size, without changing the design, as to develop 1 hp. under 1-ft. head. In Europe, the units are 1 metric hp. and 1 meter head.

Characteristic speed is a complete measure of the possible performance of a runner under any head, both as to power and speed. It is not a measure of its efficiency, but aside from that consideration, it is an absolute type characteristic, and, given the characteristic speed of a runner, it is possible to decide whether that type of wheel is suitable for a given set of conditions.

Let

P = horsepower
N = revolutions per minute
H = head
N_s = characteristic speed.

Then

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} \tag{240}$$

or, more conveniently,

$$N_s = \frac{N\sqrt{P}}{H \times \sqrt[4]{H}}$$

The fourth root of H may be found by extracting the square root twice, but the numerical value of $H^{5/4}$ is more difficult to determine unless logarithms are used.

The horsepower usually considered in figuring N_s, is the maximum horsepower which the wheel can deliver at a reasonably good efficiency, say 80 per cent., or sometimes more. For example, if a test runner shows 150 hp. at 300 r.p.m. under 18-ft. head, its characteristic speed is

$$N_s = \frac{300\sqrt{150}}{18^{5/4}} = 99$$

If it is desired to select a runner for an installation requiring units with a capacity of 12,000 hp. each, under 30-ft. head, at 63.5 r.p.m.

$$N_s = \frac{63.5\sqrt{12,000}}{30^{5/4}} = 99$$

The characteristic speed is the same, and, therefore, it follows that a wheel of the same type as the test wheel is suitable for this installation, provided its efficiency is satisfactory.

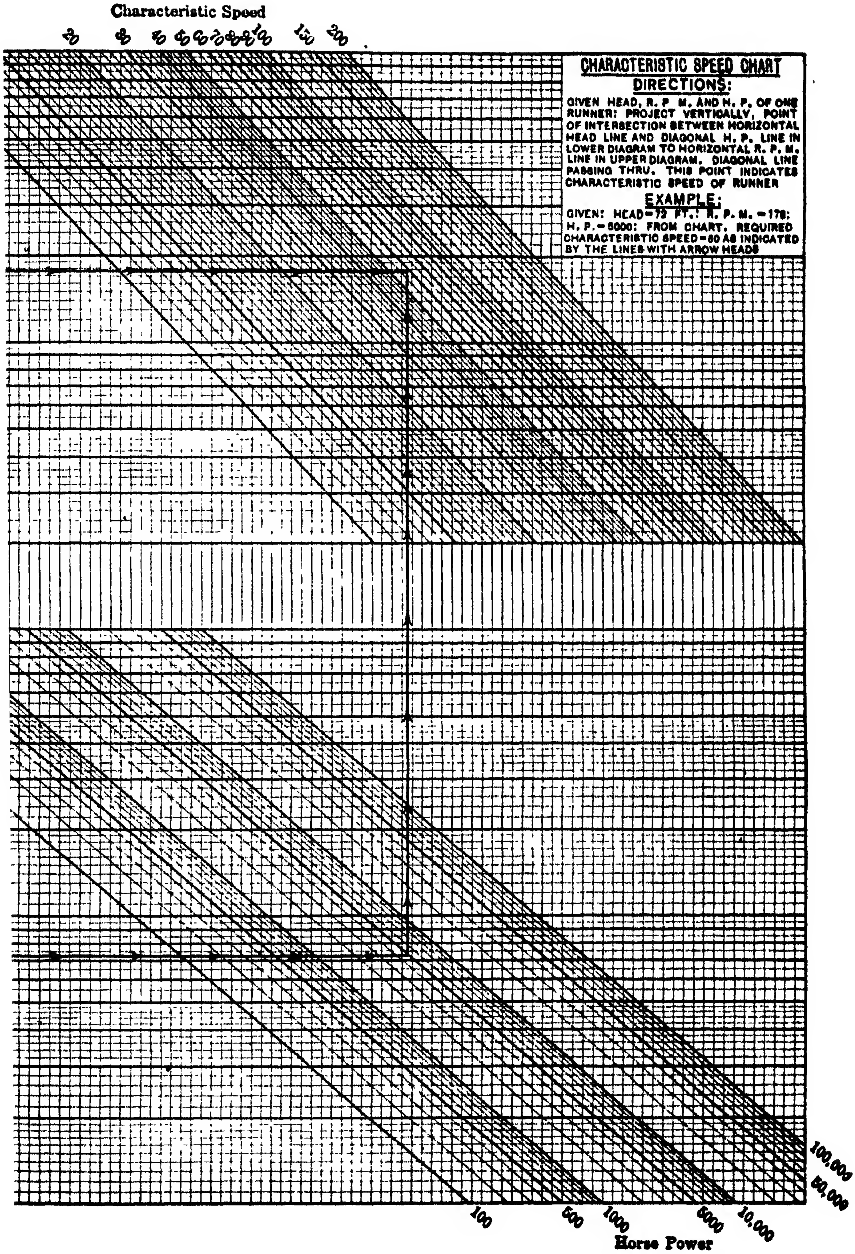

Characteristic Speed
20
100
150
200
CHARACTERISTIC SPEED CHART
DIRECTIONS:
GIVEN HEAD, R. P. M. AND H. P. OF ONE RUNNER: PROJECT VERTICALLY, POINT OF INTERSECTION BETWEEN HORIZONTAL HEAD LINE AND DIAGONAL H. P. LINE IN LOWER DIAGRAM TO HORIZONTAL R. P. M. LINE IN UPPER DIAGRAM. DIAGONAL LINE PASSING THRU. THIS POINT INDICATES CHARACTERISTIC SPEED OF RUNNER
EXAMPLE:
GIVEN: HEAD=72 FT.: R. P. M. =178: H. P.=5000: FROM CHART. REQUIRED CHARACTERISTIC SPEED=60 AS INDICATED BY THE LINES WITH ARROW HEADS
100,000
50,000
100
500
1000
5000
10,000
Horse Power

FIG. 176.

It should be noted that characteristic speed is always based on the capacity of a single runner. If a unit has two or more runners, N_s should be calculated from the power of one runner and not the total power of the unit.

By the use of the diagram, Fig. 176, the characteristic speed of a runner may be determined for any given set of conditions without the use of the formula.

Efficiency and Size of Wheels.—Higher efficiency of the turbines means more power, and hence, more income from the same investment. The difference between a turbine of 90 per cent. efficiency and one of 75 per cent. may mean the difference between an investment paying normal dividends and one paying nothing. It is not unusual, therefore, for the purchaser to offer a bonus for additional efficiency.

Considered by itself, it is manifest that high efficiency is always desirable. It often happens, however, that efficiency must be subordinated, more or less, to other considerations. For example, it may be desirable to use a wheel of lower efficiency in order to secure higher speed, or more power at a given speed than would be possible with a more efficient wheel, or it may be necessary to sacrifice low-load efficiency in order to improve high-load efficiency, or *vice versa.*

To determine the efficiency of a wheel and to adjust the question of character of load on a station and the wheel efficiency, it is necessary to refer to tests made on a wheel of exactly the same design—though not necessarily the same size—as that which is under consideration.

As an example, consider the tests of a 23-in. wheel which are shown graphically in Fig. 177.

This is a test of a very high speed runner, the term "high speed" being used in the sense of high characteristic speed. In the design of this runner, peak efficiency has been sacrificed somewhat in favor of other characteristics. The curves are plotted directly from the test, the speed and power being reduced to a basis of 1-ft. head, according to the principle that, for constant efficiency, the speed varies as $\sqrt{H}$ and the power as $H^{3/2}$. These curves show the horsepower and efficiency of the wheel as abscissæ and corresponding speeds as ordinates. Speed, however, is expressed, not in r.p.m., but in terms of ϕ, which is the ratio between the peripheral speed of the runner

(based on its "rated" diameter) and the spouting velocity of the water due to the head.

$$\phi = \frac{N\pi D}{60\sqrt{2gH} \times 12} \tag{241}$$

D being the rated diameter of the runner, in inches.
N = revolutions per minute.
If ϕ be known for one head, say H_1, the corresponding value of ϕ, or ϕ_2, for some other head H_2 is

$$\phi_2 = \frac{\phi\sqrt{H_1}}{\sqrt{H_2}} \tag{242}$$

One of the curves shown in Fig. 177 gives the maximum characteristic speed of the runner for each value of ϕ. The characteristic speeds are calculated from the power, shown by a dotted curve forming an envelope of the horsepower curves. This curve represents the maximum power obtainable for each speed, and is the result which would be obtained if an infinite number of tests had been run, with varying gate openings up to the limit of full gate opening.

It will be observed that the maximum efficiency of this wheel occurs at a speed for which ϕ = 0.780. This value is taken from the dotted line which forms an envelope of the efficiency curves, for the reason that the actual gate openings used in the test do not appear to show the maximum efficiency of the wheel. This value ϕ = 0.780 should, therefore, be regarded as the "normal" speed of the wheel. It is simply the speed which produces maximum efficiency. It does not necessarily follow, however, that this wheel will always be selected to run at that exact speed under normal plant conditions. Some other speed may be selected, for special reasons, and it should be observed that any speed between ϕ = 0.640 and 0.874 may be selected without sacrificing more than 2 per cent. of the peak efficiency. Other considerations aside, however, the normal speed of this runner is ϕ = 0.780.

Suppose it is desired to consider the application of this runner to a plant in which single-runner wheels having a capacity of 15,000 hp. each under normal head of 50 ft. are to be installed and that the head is, at times, reduced to 30 ft. by flood conditions. In the latter case water is plentiful, and the main consideration is power, regardless of efficiency. It is desired to determine whether this runner is applicable to these conditions, and, if

so, the required diameter and the speed at which it should run.

There are two fixed conditions desirable to fulfill:

(*a*) The wheel should develop its maximum efficiency at 50-ft. head. The test shows that in order to do this ϕ must be about 0.780.

(*b*) The characteristic speed of the wheel should be a maximum at 30-ft. head. The test shows that in order to accomplish this it must run at a speed such that $\phi = 1.100$.

It now remains to be seen how nearly this particular wheel will satisfy both conditions.

If $\phi = 0.780$ at 50 ft., then it follows that at 30 ft.

$$\phi \times 0.780 \times \frac{\sqrt{50}}{\sqrt{30}} = 1.007$$

This value is not as high as it should be to satisfy (*b*), but the characteristic speed, which has a maximum value of 130.5 for $\phi = 1.100$, drops to only 129 when $\phi = 1.007$. In order to satisfy (*b*) it would be necessary to increase ϕ, at 50 ft., to

$$\frac{0.780 \times 1.100}{1.007} = 0.852$$

which would entail a loss of about 1.5 per cent. efficiency under normal operating conditions. It would seem better to make a slight sacrifice of power under extreme flood conditions, and let $\phi = 0.780$, at 50 ft.

$$\text{Now, } \phi = \frac{N\pi D}{720\sqrt{2gH}};$$

therefore,

$$N = \frac{720\sqrt{2gH} \times \phi}{\pi D} \qquad (241\ a)$$

The maximum unit power shown by the test at $\phi = 0.780$, is 2.22 hp., and, therefore, at 50 feet head, its power would be, from equation 237,

$$Hp = 2.22 \times 50^{3/2} = 784.77\ hp.$$

The diameter of the test wheel is 23 in., so that the diameter of a similar wheel to give 15,000 *hp.* under 50 ft. head, would be, from equation 239,

$$D = 23\sqrt{\frac{15,000}{784.77}} = 100.3 \text{ in.}$$

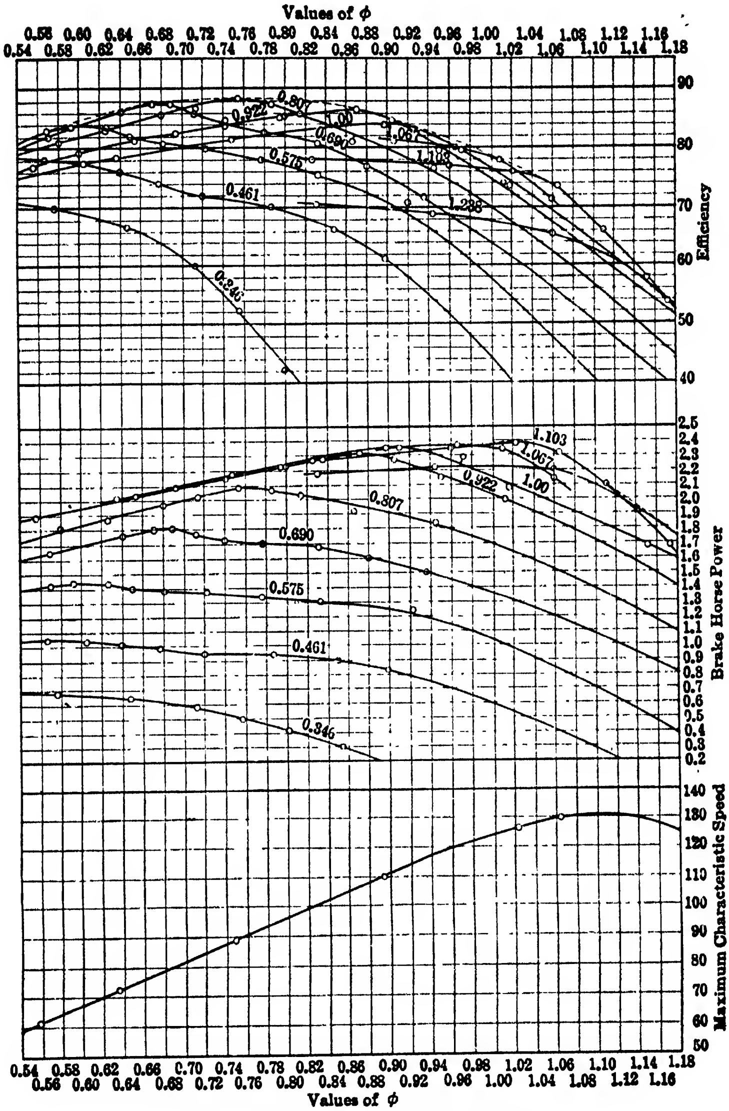


FIG. 177.—Curves of tests of turbine.

Also, from equation 241*a*, the speed of the large wheel is

$$N = \frac{720 \times \sqrt{2g} \times 50 \times 0.780}{\pi \times 100.3} = 101 \text{ r.p.m.}$$

Since at 30-ft. head, $\phi = 1.007$ and the horsepower curve, Fig. 177, shows a maximum value of 2.37 hp. for the 23 in. wheel at this speed, then the power of the 100.3 in. wheel at 30 ft., will be

$$Hp. = \frac{2.37 \times (30)^{3/2} \times (100.3)^2}{(23)^2} = 7440 \text{ hp.}$$

Figure 178 shows the power-efficiency curves for the heads of 50 ft. and 30 ft. and an additional curve for 60 ft., assuming this to be the maximum head.

The foregoing analysis serves to illustrate the method of using Holyoke, or other standard, tests. The best speed of the wheel is shown to be 101 r.p.m., and, even though it might be advisable

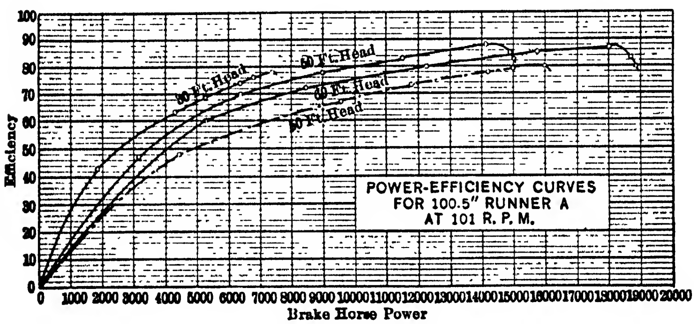


Fig. 178.

to sacrifice efficiency somewhat in order to increase the speed, it can not be done to any material extent without considerably reducing the output under 30-ft. head, and the power at this head will, in all probability, be the limiting condition.

If, however, the minimum head were only slightly lower than the normal, the problem might be changed. The peak efficiency drops to 80 per cent. at $\phi = 0.978$, and if it were permissible to have the efficiency that low, the speed might be increased to $N = \frac{101 \times 0.978}{0.780} = 126.6$ r.p.m. The curve for this speed is shown by broken lines in Fig. 178.

Thus far, only peak efficiency has been considered. Naturally,

however, the shape of the efficiency curve must be carefully studied in relation to the load characteristics of the plant. If it is dependent on storage, and the load fluctuates, the efficiency at low heads may be of importance. If the number of units is small, part-load efficiency would be necessary; but if the number of units is sufficient to permit enough adjustment, the turbines may be kept loaded close to their point of maximum efficiency, even under small plant loads, and part-load efficiency would become, relatively, unimportant. Again, if the load is constant, low-gate efficiency may be of small moment, regardless of the number of units in the plant. Sometimes it is advisable to have maximum efficiency occur at practically full load. It depends entirely on whether, or not, plant conditions are such

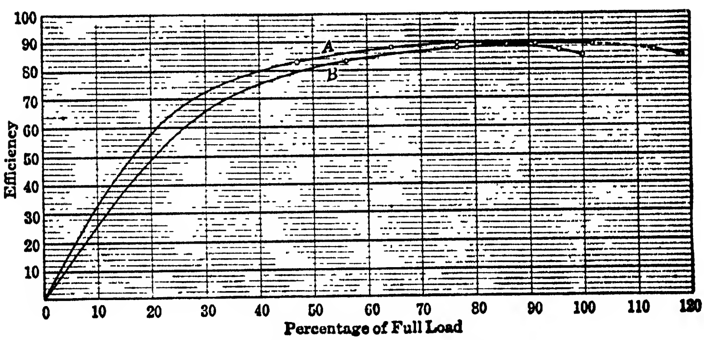


FIG. 179.—Efficiency curves of turbine.

that the turbines can be operated most of the time at, or near, full load.

Figure 179 shows two curves illustrating the application of the same type of runner to the same plant. Curve (*A*) shows the wheel adapted in the usual way, with maximum efficiency occurring at about 0.8 load. Curve (*B*) shows the same runner applied with maximum efficiency occurring at about 0.95 load. The only difference is that (*B*) is a larger runner, and the gate stroke is blocked to cut off the drooping end of the efficiency curve. If the gates are opened wide, the rest of the curve would be as shown by the dotted line. It is quite apparent that, from 0.90 to 1.00 load, (*B*) is superior to (*A*).

The influence of characteristic speed on efficiency may be observed by a comparison of the various tests illustrated. In

general, an increase of characteristic speed moves the point of maximum efficiency closer to full load, and reduces the efficiency at low loads. If it is forced to the extreme, the result is lower peak efficiency also.

Much has been said and written about the selection of a water wheel so that it will have its maximum efficiency when operated at some predetermined load. All of these statements and writings would have some value, outside of being mere academic exercises, if a pre-determination of load were possible. No engineer can, with any surety, predict the magnitude or character of the load that will exist on any power station within 3 years after it begins operation. Every hydro-electric development is, in a measure, a public service utility, and the first rule in designing is to provide for continuity of service. Consequently, the first factor to fix for a water wheel is that it shall have power enough to carry any load which may be imposed on it by the generator which it drives. The power of the wheel, at full gate opening, under normal head, should be equal to the maximum overload capacity of the generator, when operated at this capacity for a half hour. The normal and most efficient point of operation of the water wheel should range between the normal full load and 80 per cent. of normal full load of the generator. The efficiency, when delivering any greater amount of power than that required by the normal full load of the generator, is not worth considering, as loads in excess of this will occur but seldom. It, however, will surely happen that this excess power will be needed occasionally, and without it the service and the consumers connected with the company's plant, will suffer.

The efficiency curve should be as flat as possible consistent with fulfilling these first two conditions. A wheel having a peaked curve, giving an abnormally high efficiency over a short range of load, can seldom be worked to advantage in a hydro-electric power station. It is impossible to get operators to start or stop units with load variations and in such a manner as to work always at the most efficient point on the curve and, in practice, the water wheel best adapted to any plant is one which will surely carry the maximum load, which has its best efficiency between 80 and 100 per cent. of normal generator capacity and having an efficiency curve as flat as can be obtained from this wheel.

Manufacturers Ratings.—It has become customary to list the wheels offered by manufacturers under classifications of characteristic speeds and values of ϕ (ratio of spouting velocity to peripheral velocity at entry), and under each type are given the unit speed, power and discharge for each size of wheel. From these data the performance of any wheel, under any set of conditions, can be computed.

The efficiencies can be determined only by test, as has been previously explained. Hence, in order to make the data complete, the tables must be accompanied by efficiency curves—either a curve for each type of wheel, or one showing the efficiency of one type, with curves of comparative efficiencies of all of the types.

Table 35 gives unit power, speed and discharge of wheels built by one of the prominent American makers.

Wheel Setting

The runners are set in a number of different ways, and one of the controlling factors in water-wheel installations is the

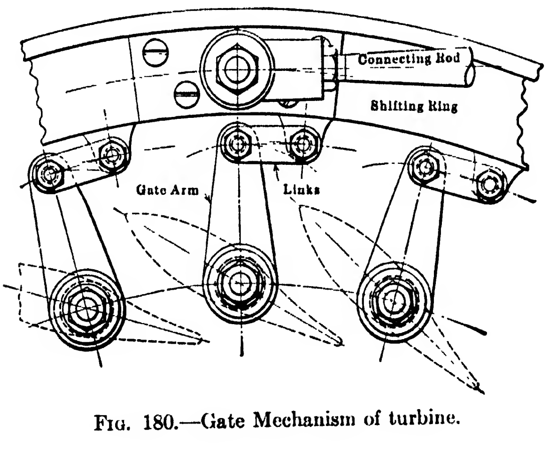


Fig. 180.—Gate Mechanism of turbine.

manner in which the wheels are set within surrounding cases or chambers.

The movable guide vanes are placed in a ring surrounding the wheel, and each of them is connected to a shifting ring through links as shown in Fig. 180. Obviously, movement of the con-

TABLE 35.—VICTOR WHEEL, *Platt Iron Works Co.*

Size	Type-1 $N_s = 84$ $\phi = 0.88$ Max. head = 40			Type-2 $N_s = 75$ $\phi = 0.84$ Max. head = 60			Type-3 $N_s = 60$ $\phi = 0.76$ Max. head = 125			Type-4 $N_s = 45$ $\phi = 0.70$ Max. head = 200			Type-5 $N_s = 32$ $\phi = 0.66$ Max. head = 400			Type-6 $N_s = 20$ $\phi = 0.62$ Max. head = 600			Type-7 $N_s = 12$ $\phi = 0.59$ Max. head = 800		
	Hp.	R.p.m.	Q	Hp.	R.p.m.	Q	Hp.	R.p.m	Q	Hp.	R.p m	Q	Hp.	R.p.m.	Q	Hp	R.p m	Q	H p.	R.p.m.	Q
15	.....	......	....	0.531	103.0	5.84	0.414	93.3	4.56	0.276	85.7	3.04	0.157	80.9	1.73	0.0693	76.0	0.836	0.0274	72.4	0.302
18	.	....	.....	0.762	85.9	8.38	0.593	77.8	6.53	0.396	71.5	4.36	0.226	67.4	2.49	0.0996	63.3	1.10	0.0396	60.3	0.436
21	...	.....	.....	1.040	73.6	11.45	0.810	66.7	8.92	0.540	61.3	5.95	0.306	57.8	3.37	0.1355	54.3	1.49	0.0540	51.6	0.595
24		......	.....	1.36	64.3	14.95	1.060	58.3	11.65	0.705	53.6	7.75	0.402	50.5	4.43	0.1770	47.5	1.95	0.0704	45.2	0.775
27	1.985	60.0	21.95	1.715	57.2	18.90	1.335	51.8	14.70	0.890	47.7	9.80	0.508	44.9	5.59	0.2250	42.2	2.48	0.0890	40.2	0.979
30	2.42	54.0	26.70	2.12	51.5	23.30	1.640	46.7	18.05	1.100	42.9	12.10	0.625	40.5	6.87	0.277	38.0	3.05	0.1110	36.2	1.22
33	2.94	49.0	32.40	2.56	46.8	28.20	2.000	42.4	22.00	1.330	39.0	14.65	0.757	36.8	8.33	0.307	34.5	3.38	0.1330	32.9	1.46
36	3.48	45.0	38.40	3.06	42.8	33.70	2.375	38.9	26.15	1.585	35.7	17.45	0.901	33.7	9.91	0.398	31.7	4.38	0.1575	30.2	1.74
39	4.09	41.5	45.10	3.56	39.6	39.20	2.790	35.9	30.70	1.850	33.0	20.35	1.060	31.1	11.65	0.469	29.2	5.17	0.1860	27.8	2.05
42	4.74	38.6	52.20	4.16	36.8	45.70	3.24	33.3	35.65	2.160	30.6	23.80	1.220	28.9	13.40	0.544	27.1	5.99	0.2160	25.8	2.38
45	5.57	36.0	61.40	4.77	34.3	52.50	3.72	31.1	40.90	2.480	28.6	27.30	1.410	27.0	15.50	0.625	25.3	6.88	0.2475	24.1	2.72
48	6.20	33.7	68.30	5.43	32.2	59.70	4.22	29.2	46.40	2.810	26.8	30.90	1.700	25.3	18.70	0.706	23.8	7.77	0.2820	22.6	3.10
51	7.01	31.7	72.20	6.13	30.3	67.50	4.75	27.5	52.30	3.190	25.2	35.10	1.810	23.8	19.90	0.798	22.4	8.60	0.3170	21.3	3.49
54	.....	......	.....	6.87	28.6	75.60	5.36	25.9	59.00	3.570	23.8	39.30	2.020	22.5	22.20	0.897	21.1	9.87	0.3560	20.1	3.92
57	.....	......	....	7.62	27.1	83.80	5.95	24.6	65.50	3.96	22.6	43.60	2.250	21.3	24.70	1.000	20.0	11.10	0.3990	19.0	4.39
60	.....	......	.....	8.50	25.7	93.50	6.58	23.4	72.40	4.42	21.4	48.60	2.505	20.2	27.60	1.105	19.0	12.20	0.4400	18.1	4.84
64	.....	....	.....	9.67	24.1	116.5	7.50	21.9	82.50	5.00	20.1	55.00	2.840	19.0	31.20	1.260	17.8	13.85	0.4980	17.0	5.48
68	. ..	....	....	10.89	22.7	120.00	8.48	20.6	93.30	5.67	18.9	62.40	3.230	17.8	35.50	1.415	16.8	15.60	0.5630	16.0	6.20
72	....	......	...	12.20	21.5	134.20	9.55	19.4	105.00	6.32	17.9	69.50	3.610	16.8	39.70	1.580	15.9	17.50	0.6350	15.1	6.99
76	... .	..	..	13.60	20.3	149.50	10.65	18.4	117.00	7.09	16.9	78.00	4.050	15.9	44.50	1.770	15.0	19.50	0.7150	14.2	7.86

Hp. at any head = Hp. in table × $H^{3/2}$. R.p.m. at any head = R.p m. in table × $\sqrt{H}$. Discharge at any head = Q in table × $\sqrt{H}$.

necting rod will cause the shifting ring (only a part of which is shown) to rotate and thereby change the angular position of each of the vanes, which is connected with it. Movement of the connecting rod toward the right will increase the area of opening between the gates, while if the rod be moved toward the left, the gates will approach nearer to each other until the extreme limit of contact between adjacent gates is reached, which entirely closes all the water passages to the wheel.

Fig. 181 shows a section through a wheel with a name given to each part shown in the section.[1] While the details of construction vary among the different builders, this is a good example of American turbine practice and the generally accepted nomenclature for the individual parts.

By the term "wheel" is meant the stationary guide vanes with supporting crown and curb plates, as well as the wheel itself, unless some specific statement to the contrary is made.

The diagrams in Fig. 182 and numbered *A* to *F* inclusive, and some succeeding figures, are indicative of some of the general types of settings. *A* is the simple, so-called, "vertical" wheel. In reality, the wheel is set horizontally while its shaft is vertical. It is customary to refer to the direction of setting as fixed by the shaft instead of by the position of the wheel. Therefore, under the accepted practice, this horizontal wheel is termed a "vertical" wheel. As shown, the wheel rests on the bottom of an open pit and water enters it around its whole circumference, passing through the guide vanes to the wheel, through the wheel and the central discharge opening into the vertical draft tube, through which it passes down to tail water. A setting in an open pit, in which the water level is the same as that of the source of supply, is called an "open penstock," or "open wheel-pit," setting.

A somewhat similar setting is shown in *B*, except that the turbine is set horizontally, and the discharge water has to make a right angle turn through a large 90° elbow in order to pass down to tail water. In water-wheel parlance, these elbows are usually designated as "quarter turns." The left-hand end of the shaft, which passes through the wall of the penstock, is, of course, provided with a stuffing box, while another stuffing box is fastened to the right-hand side of the quarter turn, so that the

[1] Hydraulic Turbine Corporation.

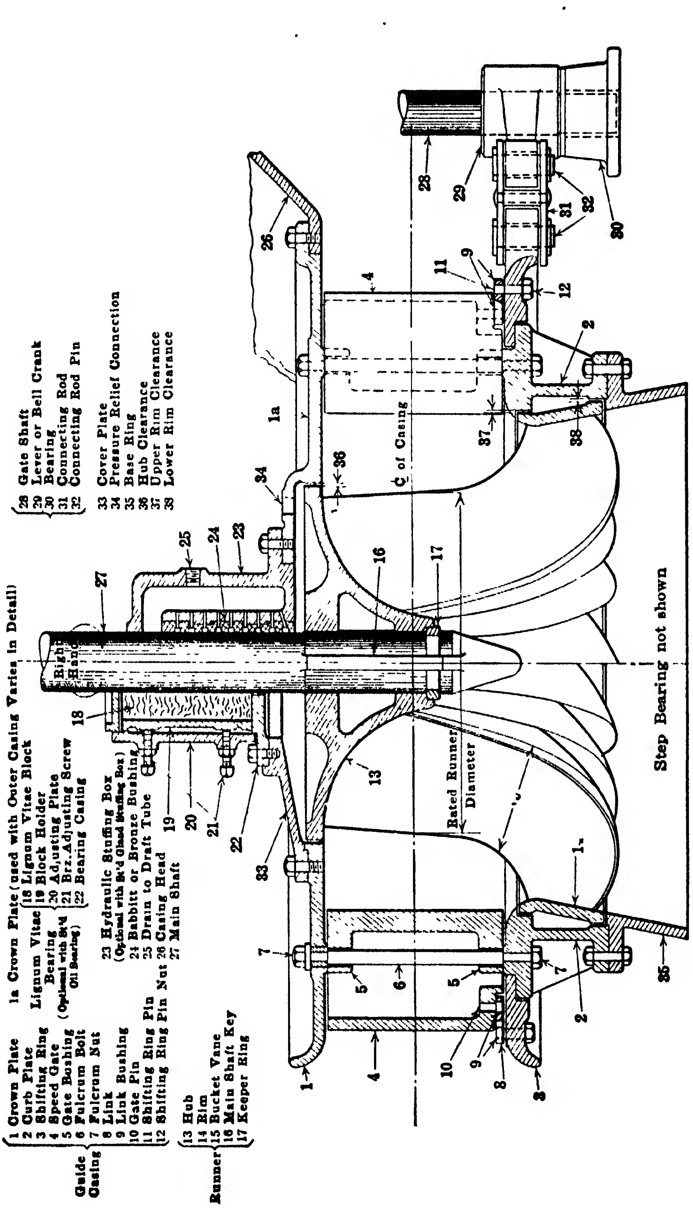

Guide Casing
1 Crown Plate
2 Curb Plate
3 Shifting Ring
4 Speed Gate
5 Gate Bushing
6 Fulcrum Bolt
7 Fulcrum Nut
8 Link
9 Link Bushing
10 Gate Pin
11 Shifting Ring Pin
12 Shifting Ring Pin Nut
1a Crown Plate (used with Outer Casing Varies in Detail)
Lignum Vitae Bearing (Optional with Std Oil Bearing)
18 Lignum Vitae Block
19 Block Holder
20 Adjusting Plate
21 Brz. Adjusting Screw
22 Bearing Casing
23 Hydraulic Stuffing Box (Optional with Std Gland Stuffing Box)
24 Babbitt or Bronze Bushing
25 Drain to Draft Tube
26 Casing Head
27 Main Shaft
Runner
13 Hub
14 Rim
15 Bucket Vane
16 Main Shaft Key
17 Keeper Ring
28 Gate Shaft
29 Lever or Bell Crank
30 Bearing
31 Connecting Rod
32 Connecting Rod Pin
33 Cover Plate
34 Pressure Relief Connection
35 Base Ring
36 Hub Clearance
37 Upper Rim Clearance
38 Lower Rim Clearance
Right Hand
℄ of Casing
Rated Runner Diameter
Step Bearing not shown

Fig. 181.

shaft may also pass out through it and reach the external bearing, which is likewise supported by the quarter turn.

C shows the setting of a pair of turbines, horizontally, in an open penstock. The discharge from these two wheels moves through a specially shaped chamber toward a point of common efflux, the area of the discharge pipes and chambers being properly proportioned to carry the quantities of water which pass through them. The chamber which receives the discharge from the two turbines and to which the draft tube is connected is called a "draft chest." The method of supporting the shaft in bearings, mounted on heavy floor stands, is shown. Wherever the size of the wheels is such that a large draft chest is required, which necessitates separating them a considerable distance apart, say 18 ft. or more, it is customary to place a middle bearing in the draft chest to prevent vibration of the shaft.

D shows an arrangement of four wheels on a single shaft in an open penstock setting. The adoption of more than one wheel on a shaft is for the purpose of obtaining a given amount of power at a higher speed than a single wheel can produce. The arrangement shown in *D* is simply a duplication of that shown in *C*, four wheels being on a common shaft.

In *E* is shown another arrangement of open penstock setting, which is similar to that in *C*, except that the wheels are set vertically instead of horizontally.

Open penstock settings are not suitable nor economical for installations where the head of water exceeds 30 ft., except under special conditions which may at times justify this method of installation. Where the head is greater than 30 ft., it is customary to enclose the wheels in a pressure chamber of steel or concrete. A setting of this kind is shown in *F*. This arrangement is similar to that shown in *C*, except that the wheel and draft chest are enclosed in a steel casing. The conical-shaped cover at the end of the wheel, on the right, is to diffuse the entering water and cause it to pass into the chamber and around the guide vanes without eddy swirls which would be formed if the incoming water should impinge directly on a flat surface at right angles to its direction of flow.

Figure 185 shows a setting in which the water enters through the middle of the casing from above, passes through the wheels and out through the draft tube, which latter also passes out

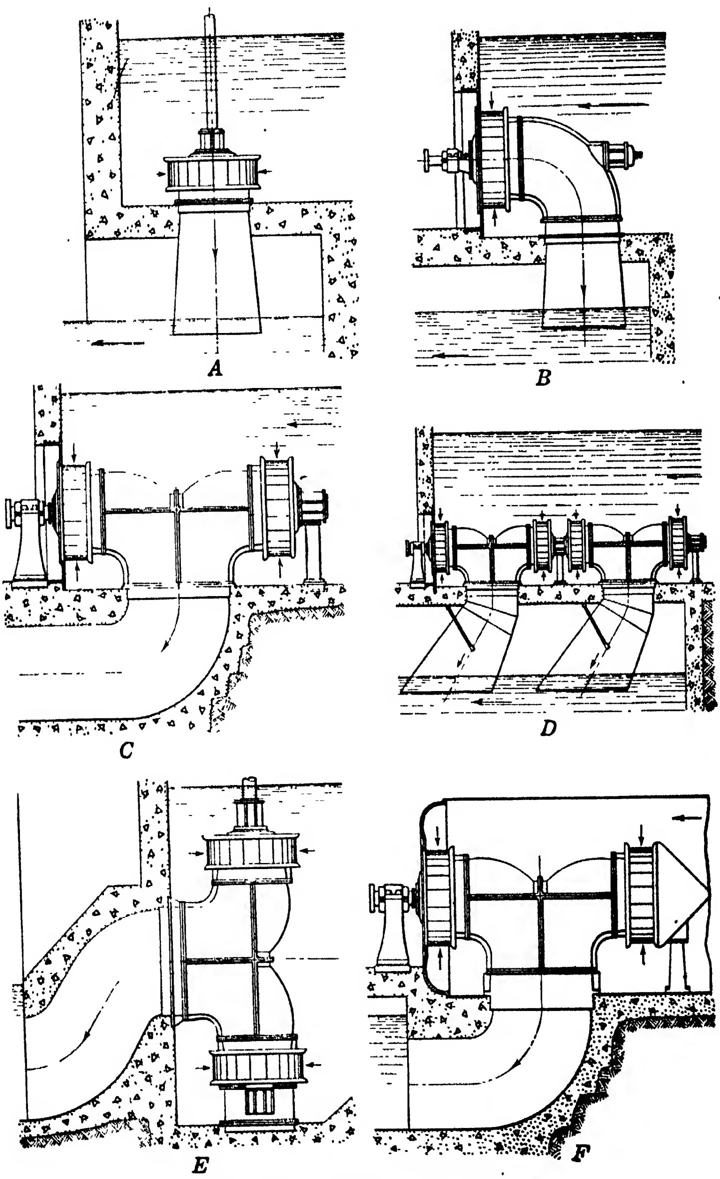


FIG. 182.—Types of turbine settings.

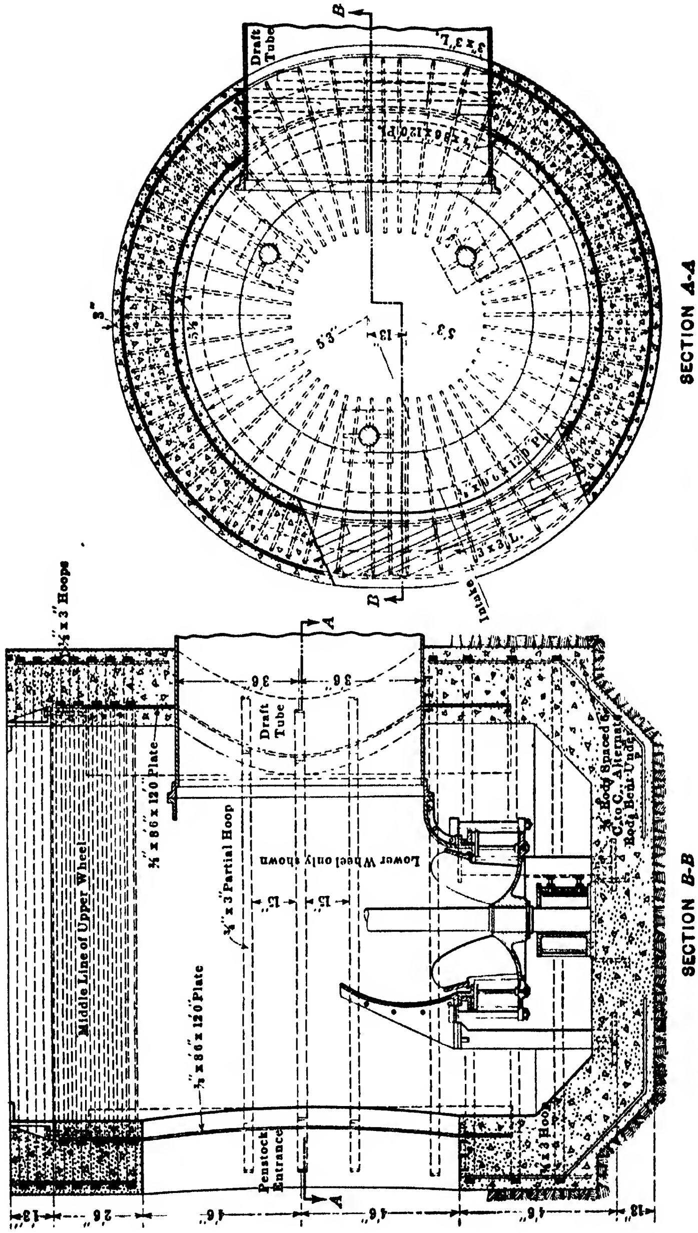


Fig. 183.—Concrete turbine case.

through the middle of the casing and on the opposite side from the entry opening.

The arrangement shown in Fig. 185 will operate just as satisfactorily when set vertically as when horizontally placed. Also, the vertical double wheel shown in *E*, Fig. 182, is adapted to setting in a pressure chamber, as well as in an open penstock. The pressure chambers for settings of units such as shown in *E* and *F*, and in Fig. 185, are usually made up of steel plates, although, in one or two instances, a chamber for this character of setting has been made of re-enforced concrete, notably in the case of the power installation at Austin, Tex., where vertical units of 3000 hp., and similar to that one indicated in Fig. *E*, were so installed.

The details of the design of this setting are shown in Fig. 183.

It has been found that the efficiency of water wheels is highest when set in spiral-shaped cases; that is, cases so shaped that the water enters the chamber tangentially and flows around the wheel, as indicated in Fig. 184. As the water enters, a portion of it passes into the first guide vanes it encounters, so that the amount of water moving on to the next point in its travel is diminished, becoming less and less as it circles around the wheel. The radius of the case and, consequently, the cross-section of the moving water, is diminished in exact proportion to the diminution in the amount of water flowing at each point in its path of travel. In other words, the pressure produced by the water in the case is constant at every point in its travel around the guide vanes, or "speed ring."

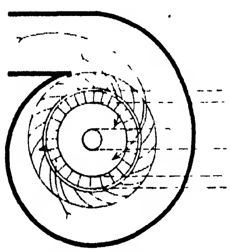


FIG. 184.—Spiral turbine case.

One of the main advantages of this setting is that the initial velocity of the water entering the case is transformed into energy, while with other forms of setting but little, if any, of this energy is recovered. Spiral cases are made both of iron and concrete and, practically, every large water wheel in an important installation is now set in this type of chamber.

Figure 186 shows a cross-section through a single wheel set vertically in a concrete spiral case, the draft tube being a continuation of the concrete chamber and monolithic with it.

The difference in area of the water passage on the two sides of the wheel is clearly indicated.

Figure 187 is a vertical wheel also set in a spiral, or scroll, case made of cast iron. Both of these last two settings are for single discharge wheels.

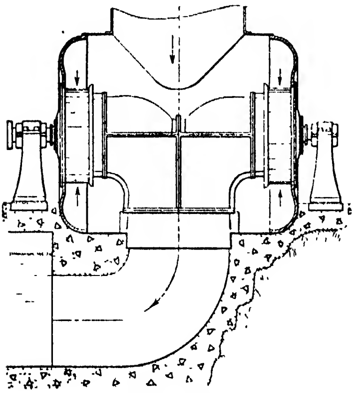


FIG. 185.

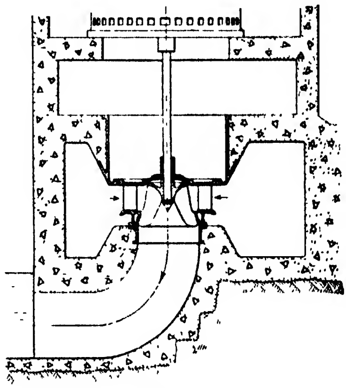


FIG. 186.

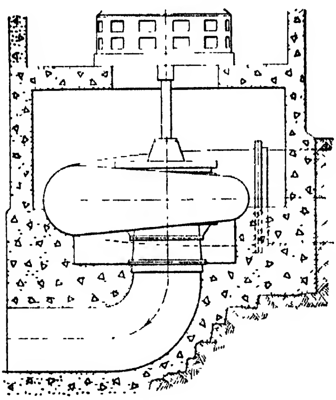


FIG. 187.

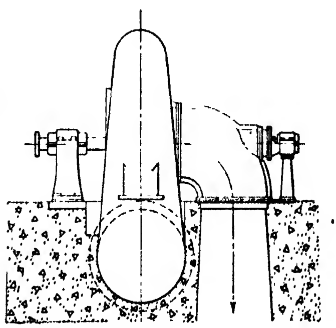


FIG. 188.

FIGS. 185 TO 188.—Types of turbine cases.

Obviously, either setting is just as well adapted to horizontal as to vertical turbines. Fig. 188 shows a horizontal wheel in cast-iron scroll setting with the discharge through a quarter turn down to the draft tube.

A pair of wheels like the single one shown in Fig. 188, may be coupled together in a single shaft, each having its scroll case

around it. The two wheels discharge into a draft chest, the water from them passing out through a common draft tube. This arrangement is shown, in section, in Fig. 189.

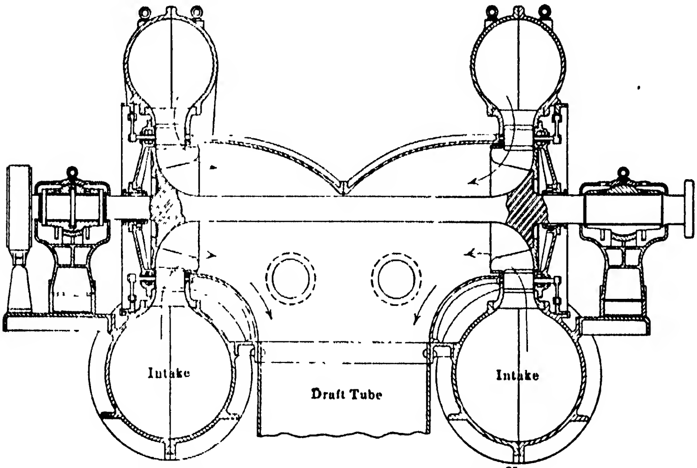


Fig. 189.—Twin turbines in iron scroll cases.

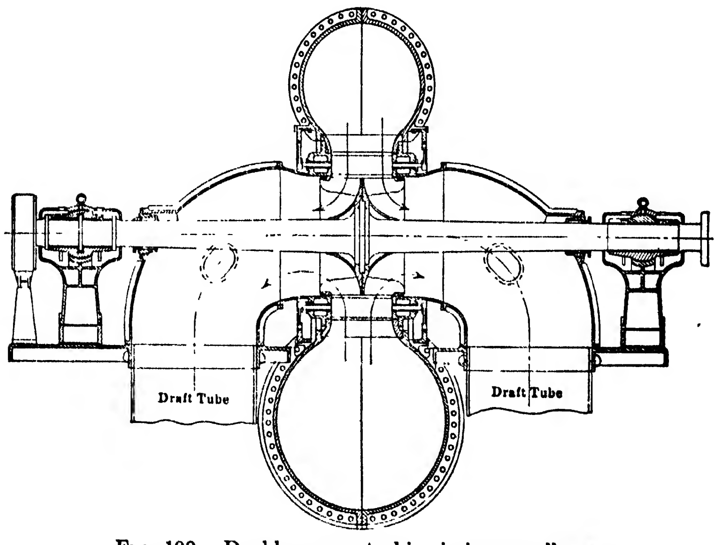


Fig. 190.—Double runner turbine in iron scroll case.

One of the best forms of setting is shown in Fig. 190, which depicts a scroll case surrounding a double wheel. The runner

for this form of water wheel is shown in Fig. 191 and, as may be seen, it consists of a single wheel, approximately twice as long as the ordinary single wheel, and it is divided through its middle by a disk perpendicular to the axis, making it, in effect, two separate wheels. Water enters the periphery and discharges at both ends so that two draft tubes are needed, as indicated in the sectional drawing. Wheels of this design are called "double discharge" runners. Pictures of a pair of wheels and of a double discharge wheel, both in cast-iron scroll cases, are shown in Figs. 192 and 193, respectively.

Wherever water wheels are of a considerable size, and the head does not exceed 65 ft., they are, invariably, set vertically in concrete chambers, and the entrance opening is divided into two or more sections to prevent the formation of transverse eddies.

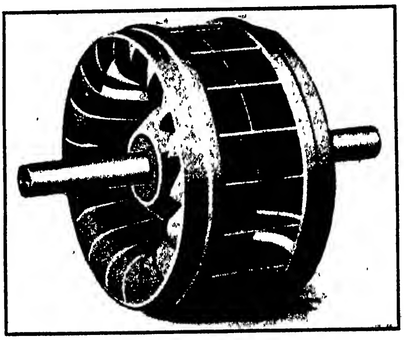


Fig. 191.—Double Victor-Francis runner.

A section of a chamber of this kind is shown in Figs. 201, 204 and 205. It should be observed that the chamber is finally contracted in cross-section until there is no space for the water to pass further, circumferentially, around the wheel, so that the even flow of incoming water is not disturbed by any circular motion derived from small quantities of water circulating around the wheel case and past the point of entry. Iron scroll cases are used for heads above 70 ft. The design of concrete turbine cases is discussed in detail elsewhere in this chapter.

A special form of turbine casing, such as indicated in Figs. 194 and 195, has recently been brought into use and is giving excellent results.[1] This is the so-called "cone flume" wheel, which com-

[1] Platt Iron Works.

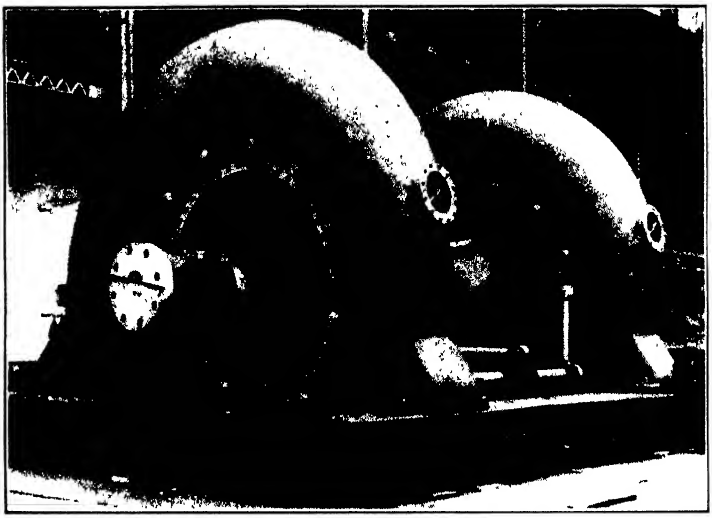


FIG. 192.—Pair of turbines in iron scroll cases.
(Wellman Seaver Morgan Co.)

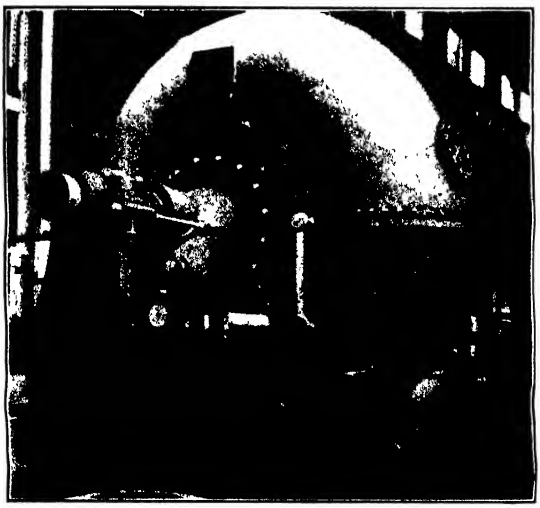


FIG. 193.—Double discharge runner turbine in iron scroll case.
(Wellman Seaver Morgan Co.)

prises a single runner set in a cone-shaped case, having at every point a cylindrical cross-section, but the diameter of the case increases from the intake opening at one end, to the maximum diam-

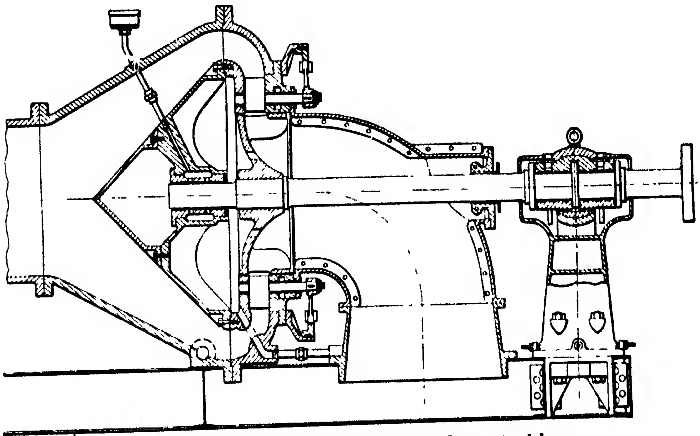


FIG. 194.—Section through cone-flume turbine.

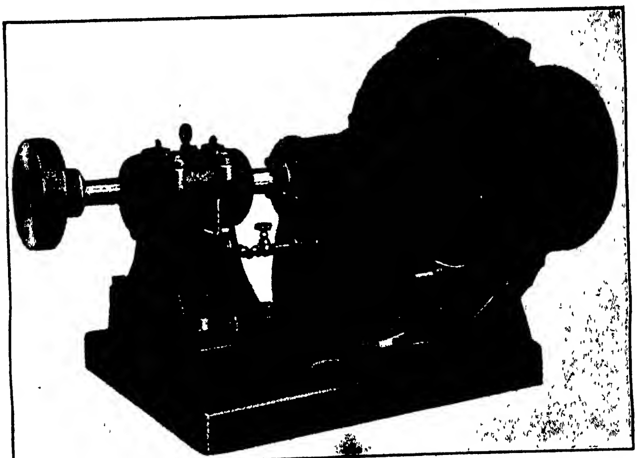


FIG. 195.—Cone-flume turbine.
(Platt Iron Works.)

eter which is reached just before the water enters the guide vanes. The interior cone, placed with its apex toward the entry opening and forming the end covering of the wheel, is fastened to the crown plate, and by this arrangement the water is gradually

changed from an axial to a diverging direction without shock or eddy swirls.

The area of the section through which the water passes in this cone chamber is constant, as is, of course, the velocity of flow until the water actually enters the guide vanes. Tests indicate that efficiencies obtainable with this character of setting are practically equal to those for scroll-case enclosed wheels. This arrangement is principally adapted for high heads, that is, 100 ft. and greater.

Details of Turbine Design

Runners.—Runners may be cast as a single piece, the vanes, hubs and rims being of one single casting, or they may be built up by making the vanes of steel plates pressed into form, placed in a mould and the hubs and rims then cast around them, so that a composite structure results. Theoretically, the latter form is more nearly true to design, the buckets are smoother and the efficiency higher than for the solid cast runner. In practice, however, it is found just as difficult to set the vanes in the mould with exactness as it is to place the cores in the mould for the solid cast wheel, and there is no real difference in the results obtained between the composite and the solid wheels. The composite wheel has the disadvantage that some of the vanes may not weld perfectly with the cast metal which is poured around them, in which case the wheel is defective, and the defect does not usually show itself until after the wheel has been in service. American manufacturers have adopted, almost exclusively, the solid cast wheel as standard. For high heads the solid wheel is sometimes made of cast steel and occasionally of bronze. Bronze runners are too expensive, however, for general service and are seldom used. Their single advantage consists in the fact that they are not easily corroded. Water wheels which are subject to working at part gate, have the vanes corroded in the course of time by an alternate action of air and water on their rear surfaces. At part gate, the spaces between the runners may not be solidly filled with water and any air entrained in the water accumulates at the point of least pressure, which is the rear surface of the vanes. This air oxidizes the vanes at these points of air contact. Later, when the buckets are completely filled and under considerable pressure, the oxidized

metal will be washed off clean, leaving a fresh, bright surface for the next accumulation of air to attack. This action causes pitting and erosion which will, eventually, eat through the vanes.

The total torque of the wheel is transmitted through the hub and this, therefore, must be of ample dimensions to resist the stresses which may be imposed on it. The vanes should be made as thin as is consistent with reasonable strength. When the wheels are first installed, and in the preliminary stages of operation, before the site has been cleared of the débris around the power station, small pieces of timber, planking, blocks and materials of like character are liable, at one time or another, to get into the intake in the forebay and be carried into the wheel.

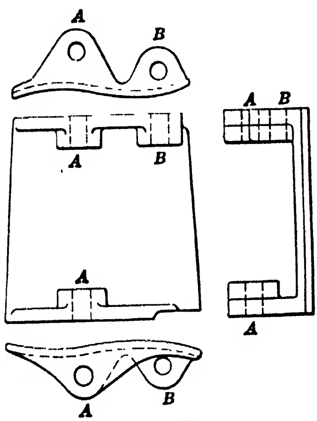


Fig. 196.

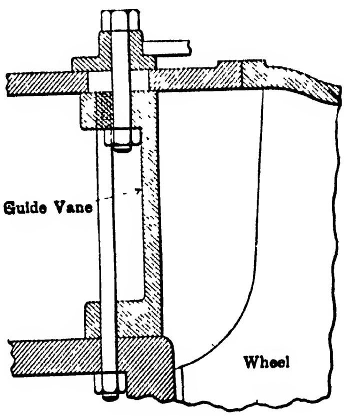


Fig. 197.

Figs. 196 and 197.—Details of wicket gates for small turbines.

They will pass through the guide chutes, and the runners will strike against these projecting pieces, in which case, something must give way. The guide vanes are removable, and it is much better that a few of these should fail than for one of the runner buckets to break and, therefore, the runner vanes should always be much stronger than the guide vanes.

Gate Rigging.—The form of gates, which are, in reality, movable guide vanes, varies considerably with different manufacturers. One form is shown in Figs. 196 and 197. Each gate, or vane, consists essentially of a thin cast plate which is curved in one direction, namely, when viewed from either of the ends which work between the crown and the curb plates. These

vanes are provided with projecting lugs at the top and bottom, which are drilled to receive fulcrum bolts, about which the gates may turn within the limits of their motion. Another lug on one end of the vane, is provided to receive a stud which moves the vane. Fig. 196 shows one of the guide vanes, the side and front elevation, and top and bottom views being given, while Fig. 197 shows a section through a vane in place in the wheel housing. Lugs *AA* receive the fulcrum bolt, while in the lug *B* the gate pin is placed. Movement of the gate pin results in a corresponding movement of the vane around the fulcrum bolt.

This type of gate is operated by a link outside the crown plate, all the links connecting with a small disk which surrounds a central neck on the cover plate. The gate pin passes from lug *B* through the crown plate and is there attached to a stub which forms the end of the gate link. A slot is formed in the crown plate through which the gate pin passes, the length of the slot representing the length of travel of the pin between complete opening, and complete closing, of the gates. The stub end has a thin, flat portion which rests on the crown plate, and its dimensions are such that the slot is completely covered in any position of the link and gate pin. The disk, which has the same function as the ordinary shifting ring, is operated through its small angular motion in one direction or the other, by means of an arm which projects out toward the periphery of the wheel, and at the outer end of which is a segmental section of toothed rack. The gate shaft is provided with a pinion which coöperates with this rack section and, therefore, movement of the gate shaft results in movement of the gates through the intermediary means of rack, disk, gate links and pins. This general arrangement is indicated in Fig. 198. Obviously, this gate rigging with the slots through the crown plate is not water-tight and, therefore, cannot be placed outside the water-wheel chamber.

Another of the different forms of gate is shown in Fig. 181, where the relations of the moving parts are more clearly indicated. In this case the gate is undercut on its lower and outer edge, and the link rests against the shift ring and lies underneath the end of the gate vane, the dimension of a link being such that it will just fill the space left by undercutting the vane. Motion is communicated to the gate links by means of a shifting ring. For large turbines, the shifting ring is of considerable diameter, and the links are short, as shown in Fig. 180. When the shifting

ring is rotated through a small angle, it moves each of the links, and, thereby, causes an exactly equal motion of all of the gate vanes. The governor or regulating connection is attached to the shifting ring, as shown in Fig. 180. In this latter figure is shown a type of gate in which the vane has a thickness greater than the diameter of the fulcrum bolt, and the latter passes directly through the vane itself. The vane is either keyed to the fulcrum bolt, or has a portion of the hole through it made square, and a corresponding portion of the fulcrum bolt is also of square section, fitting into the square hole. Hence,

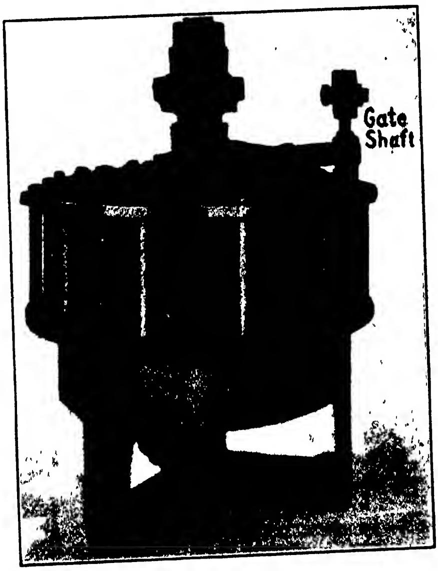


Fig. 198.—Water wheel showing gate rigging.

movement of the fulcrum bolt will cause a corresponding movement of the vane.

A short link, called a gate arm, is attached to the fulcrum bolt, either by keying, or by a square hole which fits on a square shoulder of the bolt, so that movement of the gate arm will cause rotation of the bolt and vane. To the gate arm is attached the gate link while the other end of the gate link is connected with the shifting ring. This construction is generally used for iron-encased water wheels having only one runner in a casing. The operating parts are all on the outside of the casing, and,

in some of the pictures of water wheels which appear elsewhere, the parts of this rigging are shown.

In some cases, the fulcrum bolts take the form of trunions or extensions at the ends of the gates which are either cast or forged integral with the gate itself. Such construction is to be avoided, as the entire wheel must be dismantled in order to replace a single gate, which, sooner or later, inevitably becomes necessary in any installation.

The ability to put all of the working parts of the gates outside the casing, where they are open for inspection and can be readily adjusted when subject to deterioration by reason of sand or silt which may be entrained with the water, is one of the chief advantages of using single wheels in iron or concrete casings. In every instance, a water wheel should be arranged so that any individual vane may be removed without having to dismantle any part, or doing any work other than removing the fulcrum bolt and disconnecting the gate ring. It is also especially desirable that the crown and curb plates of a wheel be held together by bolts other than the fulcrum bolts, and not depend on these to keep the parts of the wheel housing together.

Holes in the housing through which the gate pins and the fulcrum bolts pass should, in every instance, be bushed with bronze. Where the construction is as shown in Fig. 180, and the fulcrum bolts pass through stuffing boxes in the side of the wheel casing, either the bolts should be of bronze, or the interior of the stuffing boxes should be lined with this material. In no case should surfaces, operating one inside or against the other and subject to the action of water, be both of iron. One surface must always be of bronze, or some equivalent material which resists oxidization.

The end surfaces of the gates should be machined so that they may move freely between the crown and curb plates and, at the same time, prevent the leakage of water past the ends; also, the lines of contact where the inner end of one gate touches against the surface of the next adjacent one, when the gates are fully closed, should be machined and fitted so that leakage through the gates, when closed, is reduced as far as practicable.

It is not commercially possible to make wicket gates so that they are absolutely water-tight when closed, and in some excellent turbines, where the efficiencies of the wheels are high, the leakage through the gates has been found to be consider-

able. Under certain conditions, this will reduce the hydraulic efficiency of an entire plant. If three water wheels are required to carry the maximum load, and only one of them is in use 12 hr. a day, the total efficiency of the plant during this 12 hr. of light load, being fixed by the total flow of water through the penstocks, will be low if the leakage through the other two wheels is appreciable, even if the efficiency of the operating wheel is high.

Another difficulty arises in stopping machinery in the power house, where the gate leakage is greater than a reasonable amount. When the load is thrown off the generator, there is only the friction of the then rotating parts to overcome in order to keep it going, and in the case of vertical units with roller or ball thrust bearings the energy required to maintain the unit at its full speed is extremely small, so that a slight leakage, only, past the gates, is necessary to keep the machinery moving. In many instances it will be found that the leakage is almost entirely due to openings between only two or three gates. Of the whole number, nearly all of them may close completely, leaving two or three the ends of which do not quite touch the surfaces of the adjacent vanes. Also, if one or two of the vanes are longer or thicker than the others, the limit of motion of the shifting ring will be reached before the other gates have quite made contact. The author believes that there should be some method of adjusting the individual gates when the machine is set, although no sufficiently simple construction has yet been devised for this purpose.

In drawing specifications for turbines, the limiting value of gate leakage should be specified. No rule can be given for this because the amount of leakage which would be admissible with one form of turbine and one character of setting could not be allowed in certain others, and the fixing of this factor must always be a matter of discussion between the engineer and the constructor of the wheels. Another point in connection with the gates, is the question of the action of water thrust against them. They should always be so designed that water flowing through them sets up an unbalanced moment around the fulcrum bolt, tending to close them. This moment must be great enough to actually close the gates, if the movement is not resisted. It should, however, not much exceed the moment necessary to move them to a shut position, as otherwise, too great a force

must be applied by the governor to move them to an open position, which means that the size and cost of the governor will have to be increased.

The object of this arrangement is that in case of accident to the governor the wheel will not tend to run away, or overspeed, but will automatically shut down. Also, the gates must be easy to move and the machine work on them must be of a high order of workmanship, so that the leakage is small and the gates may be easily moved. Under proper conditions of fly-wheel effect for the moving parts and methods of compensating for variation in penstock pressure, the degree of regulation which may be attained with a governor of a given size is almost directly proportional to the ease with which the gates may be moved.

The gates should be of cast iron. The author is well aware that cast or forged steel is recommended by most water-wheel designers, the idea being that the strength of the vanes should be great enough to resist fracture whenever ice, stones, wooden blocks or other rubbish gets in between them while they are closing. If these vanes are strong enough to resist such stresses without breaking, a block or timber which might pass between them and into the runner would break the runner buckets, and it is far better to break an occasional guide vane, which may be replaced within a few hours at a nominal cost, than to ruin an entire runner, which would be the result of breaking any of its buckets. The guide vanes should be protected against breaking in case of any obstruction lodging between them, by making either the links, or the link pins, weak enough to break before the vane does, but, in any event, the gate vanes should break before the runner vanes do.

Bearings.—There are two kinds of bearings, namely, the standard shaft bearings and thrust bearings. When the wheel is vertical, the shaft bearings simply keep the runner and its shaft central with the surrounding guide vanes and casing, the weight of the rotating parts being carried on a thrust bearing, which is usually placed above the top of the generator and supports not only the shaft and runners, but also the weight of the generator rotor.

In horizontal units, the shaft bearings hold the rotating parts centrally and, also, support them. Hence, in the case of a horizontal unit, the provision for taking up wear and replacing worn

parts of the bearings must be more complete and elaborate than for the shaft bearings in vertical units.

Shaft bearings are mainly of two kinds, namely, babbitted bearings, arranged for oil lubrication, and bearings made of lignum-vitæ, which latter are generally used for all bearings that are submerged. Water circulation over the lignum-vitæ bearings is generally sufficient, although grease is sometimes used for additional lubrication. Recently, babbitted bearings have been used where submerged, and lubricated by oil fed to them under pressure, slightly exceeding that of the hydrostatic pressure of the water. With this arrangement there is always a slight, continuous passage of oil out of the bearings into the surrounding water. This prevents the entrance of grit or silt to the bearings, which would ruin them. Lignum-vitæ can continue to work after a considerable amount of abrasion, and for this reason, has been generally adopted for journals under water.

Horizontal bearings of either type are self-aligning and adjustable both in height and horizontal position. Babitted bearings, if accessible, are ring-oiling; otherwise they are lubricated by forced feed. All large, lubricated bearings are water-cooled. This feature is not usually required in ordinary operation, but is valuable in case of emergency. Lignum-vitæ bearings may be equipped with force-feed grease lubrication, if desired. Horizontal lignum-vitæ bearings are not recommended if it is practicable to use babbitted bearings. They are not safe, unless frequently and systematically inspected.

Vertical babitted bearings are ordinarily lubricated by gravity oil feed. Oil is supplied from a reservoir situated above the turbine, to which it is returned by a pump after draining from the bearings. This type of bearing is best adapted for positions intermediate between the turbine and generator, if any are required. Vertical units of customary design, however, require no intermediate bearings. Unless the shaft is exceptionally long, the only guide bearings needed are one on top of the generator and one on top of the crown plate of the turbine.

The bearing on the turbine is usually, lignum-vitæ. It may be placed closer to the runner than an oil bearing, is simpler, requires less attention, and is equally efficient. Furthermore, the duty required of a vertical guide bearing on a well-balanced turbine is so light that lignum-vitæ will not wear appreciably in many years of service.

Thrust bearings for horizontal units are usually of the marine type with the thrust collars forged on the shaft. They are self-aligning, ring-oiling and water-cooled, and are substantially braced or tied to the turbine head. These bearings are designed only to carry the residual thrust of the turbine.

Single-discharge, horizontal turbines are sometimes provided with an automatic hydraulic piston to take up the unbalanced axial pressure. This thrust piston is in the form of a large collar fixed to the shaft and working in a cylinder. The length of the cylinder is very short, as there is practically no in-and-out motion to the piston. One side of the piston is connected with the vent opening in the turbine cover plate by a small pipe. The unit pressure set up behind the piston is equal to that between the runner and the cover plate, and the piston pressure acts in a direction to oppose that of the end thrust of the wheel. By properly proportioning the area of the piston to that of the wheel diameter, the end thrust is neutralized.

Fig. 199.—Roller thrust bearing.

Several types of thrust bearings have been successfully used on *vertical turbines. The oldest type for large units and heavy loads* is the oil-pressure bearing introduced a number of years ago at Niagara Falls. In this bearing, the thrust disks have a large annular groove into which oil is forced under pressure. The pressure used is sufficient to lift the rotating parts, permitting the escape of oil between the faces of the disks which would, otherwise, be in contact. Thus, the load is carried on a film of oil. The disadvantages of this type of bearing are the expensive and troublesome auxiliaries required to furnish the oil pressure, and the certainty of serious damage if the oil pressure fails wh[illegible]ation.

an overhead reservoir. A roller thrust bearing is shown in Fig. 199.

A vertical section through this roller bearing is shown in Fig. 199a.

Roller bearings for large units are sometimes constructed to incorporate the oil-pressure feature also. The latter is combined with the rollers in such a manner that the weight may be lifted off the rollers for ordinary operation and carried by them only in the event that the pressure should accidentally fail. Or it may be carried ordinarily on the rollers, the oil pressure being

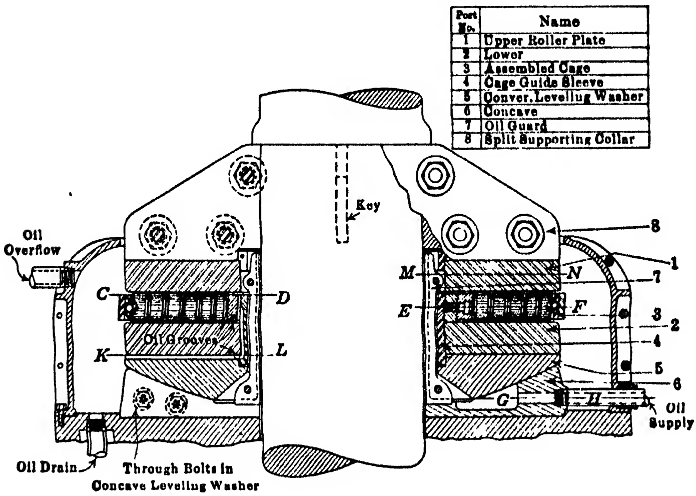


FIG. 199*a*.—Section through roller bearing. Standard Roller Bearing Co.

held in reserve in case of trouble with the rollers. Several of the largest plants in this country are equipped with this type of bearing.

An excellent form of thrust bearing is that in which the weight is carried on a series of heavy, accurately ground balls of tool steel. These balls run in annular grooves made in the thrust and supporting blocks, which are also of tool steel. The upper plate is fastened to the wheel shaft. The lower plate rests on the supporting spider of the generator, the balls being interposed between them. The balls run in an oil bath, the entire bearing being surrounded with cast-iron casing and a cast-iron cover plate which bolts down to the top of the case. The casing

is tapped for ½-in. pipe openings, and to these are connected oil gauges to show the height of oil, drain valves to draw off the oil, and a filler opening near the top through which oil can be introduced into the casing.

Ball thrust bearings were used on the generating units for the Austin, Tex., plant. The net weight carried is 44,000 lb. and the speed 300 r.p.m. To carry this load at the given speed with an ample factor of safety, required 35 balls, each 2 in. in diameter. In order to reduce the radius of their circular path they were arranged in two concentric circles, there being 16 balls in one and 19 in the other. The friction loss in the whole unit with this bearing, is less than 1 per cent. of the 3000-hp. capacity of each unit. The lower supporting plates of all ball or roller thrust bearings should be spherically shaped on the under side and fitted into a corresponding concavity in the supporting spider so that the shaft will, automatically, take a true vertical position without the necessity of adjustments which must be made and afterwards maintained. (See Fig. 199*a*.)

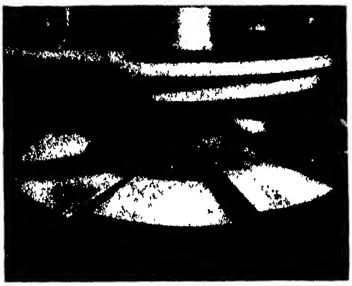


FIG. 200.—Kingsbury thrust bearing.

The latest type of thrust bearing, and one of the best, is the Kingsbury. This is a contact bearing which runs in an oil bath, the weight being carried on a set of segmental, babitted shoes. A liberal space is provided between adjacent shoes to permit free circulation of oil. Each shoe has a single pivot support located toward one end of the shoe, slightly beyond the center of gravity, in the direction of rotation. This arrangement causes the space between the shoe and the thrust block on the shaft to open slightly at the other end of the shoe, where the oil is drawn in by the rotation of the thrust block. The film of oil on the face of the shoe thus assumes the form of a very fine wedge, constantly urged forward by the rotation of the thrust block. This action insures perfect lubrication without grooves, and sustains the high oil pressure between the bearing surfaces necessary to carry the heavy weight imposed. This bearing may be operated with surface pressures of 400 to 500

lb. per square inch. A considerable excess of area must be provided, however, to take care of the starting and stopping conditions, which are much more severe than when running.

The pivots which support the shoes are individually adjustable for height. Any or all of the shoes may be removed and replaced without dismantling the unit or disturbing the shaft. It is only necessary to open up the thrust bearing housing. Fig. 200 shows the parts of the Kingsbury bearing, the shaft and thrust collar being lifted up off the slippers.

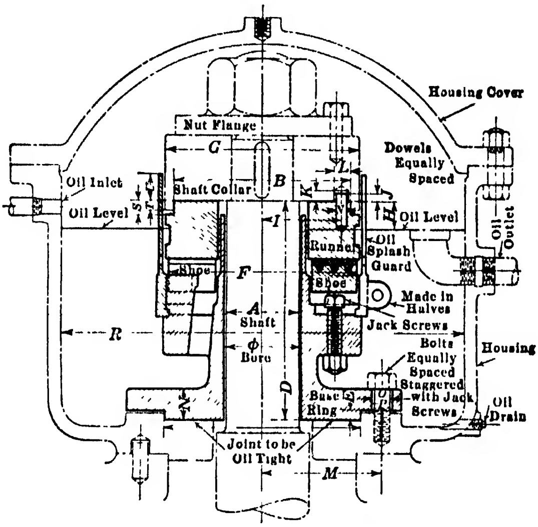


Fig. 201.—Cross-section through Kingsbury bearing.

Fig. 201 shows a vertical section through a Kingsbury bearing.

The proper position for the thrust bearing of a vertical unit is on top of the generator, supported by a spider or yoke mounted on the generator frame. This arrangement simplifies the design of the turbine, makes a more compact unit, and avoids the danger of vibration which is always present if the shaft is in compression particularly if the speed is high.

End Thrust.—There is an unbalanced axial pressure on a turbine, which tends to thrust the wheel in the direction in which the discharged water flows. The reason of this may be seen from Fig. 181. The pressures acting are:

1. An upward thrust, due to the action of the water on the under side of the hub.

2. A downward pressure, opposing the first pressure, produced by the vertical component of the discharged water against the vanes of the wheels.

3. The pressure acting on the upper surfaces of the wheel, tending to thrust it downward, acting against the first-named pressure and in the same direction as the second. This third pressure is produced by leakage of water through the hub clearance (see Fig. 181), which continues until the hydrostatic pressure between the cover plate and the upper surfaces of the wheel becomes equal to that of the pressure in the turbine at the point of entry of water to the wheel.

This pressure, in wheels of even reasonable size, attains considerable proportions; thus, in a wheel of 3-ft diameter, the area exposed to the hydrostatic pressure is slightly over 7 sq. ft., so that, if the pressure were 1000 lb. per square foot, corresponding to a head at wheel entry of 16 ft., the total pressure against the wheel would be 7000 lb. This thrust is so much greater than the first two named that they may be considered as negligible.

Where two runners are mounted on a single shaft, the pressures balance, and there is no need of providing any special means for resisting axial movement, or what is commonly called, "end thrust." Where a single wheel is used, it is necessary to provide thrust bearings such as are herein described, and, in order to reduce the size of these and diminish the pressure against them, it is customary to relieve the interior pressure between the top of the wheel and the cover plate by making a small opening in the latter, which is numbered 34 in Fig. 181. A pipe connection leads from this opening down into the draft chest, so that any leakage through the hub clearance is carried off, and no pressure can be set up against the wheel.

Another method of venting the wheel to relieve the end thrust is to drill one or more small holes through the hub and the shaft, so that the upper surface of the wheel is drained directly to the draft tube.

Where vertical turbines are set in pairs, this end thrust is made use of to diminish the weight which must be carried on the thrust bearing which supports the rotating parts. The upper runner has the space between the hub and the cover plate pro-

vided with a relief vent, while the lower wheel is placed in an inverted position, so that the cover plate is at the bottom. Under the lower wheel, therefore, the pressure tends to thrust the pair of wheels upward while there is no corresponding downward thrust from the upper wheel, and, in this manner, an upward, unbalanced pressure is obtained, which decreases, considerably, the weight that must be carried by the thrust bearing. Of course, venting the wheel means always a small loss of water, because all leakage past the hub is carried off through the vent. This loss, however, is negligible in practice.

Design of Concrete Scroll Cases.[1]—Experience has led designers to establish rules as to the best velocity of flow in spiral, or scroll, cases. In general, the velocity varies from 0.15 to 0.20 of the spouting velocity, $\sqrt{2gH}$, where H is the effective head of the plant. The lower value is used for high heads, and *vice versa*. In the speed ring this velocity is accelerated to from $0.6\sqrt{2gH}$ to $0.8\sqrt{2gH}$, at which velocity the water enters the turbine.

In a low-head, single-runner installation the passage for the water can be divided into five main parts: Intake; scroll case; speed ring; turbine and draft tube. The term intake will signify the passage from the front of the headgate piers up to the point where the scroll case begins. "Speed ring" is the name given the circular frame-work surrounding the wheel and in which the guide vanes, or gates, are placed.

In designing an intake abrupt changes in the flow must be avoided, as eddies formed at these points, will upset the flow throughout the intake and the scroll case. It is, therefore, best to divide the flow into imaginary vertical strips and regulate the velocity of each strip so that no eddies can form.

Suppose that the intake is so large that four gate openings are required, and that the speed ring of the turbine has 20 vanes. The flow, then, may be conveniently divided into either 20 or 40 strips. It is, however, usually divided into as many strips as there are vanes. Therefore, assume 20 strips in this case. As there are four gate openings, there will be five of these strips per opening. Fig. 202 shows a scroll case in which the flow has been divided in this manner. Beginning at the scroll case proper and working backward, it will be readily seen that these vertical strips or flow lines are of different lengths. The passages

[1] A. G. Hillberg, *Eng. Record*, Oct. 2 and 9, 1915.

in the concrete, obviously, must be so designed that the water in each flow line moves at the same velocity as that in the

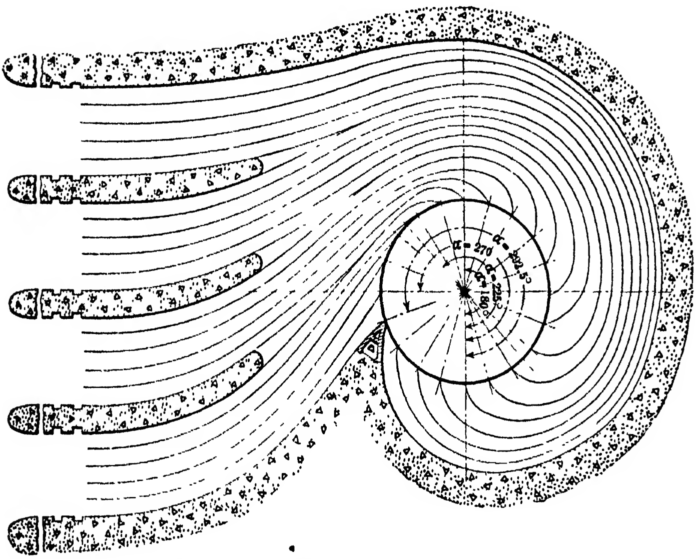


FIG. 202.—Typical concrete scroll case.

adjacent line, at points where such lines meet or run alongside each other. Otherwise eddies will form.

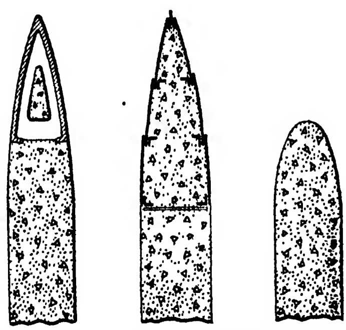


FIG. 203.—Pier ends.

As the lengths of these flow lines vary, it is necessary to separate the groups belonging to each headgate opening. This is readily done by the introduction of piers, which also serve to break up

the span of the roof of the intake. Theoretically, such piers should be carried out to a knife edge on their downstream side, but it is impossible to do this in concrete. Hence, it is necessary either to provide a knife edge of steel plate or cast iron, or to end the pier with a suitably rounded edge. Examples of such designs are shown in Fig. 203.

The velocity of the water through the racks and the headgate openings should be kept as low as possible, so as to make the loss in head due to the entry, small. Under no circumstances should this velocity exceed 5 ft. per second, in which case the loss in head due to entry would be less than 0.2 ft. For wheels operating under heads lower than 30 ft. this loss due to entry would be excessive, and a lower velocity should be used. The losses due to entry should never exceed 0.5 per cent. of the total head.

The velocity head H_v is twice the entry head h_e, so that for an entry head equal to 0.5 per cent. of the total head H, or $0.005H$, the velocity head will be 1 per cent., or $0.01H$. The entry velocity will be that due to the velocity head, or

$$v = \sqrt{2g \times 0.01H} = 0.8\sqrt{H} \qquad (243)$$

A plant operating under a head of 25 ft., therefore, should have a velocity of 4 ft. through the headgate openings, and the loss due to entry would be 0.125 ft.

In many instances, however, the roof of the intake opening lies so high that when the water level in the forebay is low, a velocity of $0.8\sqrt{H}$ per second, with the corresponding depression of the water level in front of the gate opening, might draw air into the turbine. In such cases the velocity must be kept below the critical velocity.

The velocity through the headgate openings can be maintained throughout the intake and the scroll case, but as a sufficient amount of concrete must be provided between the different units in the plant to carry the weight of the substructure, the horizontal, unbalanced part of the hydrostatic pressure and the water pressure on the partition walls when adjacent units are shut down, it is often necessary to accelerate the velocity of the flow in order to economize in space. When the flow is accelerated, the passage is made smaller and more space is available for concrete. As the velocity through the headgate openings is known, it is necessary to determine the velocity in

the scroll case and the distance between the headgates and the scroll case proper in order to determine the rate of acceleration This increase in velocity should be a constant amount per linear foot of travel of the water.

The rate of acceleration should also be maintained in the scroll case. When points of equal velocity have been located on the flow lines, these points should be connected by curves, and if any irregularities are discovered they should be adjusted. The boundary lines of the two extreme flow lines will determine the limits of the concrete.

When the flow lines are carried around the scroll case they naturally decrease in number each time a speed-ring vane is passed, as shown in Fig. 202. The maximum velocity in this part must be kept low enough so as not to cause erosion of the concrete. Either this maximum permissible velocity can be assumed at the end of the scroll case, and the velocity accelerated at a uniform rate from the headgate opening at the end of the scroll, or the velocity at the beginning of the scroll can be determined by the formula

$$v = c\sqrt{2gH_o}$$

where $c = 0.15$ to 0.18; g, 32.16, and H_o, the net operating head. As only the maximum, minimum and average heads on the plant are known, the net operating head must be guessed at. An empirical formula, which for average heads between 7 and 65 ft. gives very close results, is

$$H_o = H_a - (\sqrt{H_a})/3 \tag{244}$$

where H_o is the net operating head and H_a the average gross head.

As soon as the operating head is known, the most suitable velocity of flow at the beginning of the scroll can be calculated. Many engineers prefer to keep this velocity constant throughout the scroll, but, if the water be accelerated at a uniform rate in the intake, it is better to keep this rate of acceleration in the scroll also.

The next step in the calculation is to determine where the scroll case actually begins. Theoretically, the best place is on the transverse axis of the unit, in which case the angle α in Fig. 202 is 180°. The whole discharge, Q, then passes through the vertical plane through this line, and, at the opposite side, where $\alpha = 0$, an area sufficient to pass $Q/2$ must be provided. This, however, sometimes makes the breadth required by the unit

unduly large. To economize on space, and, consequently, on concrete, it is advisable to place the end of the scroll at another point. Calling the discharge in the scroll case to the left of the unit D_1, and that to the right D_2, Table 36 shows the discharges to be provided for.

TABLE 36.—DISCHARGES TO BE PROVIDED FOR AT DIFFERENT POINTS IN THE SCROLL CASE

a	180°	225°	270°	315°	360°
D_1	Q	$7Q/8$	$3Q/4$	$5Q/8$	$Q/2$
D_2	$Q/2$	$3Q/8$	$Q/4$	$Q/8$	0
Total	$3Q/2$	$5Q/4$	Q	$3Q/4$	$Q/2$

Not much difference in efficiency exists between the cases where $\alpha = 180°$, 225° and 270°. Where α is larger than 292.5° the curvature of the side of the intake will be so sharp that it is

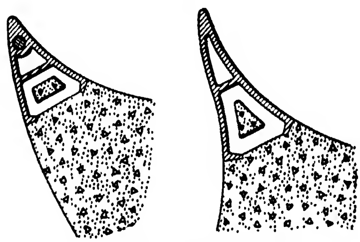


FIG. 204.—Scroll noses.

almost impossible to prevent the formation of an eddy. The effect is that less water will enter the runner at that point, thus subjecting the runner to an unbalanced thrust.

The general rule applies to all cases, that the intake masonry should be an easy curve and as nearly as possible a smooth continuation of the speed-ring vane.

The speed-ring vane at the end of the scroll case must be specially designed so that the concrete can be anchored to it. It should also be carried out far enough to protect the tapering end of the concrete and prevent the excess pressue at the end (due to the higher velocity) from breaking through the concrete. In most cases a thickness of from 18 to 24 in. will suffice. Fig. 204 shows two scroll noses of good design.

The friction losses in a scroll case can be determined with accuracy. The method, however, involves lengthy computations,

and as all losses are in proportion to certain areas, turbine designers generally use empirical rules whereby the influence of the friction can be allowed for. The method consists simply of enlarging the sections a certain predetermined amount. As the influence of the friction grows larger in proportion as the areas get smaller, the areas must be increased in accordance with a sliding scale. If the succeeding sections of a scroll case to be calculated are numbered 1, 2, 3, 4 . . . n, it is customary to enlarge them 0, 1, 2, 3 . . . n—1 per cent. Experience has shown that smaller scroll cases designed in this manner have a high hydraulic efficiency.

Good judgment, however, should be exercised in selecting a suitable sliding scale, as large sizes of the casing and low velocities of flow may, in certain cases, require the adoption of a smaller scale than that given. As the designer cannot predict the quality of workmanship of the concrete in the finished scroll case, it is always advisable to use a slightly larger increase in the scale than the theoretical computation indicates.

The vertical sections of a scroll case should, as far as possible, be symmetrical about the horizontal center line of the distributor. Consequently, this center line must be known before the scroll case can be given its shape. In determining the location of the horizontal center line of a distributor, which is a plane in which are located the horizontal center line of the wheel gates as well as the rated diameter of the turbine runner, the velocity of flow in the upper part of the draft tube must be known.

Theoretically, the best shape of the cross-sections of a scroll case is circular, as the hydraulic radius is then a maximum, and the difference between the maximum and minimum velocities of the flow in the section is a minimum. Economic reasons, however, prevent the use of such a shape, as the width of the unit and, consequently, the amount of concrete needed in the substructure would be unduly large. In order to reduce the overall width of the scroll case as much as possible, the cross-sectional areas are usually flattened, their longest dimension being vertical.

To facilitate the form work it is customary to make the periphery of the scroll case, vertical. The floor and the roof may be horizontal, but as sharp corners will create "dead" water, thus reducing the sections, large fillets must be provided. These fillets are sometimes so large that they meet at the roof

and the floor, making arcs of circles in the transverse sectional elevation.

The cross-sections at the upper and lower foundation rings of the speed ring are usually curves conforming with the arcs of

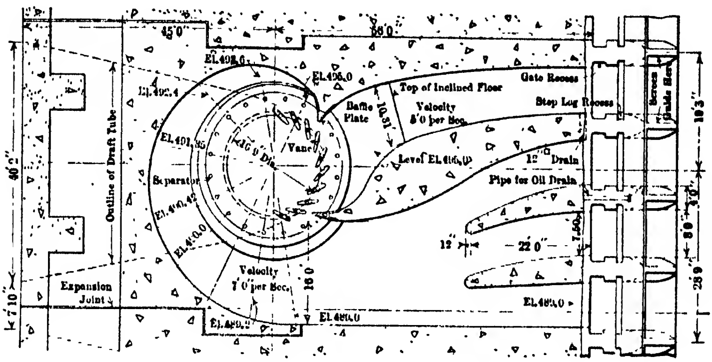


FIG. 205.—Concrete scroll case.

circles. In that way an efficient bell mouth is created. To increase the effect of this bell mouth, the scroll case is connected by sloping sides to this speed ring. These sides have a slope of from 45° to 60° with the horizontal. The effective area of the

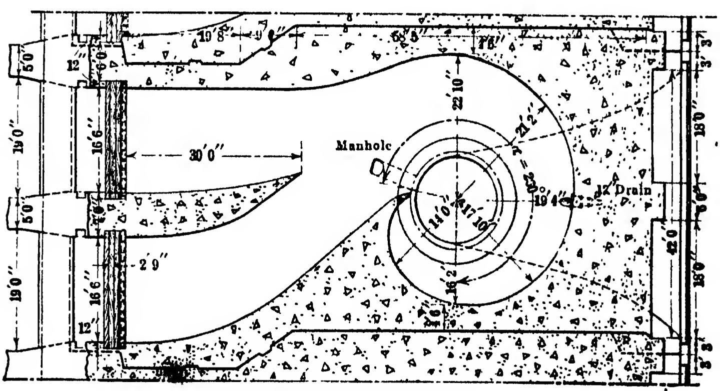


FIG. 206.—Concrete scroll case.

scroll-case sections, are to be figured up to the outer edge of the speed-ring vanes. Typical scroll-case sections are shown in the drawings, Figs. 202, 205 and 206, a vertical section being shown in Fig. 207.

As previously stated, the most advantageous location of the scroll case is when its horizontal center line conforms with the horizontal center line of the runner. Generally, such a location will place the scroll case rather high, thus requiring more concrete in the substructure than with the center line at a lower elevation. The distance to which the center line may be lowered is limited by the fact that the draft tube curves out to the stream, passing under the case, and the designer must provide enough masonry between it and the case to prevent the water

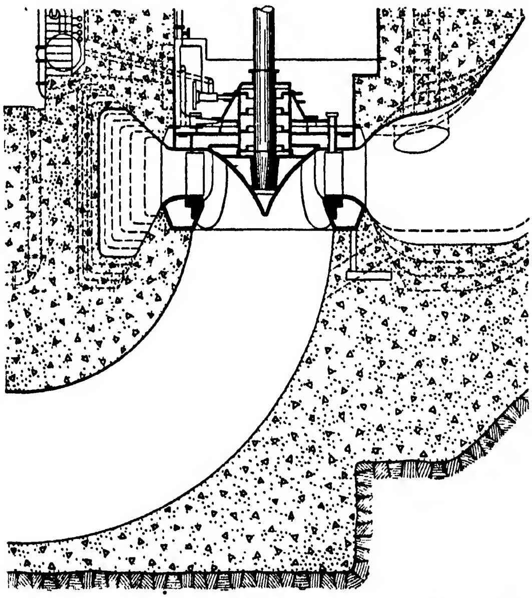


Fig. 207.—Vertical section through concrete scroll case.

from breaking through or even percolating through the masonry at the thinnest point. The critical section is generally that located on the transverse center line of the unit, or, more exactly, that located in a vertical plane laid through the axis of the draft tube.

The floor and roof of a scroll case can be parallel to each other until such a point has been reached that it is necessary to slope them toward each other in order to reduce the cross-sectional areas of the scroll; they can be given a uniform slope from the beginning to the end of the scroll, or the roof can be kept hori-

zontal while the floor is given an upward slope. For very large and deep scroll cases placed so that the horizontal center line is considerably lower than the horizontal center line of the runner, this latter arrangement is advantageous, as it brings the center line of the scroll higher and higher. As soon as a point is reached where both lines are in the same plane, the roof should also be given a slope, so that this condition is maintained for the rest of the way.

As long as the cross-sectional areas are comparatively large, and, consequently, the amount of water abundant, it does not make much difference whether or not the horizontal center lines of scroll case and runner coincide, but the smaller the area gets and the smaller the amount of the available water becomes, the more important it is to have this condition satisfied. Furthermore, the hydraulic radius should also be kept as large as possible, and in order to accomplish this it is necessary to design the smaller areas with a rapid decrease in height.

Draft Tubes.—The design of a draft tube of proper proportions for the conditions under which a turbine must operate is an important factor in power-plant engineering. The assumption, so frequently made, that a tube of any kind will suffice, is erroneous.

When the water is discharged from a turbine there is energy still left in it, due both to its elevation above tail water, and its velocity.

The upper end of the draft tube should fit accurately to the band of the runner. The wall of the tube at this point should be a continuation of the inside surface of the runner band. Abrupt bends should be avoided and the tube should be long enough to reduce the velocity to a proper value without too rapid enlargement of cross-section. At the same time it should not be so long that the inertia of the column of water in the tube is sufficient to break the vacuum when the flow is suddenly checked by the closing of the turbine gates.[1]

The bottom of the tube must be well submerged at time of lowest tail water. The lower end should, in any case, project at least 10 in. below the surface of tail water, and for large tubes, having a diameter 8 ft. and more, the depth should be at least 2 ft.

Good draft-tube design is, fundamentally, dependent on the

[1] Chester W. Larner, Wellman-Seaver Morgan Co.

proper elevation of the turbine above tail water. The runner should be so located that the total draft head at the top of the tube is well within the theoretical limits of a vacuum; namely, approximately 34 ft., for a 30-in. barometer, and proportionally less for localities above sea level, where the normal barometer is less.

The total draft head at the top of the tube, friction being negligible, is the vertical elevation, H_e, of this point above tail water, plus the head represented by the velocity of the water at that point. The latter is the so-called "velocity head," or H_v.

It is safer to base the calculations on the cross-section at the top of the runner band rather than the top of the draft tube, because the draft head actually extends well into the runner.

The difference between theoretical head, as shown by the barometer reading, and $H_e + H_v$, is the margin of safety allowed for contingencies, the most important of which is the additional vacuum caused by the inertia of the water column in the draft tube when the turbine gates close. Whenever the load on the turbine is reduced, the governor closes the turbine gates proportionately, in order to maintain constant speed. If the continuity of the water column in the draft tube is to be maintained, its velocity must be correspondingly reduced, as rapidly as the gates close. The vacuum, H_i, at the top of the draft tube necessary to overcome the inertia of the water column therein, is the same as the pressure which would be required if the retarding force were applied at the bottom of the tube. This is readily calculated on the basis of any assumed rate of governor action, and must be considered in determining the elevation of the turbine. The maximum value of $H_e + H_v + H_i$ should be less than the height of a water column which a vacuum will sustain or an equivalent water barometer reading, otherwise the column of water in the draft tube will break, returning with a surge and causing a water hammer.

It should be noted that H_e is a constant, while H_v and H_i are both variables, and the maximum value of the term $(H_v + H_i)$ is best determined graphically.

The worst condition to be considered is that of a shutdown from full-load discharge to a gate opening which will deliver only sufficient water to keep the wheel running at speed with no load. Curves of H_v and H_i should be plotted for a shutdown, with time

of gate movement as abscissæ, the corresponding values of H_v and H_t as ordinates, and from the curves the maximum value of $(H_v + H_t)$ determined. The curve of H_t increases from the beginning of the gate stroke to the end, whereas H_v decreases. The maximum values of H_v and H_t are not coincident, and hence, can not be calculated independently and added together. In practice, the sum of these three terms should be kept safely below the theoretical limit. If the vacuum in the draft tube is near the breaking point, continuity of flow may be interrupted at the discharge end of the water passages through the runner, resulting in corrosion and pitting of the vanes.

The effective length of the draft tube extends to the point where the discharge is released to atmospheric pressure. This fact may be utilized to advantage where it is impossible, for any reason, to make the draft tube as long as it should be to reduce the velocity at exit to a proper value. In such cases, the tailrace may sometimes be sealed to the atmosphere, and the residual loss is, then, the velocity at exit from the tailrace rather than from the draft tube. Of course, such an arrangement is not as efficient as a properly designed draft tube, because there is a loss at the point where the draft tube discharges into the tailrace.

The residual velocity, at the point where the discharge is released to the atmosphere, is an irreclaimable loss and should be made as small as possible.

It used to be a common practice to make all draft tubes of steel plate, but of late years they are usually moulded in the concrete foundation of the power house, except for small turbines. It is not feasible to build large draft tubes of plate, nor is it possible to obtain the smooth curves and efficient design characteristic of concrete tubes.

The reduction in the exit velocity of the water from the tube by gradually enlarging the cross-section should be in accordance with the equation.

$S^2V = A$ = constant for the area of the cross-section at any point along the length of the tube.

S = length of the path from an origin to any point measured along the middle line of the tube.

V = velocity of the water at any point.

This equation simply shows that the diameter of a round tube should increase uniformly from the draft chest to the lower end. The area increases as the square of the diameter, so that the

condition of S^2V = a constant is fulfilled. The final, or discharge velocity, V_e, is not a fixed quantity. It is usually taken at a value such that the lost energy of discharge shall never exceed 1 per cent. of the energy due to the total head and shall, preferably be kept within 0.5 per cent., or $V_e = 0.4\sqrt{H}$ to $0.8\sqrt{H}$ is the range of values of the velocity of exit. The principal loss in the draft tube is, however, not due to the exit velocity, but to eddy whirls and internal disturbances caused by the twisting motion of the water on leaving the wheel.

The diameter of a draft tube, at the bottom, is limited by its length and by the diameter where it is attached to the draft chest. The maximum "flare," or rate of increase in diameter, should not exceed 0.33 ft. increase per foot length. Of course, the length of the tube is limited by the height of the turbine above tail water. This height, measured to the middle point of the runner, should not exceed 26 ft. (at sea level) and is, preferably, fixed by the conditions before set forth.

Large draft tubes are usually flattened at the bottom to avoid the cost of the excavation that would be required if the cross-section were maintained circular These tubes are invariably built of concrete.

Concrete Draft Tubes.[1]—Vibrations in power-house substructures are often due to improperly designed draft tubes, in which the discharge does not fill the tube. In the upper part, where the change in elevation for a given distance, measured along the axis of the tube is more rapid than in the lower part, a vacuum is formed between the stream and the walls of the draft tube, resulting, naturally, in vibration of the stream. The greater the discharge and the higher its velocity, the less the vapor pressure in the vacuum and the greater these vibrations.

The total draft head, referred to the difference in elevation between the top of the runner band and the lowest tail-water level is

$$E = B - (j - u) - (v_1^2/2g) \qquad (245)$$

where E = difference in elevation, or maximum permissible draft head; B = height of water barometer = 33.9 ft. for sea level; v_1, velocity through the runner band at full load; g, 32.2; j, margin allowed for governing and vapor tension, and

[1] A. G. Hillberg, *Eng. Record*, Nov. 13 and 20, 1915.

u, allowance for all losses due to friction and curvature in the draft tube and the outflow loss at its end.

The value of $(j - u)$ should be selected at from 3 to 6 ft., depending on the physical conditions of the plant in question, j depending on the load variations, and u upon the length of the draft tube.

In dealing with the flow in curved and expanding tubes, several factors develop which prevent an absolutely correct mathematical solution of the problem. If two succeeding sections be considered, and it be assumed that the flow is normal to the first one, the expansion and the consequent reduction in thickness of the film of water, when passing through the following section, will have changed this direction of flow. The result may be an increase, or a reduction, of the pressure on the walls, or it may be the formation of eddies. In order to permit any kind of a mathematical solution, it is, therefore, necessary to make two assumptions, namely,

(a) The flow in the tube is normal to the plane under consideration.

(b) The change in velocity head should not, at any time, be greater than the change in elevation. (This is in order to get a certain amount of back pressure, so that the stream will expand and fill the tube.)

In order to analyze the problem as completely as possible, the following factors should be investigated: shape of center line of tube and its length; velocities and change in velocity heads; shape of areas and volume of draft tube.

The velocity, v_1, of the flow through the band of the runner is always known as soon as the turbine has been designed, and the desired outflow velocity v_n at the end of the draft tube can be determined from the physical conditions of the tailrace. As the discharge, Q, at full load (that is, 0.8 gate opening, wider openings being considered as overloads) is known, the areas A_1 at the top, and A_n at the lower end of the tube, can be calculated. $A_1 = Q/v_1$ and $A_n = Q/v_n$.

The draft tube must be placed deep enough to permit its being sealed at all stages of the tail water. This seal refers to the lowest point of the roof of the draft tube and not necessarily to the upper edge of the end area. As the required seal only needs to be from 2 to 3 ft., it is generally sufficient to locate the

upper edge of the end area about 8 to 15 in. below the lowest tailrace level.

As the single-runner turbine is of the mixed-flow type, that is, the flow is deflected from radial to axial, the center line of the draft tube should be vertical immediately below the runner. This vertical part is a direct extension of the center line of the turbine shaft, and its location is therefore known. The center point of the lower end is also known, as this area has been determined as to its size and location.

The center line must pass through the center point of the end area, and its direction at the point of its intersection with the plane of the end area should be horizontal, or nearly so.

The curve most generally used is the arc of a circle with tangents at both ends. In laying out a center line of a draft tube in this way, the radius of the arc should be as long as possible in order to minimize the losses due to the curvature.

One disadvantage of this curve, however, is that the change of curvature is small in the upper end of the draft tube, and, consequently, in order to make the stream of water fill the tube, a considerable back pressure is required. Part of an ellipse, or a parabola, would be better in this respect.

The change in elevation is more gradual and better distributed throughout the length of a center line of parabolic shape than of one elliptical or circular. If the draft tube is short, however, the tangent at the lower end of the curve will not be horizontal, and, as it is desirable to have a horizontal, or nearly horizontal, discharge of the water, it is sometimes necessary to insert at this end a short arc of a circle with a long radius.

As the runner band is circular, the upper end of the draft tube must also be circular. The flow of the discharge below the turbine, although axial in its general direction, follows along lines of approximately helical shape, and the areas of the draft tube should, therefore, be circular until the curvature of the tube has broken up the helical flow lines and forced the flow to become perpendicular, or approximately so, to the cross-sectional areas.

The shape of the lower end can be varied within a wide range. Its limits in the vertical direction are the depth of excavation and the water seal required, and in a horizontal direction the thickness of masonry between the draft tubes necessary to carry safely the weight of the power house. The least amount of

excavation is required if this area is a rectangle and the intervening pier is as thin as permissible in order to carry the superimposed weight of power house and foundations. For maximum hydraulic efficiency, it is better to have an area of circular cross-section. As this form, however, generally requires too much excavation in the tailrace and too much concrete in the power house, a compromise between the opposing structural and hydraulic conditions is made. An elliptical shape is the best compromise form.

Elliptical sections are difficult to build and, for this reason, it is customary to make the bottom section in the form of a parallelo-

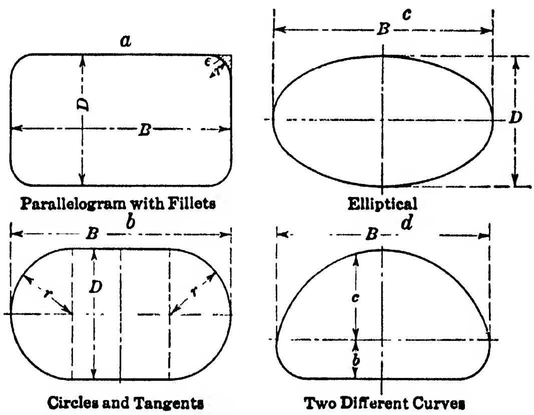


FIG. 208.—Typical discharge ends of draft tubes.

gram with large fillets in the corners. These fillets may be large enough to make a continuous semicircle across the end of the rectangle. Fig 208 shows the kind of bottom sections that have been used.

The area of the parallelogram section with fillets is

$$A = BD - 0.8584r^2 \qquad (246)$$

where r = radius of fillets.
B = extreme width of section.
D = extreme height of section.

Obviously, this equation holds for a square with fillets that meet, *i.e.* a circular cross-section.

The area of an elliptical section is $\frac{\pi DB}{4}$. (246*a*)

For a section such as shown in d, Fig. 208, the area is

$$A = bB - 0.4292r^2 + \frac{\pi cB}{4}. \qquad (246b)$$

b = distance from lower tangent to horizontal line dividing the areas enclosed by the two curves. r = radius of fillets. c = distance from the uppermost element of the elliptical portion, down to the horizontal dividing line, *i.e.*, half the minor axis of the ellipse.

All intermediate cross-sectional areas must be larger than the top area and smaller than the end area. Their shape can also be varied within the limits of the shapes of the two given areas. It is, however, customary to keep the areas circular until the diameter of the circle becomes equal to the minor axis of the end area. After that point has been reached, the circular shape is abandoned and the horizontal axis increased until it equals the major axis of the end area.

The velocity of the flow at any point in a draft tube must be related, in a certain way, to the velocities at other points. The velocities at two points, however, are known at the outset; these are v_1 and v_n, the velocities at the top and the bottom ends of the tube, respectively. For the former,

$$v_1 = 2.4 \text{ to } 3.2\sqrt{H_o} \qquad (247)$$

where

$$H_o = H_a - (\sqrt{H_a}/3) \qquad (244)$$

H_a = average gross head, and H_o is the operating head of the plant.

The turbine builder will give the velocity of flow as soon as the runner of the wheel has been designed. If it be required to design a draft tube for estimating purposes before the turbine has been designed finally, it is better to assume too high than too low a value.

The outflow velocity v_n should be kept as small as possible in order to waste a minimum of the head. Usually

$$v_n = 0.4 \text{ to } 0.8\sqrt{H_o} \qquad (248)$$

It is not always advantageous to use a small value of v_n, as the flow in the tailrace must be rapid enough to carry away the water. A low value of v_n might, therefore, result in an unnecessarily expensive excavation for the tailrace. As a low velocity requires a larger area for the outflow, too low a velocity

might result in too great a spacing of the wheel centers, thus increasing materially the cost of the power house.

By means of these velocities the corresponding cross-sectional areas, A_1 and A_n, can be calculated, as the discharge Q is known. The intermediate areas can be calculated only after the velocities through the planes in which these areas are located have been determined. The problem, therefore, is to find the velocity change per length unit in the draft tube, measured along the center line.

It must be remembered, however, that this change in velocity must be such that the change in velocity head becomes somewhat smaller than the change in static head. The theory is that on one side, the film of water is subjected to a pressure corresponding to the change in velocity head, and, on the other, to a pressure corresponding to the change in static elevation. If the difference in pressures be such that the film is subjected to a back pressure, the film will obviously expand sideways and thus fill the cross-sectional area of the tube.

The water flows through the tube at a constantly diminishing velocity, and a mathematical analysis shows that the velocity at any point in its path of flow is related to the distance traversed, and measured along the center line of the tube, by a parabolic equation which is

$$v = b - \sqrt{kz} \tag{249}$$

in which z is the distance of any point along the center line of the tube from x, the vertex of the curve; v is the velocity at that point, and

$$b = v_1 + v_n$$

$$k = \frac{v_1^2 - v_n^2}{l}$$

v_1 and v_n being, respectively, the velocities in the upper and lower ends of the tube, and l the length of the tube. After l and k are found, the distance from the vertex of the first and last points on the curve can be determined by the following formulæ:

$$z_1 = \frac{v_n^2}{k} = \frac{l}{\left(\frac{v_1}{v_n}\right)^2 - 1} \tag{250}$$

$$z_n = \frac{v_1^2}{k} = l + z_1 \tag{251}$$

Figure 209 shows the general relationships of these quantities. If the axis of the tube is parabolic in form, the length along the center line, l, is

$$l = \frac{a}{2} + \frac{2.3y^2}{4x} \log\left(\frac{2x + a}{y}\right) \text{ ft.} \tag{252}$$

in which

$$a = \sqrt{4x^2 + y^2}$$

From these equations all the quantities necessary to design a tube may be found.

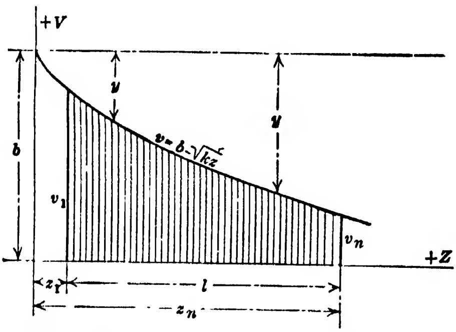


FIG. 209.—Diagram of velocity retardation in draft tube.

In computing the volume of concrete in a power station, the volume of the draft tubes must be known. This is

$$Vol. = \frac{2Qb}{k}(m - 2.3 \log m) \text{ cu. ft.} \tag{253}$$

Q = discharge through the tube, in cubic feet per second.

$$b = v_1 + v_n$$

$$m = 1 + \frac{\sqrt{k}}{b}(\sqrt{z_n} - \sqrt{z_1}) \tag{254}$$

As an example of the use of these formulæ, design a draft tube for the following conditions.

A 15,000-hp. wheel operates under a gross average head of 54 ft. The model of the runner has given a maximum efficiency of 88 per cent. under test, and it has been estimated that the full-size wheel will give close to 90 per cent. efficiency. The point of maximum efficiency on the curve has been located at 0.8

gate and the best speed determined accordingly. The net operating head is

$$H_e = H_a - \frac{\sqrt{H_a}}{3} = 54 - \frac{\sqrt{54}}{3} = 51.5 \text{ ft.}$$

$$Q = \frac{15{,}000 \times 550}{62.5 \times 51.5 \times 0.9} = 2850 \text{ sec.-ft.}$$

$$v_1 = 2.4 \text{ to } 3.2\sqrt{51.5} = 17.3 \text{ to } 23 \text{ ft. per second,}$$

v_1 being velocity of flow through the runner band. Assume, in this case, the turbine manufacturer has stated that the design of his wheel is such that $v_1 = 20$ ft. per second which fixes the value of v_1.

The velocity of discharge is

$$v_n = 0.4 \text{ to } 0.8\sqrt{51.5} = 2.9 \text{ to } 5.75 \text{ ft. per second.}$$

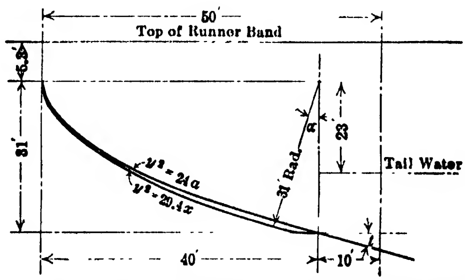


FIG. 210.—Diagram locating center line of draft tube.

Suppose the river channel at the power house, however, is narrow and comparatively deep even at the lowest stages of the river. Practically no tailrace, therefore, has to be excavated, so that the outflow velocity is assumed at 3 ft. per second.

By means of equation (245) the allowable difference in elevation between the top of the runner band and the lowest tail-water level can be determined. The equation is

$$E = B - (j - u) - (v_1^2/2g)$$

In this case the draft tube will be assumed an ordinary length justifying the use of a value of 3 for $(j - u)$, but the power will be used for traction purposes and the load variations will be severe. The term $(j - u)$, therefore, is taken = 4.68. Assuming for B, its value at sea level, or 33.9 ft.

Then

$$E = 33.9 - 4.68 - 6.22 = 23.0 \text{ ft.}$$

The top area of the draft tube, that is, at the runner band, is

$$A_1 = Q/v_1 = 2850/20 = 142.5 \text{ sq. ft.}$$

and the area at the outflow end is

$$A_n = Qv_n = 2850/3 = 950 \text{ sq. ft.}$$

The top area, as explained previously, must be circular, and its diameter, therefore, is 13.46 ft. The end area can be one of several shapes, and it is here assumed elliptical, as a circular area would require too much excavation both in the foundation and the tailrace. The horizontal axis is assumed twice the vertical.

Consequently, $\frac{\pi BD}{4} = 950$, and, since $2D = B$,

$$D = \sqrt{2 \times 950/\pi} = 24.6$$

The minor axis (vertical) is, therefore, 24.6 ft., while the major axis (horizontal) is 49.2 ft. Should it be found that this width is too great, the permissible width can be determined by means of the spacing of the units, and the width of the concrete pier between the draft tubes required to carry the load safely. The calculation is then repeated backwards and the length of the minor axis determined. In this case it is assumed that the length of 49.2 ft. is satisfactory.

As the outflow velocity is small, the highest point of the end area will be located 1 ft. below the lowest tail-water level.

The distance from the center line of the unit to the downstream wall of the power house has been determined previously, in connection with the design of the scroll case, and it is also dependent upon the amount of space required on the generator-room floor. In this case, it is assumed that this distance is 50 ft.

The two points on the center line required, in order to locate it, are now known. Taking the lowest tail-water level as one axis, and the center line of the wheel as the other, one point is at elevation 23, above low water, while the other is at elevation 13.3, below low water. The design of the turbine, however, is such that the bottom flange of the lower foundation ring is about 4 ft. lower than the runner band. It is, therefore, determined to begin curving the draft tube at a point 5.3 ft. below the runner

band. Hence the difference in elevation between top and bottom of the center line is 23 + 13.3 − 5.3 = 31, and these two points are 50 ft. apart, horizontally.

Looking at the diagram, Fig. 211, it is seen that the center line of the draft tube is too long. As the actual length across the power station of 50 ft. can not be changed, since this is determined from other requirements than those of the draft tube, the length of the tube is shortened to 40 ft. by building an arch 10 ft. wide inside the wall of the substructure. The intrados at the

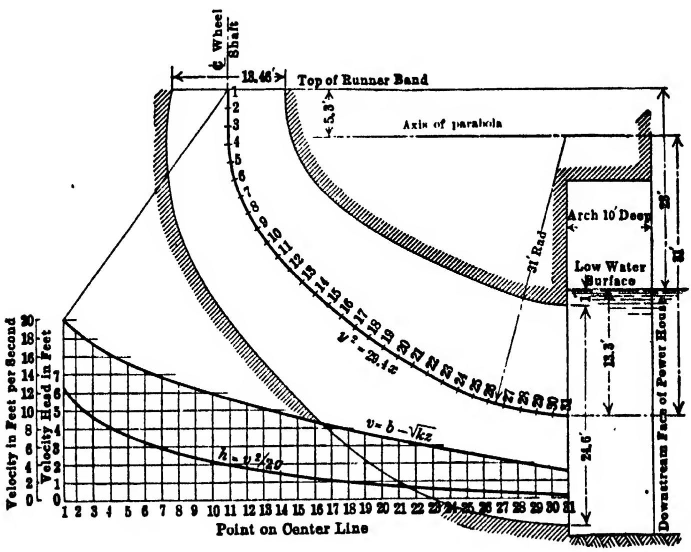


FIG. 211.—Section through draft tube and curves showing velocity and velocity head at any point in tube.

crown of this arch should project above ordinary stages of the water, and, if possible, above the highest water level, in order to effect the desired reduction in length of the draft tube. In many instances, however, tunnels located in the substructure, and, in some cases, the scroll case itself, prevent this arch from being carried as high as might be desired.

A parabolic center line will be used. In order to get as simple an equation as possible, the system of axes will have its origin at the apex of the parabola. The equation then reads $y^2 = ax$,

and, as the center point of the end area must lie on this parabola, $y = 31$ when $x = 40$, or

$$a = (31)^2/40 = 24$$

making

$$y^2 = 24x$$

The center line can now be plotted. When this has been done, it appears that the angle between the lower end of the parabola and the horizontal, is 21° 10′. This is too great, and an arc of a circle of 31-ft. radius is, therefore, inserted, as shown in Fig. 210.

A new parabola must now be calculated, and it must have its apex at the origin and its lower branch must be tangent to this circular arc. For this condition, $y^2 = 24x$ will no longer hold. The new parabola will end at the arc of the circle, and at such a point that the radius of the circle will be normal to both curves at their point of intersection, this being the condition for tangency of the two adjoining ends of the curves.

Let the equation of the new parabola be $y^2 = ex$ and

$$e = 2\left[c \pm \sqrt{c(c - a)}\right] \tag{255}$$

c = distance from apex of the parabola to center of the circle = 40 ft. in this case.

a = previous value of coefficient of x in equation $y^2 = ax$ = 24 in this case.

α is the angle between a vertical line and the radius of the circular arc to the point where arc and parabola join. Its value is computed from the equation

$$\sin \alpha = \sqrt{\frac{c}{a}} \pm \sqrt{\frac{c - a}{a}} \tag{256}$$

Obviously, there are two values that will satisfy these equations, but as the circular arc shortens the length of the parabola, the negative signs only must be used.

For this case

$$e = 2\left[40 - \sqrt{40(40 - 24)}\right] = 29.4$$

So that $y^2 = 29.4x$.

Also

$$\sin \alpha = \sqrt{\frac{40}{24}} - \sqrt{\frac{40 - 24}{24}} = 0.474$$

$$\alpha = 28° \, 20' = 28.33°$$

$$r \sin \alpha = 31 \times 0.474 = 14.7$$
$$x = 40 - r \sin \alpha = 25.3$$
$$\left.\begin{aligned} y &= \sqrt{29.4 \times 25.3}, \text{ or} \\ y &= r \cos \alpha = 31 \times 0.88 \end{aligned}\right\} = 27.3.$$

In figuring the true length of the center line, it must be borne in mind that two different curves must be considered, one a parabola and the other the arc of a circle. The length of the arc l_1 is

$$l_1 = \frac{\pi r \alpha}{180} = \frac{\pi \times 31 \times 28.33}{180} = 15.3 \text{ ft.}$$

The length of the parabolic curve, l_2, can be determined by formula (252).

$$a = \sqrt{4 \times (25.3)^2 + (27.3)^2} = 57.5$$
$$l_2 = \frac{57.5}{2} + \frac{2.3 \times (27.3)^2}{4 \times 25.3} \log \frac{2 \times 25.3 + 57.5}{27.3}$$
$$= 28.75 + 16.928 \log 3.97$$
$$= 38.88, \text{ say } 38.9 \text{ ft.}$$

The total length of the center line, therefore, is

$$L = 5.3 + l_1 + l_2 = 5.3 + 38.9 + 15.3 = 59.5 \text{ ft.}$$

Instead of the length of the center line being calculated, it can be plotted to a large scale and measured with sufficient accuracy.

In determining the shape of the draft tube, the areas should be calculated at points spaced from 2 to 3 ft. apart. As neither of these spacings, multiplied by any whole number, will equal the length 59.5, the first space will be made to suit. In this case, a 2-ft. spacing will be used, and the first space will, therefore, be 1.5 ft.

Previously, it has been determined that $v_1 = 20$ and $v_n = 3$ ft. per second.

$$k = \frac{v_1^2 - v_n^2}{l} = \frac{(20)^2 - (3)^2}{59.5} = 6.57$$
$$\sqrt{k} = 2.557$$
$$v = v_1 + v_n - \sqrt{kz} = 23 - 2.557\sqrt{z}$$
$$z_1 = \frac{l}{\left(\frac{v_1}{v_n}\right)^2 - 1} = \frac{59.5}{\left(\frac{20}{3}\right)^2 - 1} = 1.37 \text{ ft.}$$

and

$$z_n = 1.37 + 59.5 = 60.87 \text{ ft.}$$

By taking the different points along the center line of the

draft tube the values of z are found, and from them the values of v.

The area for each point is Q.

Having these quantities, all others required for the design, are calculable.

A table is computed of the general form shown (Table 37) and from the data in it curves are plotted, as shown in Fig. 211 The table is abbreviated, only a few points being given.

TABLE 37.—VALUES OF DIFFERENT ELEMENTS IN DRAFT TUBE DESIGN

(1) Point on center line	(2) Dist. from upper end of draft tube, ft.	(3) z, ft.	(4) v, ft. sec.	(5) $v^2/2g$, ft.	(6) $\frac{v_a^2}{2g} - \frac{v_{a+1}^2}{2g}$, ft	(7) h, ft.	(8) $h_{a+1} - h_a$, ft.	(9) Area, sq. ft.	(10) Remarks
1	0	1.37	20.00	6.22		0		142.5	Circle, $D = 13.46$
					0.82		1.5		
2	1.5	2.87	18.65	5.40		1.5		152.8	Circle, $D = 13.97$
					0.74		2.0		
3	3.5	4.87	17.33	4.66		3.5		164.4	Circle, $D = 14.49$
					0.54		2.0		
4	5.5	6.87	16.27	4.12		5.5		175.2	Circle, $D = 14.93$
					0.46		2.0		
5	7.5	8.87	15.35	3.66		7.5		185.7	Circle, $D = 15.38$
					1.48		9.0		
10	17.5	18.87	11.83	2.18		16.5		240.8	Circle, $D = 17.53$
					0.87		7.0		
15	27.5	28.87	9.20	1.31		23.5		309.6	Circle, $D = 19.86$
					0.555		5.8		
20	37.5	38.87	6.98	0.755		29.3		408.8	Circle, $D = 22.80$
					0.360		4.7		
25	47.5	48.87	5.05	0.395		34.0		564.0	Ellipse, $a = 12.3$; $b = 14.6$
					0.226		2.24		
30	57.5	58.87	3.30	0.169		36.24		864.0	Ellipse, $a = 12.3$; $b = 22.4$
					0.029		0.06		
31	59.5	60.87	3.00	0.140		36.30		950.0	Ellipse, $a = 12.3$; $b = 24.6$

Note: The dimensions a and b of the elliptically shaped areas are half the length of the minor and major axes of the ellipses.

The points on the center line of the draft tube are given consecutive numbers for identification. These are shown in the first column.

Their distances from the upper end of the draft tube, measured along the center line of the tube, are given in the second column.

Values of z, in the third column.

Values of v, in the fourth column.

Values of velocity heads corresponding to values of v, in the fifth column.

Change in velocity heads between consecutive points, in the sixth column.

Drop in elevation of the center line below the upper end of the tube, in the seventh column.

Drop in elevation of center line between consecutive points, in the eighth column.

Areas, in the ninth column.

Volume of displacement of the tube is from formula 253.

$$Vol. = \frac{2 \times 2850 \times 23}{6.62} (m - 2.3 \log m)$$

From formula 254

$$m = 1 - \frac{2.57}{23} \times (\sqrt{60.87} - \sqrt{1.37}) = 1 - 0.741 = 0.259$$

$$\log 0.259 = \bar{1}.4133 = -0.5867$$
$$2.3 \log 0.259 = 2.3 \times -0.5867 = -1.349$$
$$\therefore m - 2.3 \log m = 0.259 - (-1.349) = 1.608$$
$$V = \frac{131,100}{6.62} \times 1.608 = 31,800 \text{ cu. ft.}$$

A simple method of laying out draft tubes of this kind graphically, has been devised by R. Dubs.[1]

The curves of the top and bottom of the tube, as shown by a longitudinal section, are involutes of circles, which are curves generated by the end of a cord unwinding from a cylinder.

The top of the tube where it joins to the turbine casing is, of course, circular, and its diameter is fixed by the size of the opening in the casing. The area of the bottom end is fixed by the designer, and for this method, should be a rectangular, filletted section, or circular—not elliptical.

Fig. 212 shows the longitudinal section through the tube, as well as the locations and diameters of the generating circles.

Refer to Fig. 208, *a* or *b*. B is the width of the cross-section through the bottom end of the tube, while D is the height.

B is fixed by the designer, as flat as possible, as the depth of excavation is proportional to B. It, however, can not be reduced indefinitely, and hydraulic conditions require that B be not less than $\frac{D_1}{2}$, D_1 being the diameter of the upper end of the tube. The fillet radius, r, should be not less than $\frac{D}{6}$.

[1] "La Houille Blanche," April, 1913.

Consider the large circle with radius R. Its lower edge is tangent to the horizontal line through the top of the draft tube. Assume that a cord is wrapped around a cylinder, having the radius R, and located in the same position as the large generating circle, the free end of the cord being at P, where a horizontal diameter intersects the circle. If the cord be unwound, the end will reach the point Q when the free portion of the cord is in a horizontal position. This point Q must correspond with the top element of the section of the tube. As the

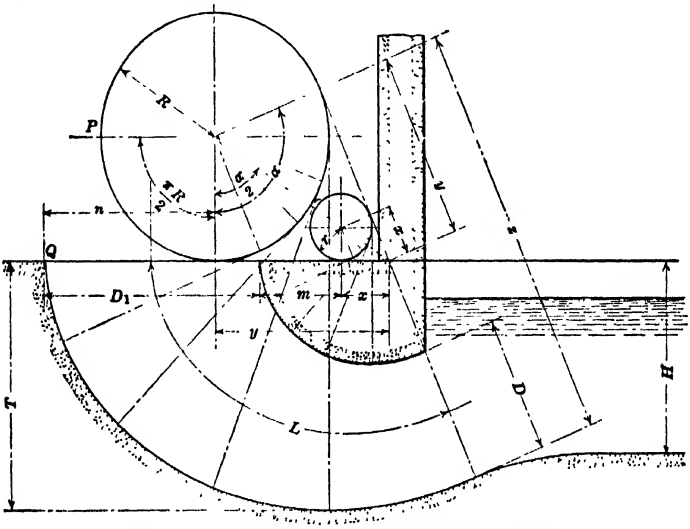


FIG. 212.—Graphical method of draft-tube design.

cord is unwound, its end describes the lower curve of the longitudinal section. In the same way the upper curve is fixed by the smaller circle.

The radius of the larger circle is fixed by the conditions

$$R = \frac{H}{2.1416} \tag{257}$$

H being the vertical distance from the top of the draft tube to the bed of the tailrace where the lower end of the tube debouches.

The position of this circle is fixed by the equation

$$n = \frac{H}{1.3634} \tag{258}$$

n being the horizontal distance of the center of the circle from the upper point of the lower longitudinal section of the tube (*i.e.*, point Q) as shown.

These formulæ are applicable only to the condition of $H = T$ or $H > T$, T being the vertical distance from the top of the tube to the lowest point in the curvature of a tube—in other words, where the curve of the bottom of the tube does not turn upwards, which means that the furthest movement of the generating cord is to a vertical position.

Where there is an upward curve to the lower end of the tube, as shown in the figure, the formulæ become

$$R = \frac{T}{2.1416} \tag{259}$$

and

$$n = \frac{T}{1.3634} \tag{260}$$

That is, to determine R and n, use as the numerator the value of the vertical distance from the top of the tube to its lowest point.

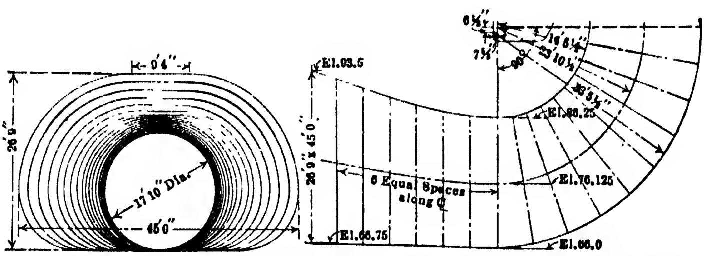


FIG. 213.—Typical longitudinal section and cross-sections of concrete draft tube.

For the smaller circle the radius is

$$r = R - \frac{D_1 - D}{2\left[\tan\frac{\alpha}{2} - \frac{\pi\alpha}{360}\right]} \tag{261}$$

and the position of its center is determined by the formula

$$m = \frac{\pi R}{2} + (R - r)\tan\frac{\alpha}{2} - D_1 + D \tag{262}$$

m being the horizontal distance of the center of the circle from the upper point of the upper longitudinal section of the tube

which lies opposite to that used as a reference point for location of the larger circle.

α = the angle between a vertical drawn from the center of the large circle downwards, and a radius to the point of tangency of the generating cord in its final position. (See Fig. 212.)

Since both circles are tangent to the horizontal line, passing through the upper end of the tube, their locations are fully known when n and m are known.

The top and bottom curves are traced graphically.

An excellent example of a concrete draft tube is that of the Cedar Rapids plant, shown in Fig. 213 which shows the longi-

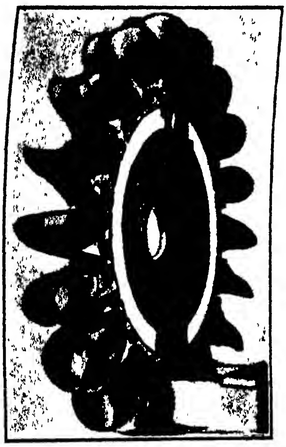


FIG. 214.—Tangential wheel.

FIG. 215.—Pelton wheel bucket.

tudinal section and a series of cross-sections taken at different points along the length of the tube.

THE IMPULSE WHEEL.—A general form of a tangential water wheel is shown in Fig. 214. A number of buckets, or vanes of a special form, are fastened to the rim of a wheel at equal distances apart. A jet of water, issuing from a nozzle, with a velocity $= C\sqrt{2gh}$, strikes against the vanes and, by impact against them, causes rotation of the wheel. The form of the buckets is shown in Fig. 215, and a section is shown in Fig. 216. The working surface comprises two curved sections, or lobes, separated by a sharp "backbone" placed in the middle of the vane, its direction

being perpendicular to the axis of the wheel. The jet is split by the sharp edge, one half passing to the curved surface on one side, and the other half to the similar surface on the other side of the backbone.

The half jets enter the lobes of the bucket axially and are gradually changed in direction until they emerge from the outer edges of the bucket in a direction (relative) almost opposite to that in which the jet discharges, the effect being, substantially, the same as that of a jet acting on a curved vane, as before discussed.

These wheels are adapted for high heads only—300 ft. and greater. The characteristic speed is low, ranging from 4 to 10 so that the actual working speeds under high heads are within the limits of mechanical possibility, and adapted for commercial electric generators. The power is proportional to the number of jets employed but the efficiency is slightly reduced when more than one jet is used, and the number of jets per wheel should never exceed two.

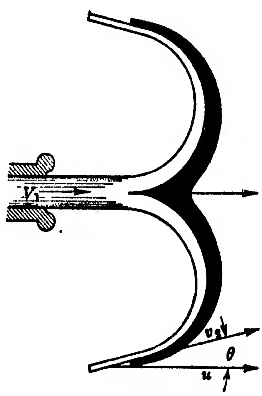


FIG. 216.—Diagram of forces acting on curved wheel-buckets.

Impulse wheels must discharge to the atmosphere. No draft tubes can be used, because the wheel can not operate submerged. Hence, the head from the wheel to tail water is lost. This means, of course, that the wheel must be set as close to tail water as it can be placed and, at the same time, avoid flooding from backwater.

The efficiency of the tangential impulse wheel is, approximately the same as that of a reaction wheel at, or near, full load. On partial load, however, the efficiency of the impulse wheel is considerably higher than that of a reaction wheel, as will appear from the general theory of the two forms of water wheels.

General Theory.—The theory of the impulse wheel is simpler than that of the reaction turbine, as the question of relative velocity does not affect the discharge from the nozzle, which corresponds to the velocity of entry of a reaction wheel.

The velocity of the water through the nozzle is always $C\sqrt{2gh}$, in which C has a value between 0.95 and 0.98. In the regulation

of the energy delivered to the wheel there are only two methods; one is to vary the area of the nozzle opening, the other, to deflect the nozzle so that only a certain portion of the water impinges on the vanes of the wheel.

Equation (217) shows that the pressure produced by a jet acting against a succession of curved, moving vanes, the vanes and jet following the same path, is

$$P = M(V_1 - U)\,(1 - \cos\theta)$$

in which P = pressure, in pounds

M = mass of water per second $= \frac{W}{g} = \frac{62.5\,V_1F}{g}$

V_1 = velocity of entering water, absolute.

U = velocity of the vane.

θ = angle between path of water at discharge and the direction of motion of the vane.

F = area of jet, in square feet.

If r = radius of wheel measured to point where the jet strikes the buckets.

Pr = torque, or turning moment, on wheel.

and $Pr\omega$ = foot-pounds of energy, per second

$\omega = 2\pi n$.

n = revs. per sec.

$$\text{Horsepower} = \frac{Pr\omega}{550} = \frac{62.5\,V_1Fr\omega}{550\,g}\,[(V_1 - U)\,(1 - \cos\theta)] \qquad (263)$$

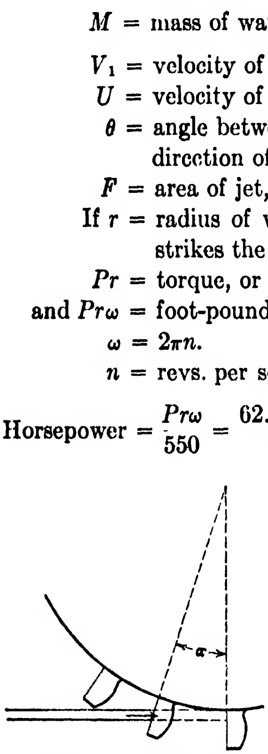


FIG. 217.—Diagram for location of α.

These are the approximate formulæ for the tangential impulse wheel.

The formulæ given are inaccurate in that they assume:

1. The angle of the entering water, α, is zero, and $\cos\alpha = 1$. The error thus introduced is, usually, very small. Fig. 217 shows the approximate value of α which is to be taken as the angle between the point on the circumference corresponding to the place where the jet first strikes, fully, against the bucket, and the point at which the axis of the jet is perpendicular to the wheel radius.

2. The hydraulic friction loss in the buckets is appreciable.

This is denoted by $\frac{kv_2^2}{g}$, in which k is an empirical constant ranging in value from 0.5 to 1.5, depending on the design of wheel.

The more accurate formula for the power is

$$\text{Hp.} = \frac{62.5V_1Fr\omega}{550\,g} \times \left[V_1 \cos\alpha - U - \left(\frac{\cos\theta}{\sqrt{1+k}}\right)\sqrt{V_1^2 + U^2 - 2V_1U\cos\alpha}\right] \quad (264)$$

As a matter of fact, the angle α is not definitely determinable for an impulse wheel.

The jet first strikes the lower edge of a bucket, and, as the wheel rotates, more and more of the bucket comes into the field of the jet, until a position is reached at which the maximum jet area acts against the bucket. The water then begins to be cut off from the bucket by the one behind it coming into the field of the jet and beginning to take a portion of the water, which increases with the moment of the wheel, producing a corresponding diminution in the amount of water which reaches the first vane. Finally, a position is reached at which the water is completely cut off from the first bucket. But the impulse of the water does not cease immediately, The time during which a vane receives water from a jet is so short, that the time of the passage of the water through the bucket is appreciable as compared with it. Hence, the bucket continues to discharge water for a short time after the supply to it has ceased. Obviously, the higher the speed of the wheel, the greater will be the distance through which the discharge continues. All of which indicates the difficulty in fixing exactly, the entrance angle of the jet. Daugherty suggests[1] that the mean point between the positions where the water first begins to impinge on a vane, and where the last particle of discharge emerges from the bucket, be taken as the point of initial impact. It appears, however, more logical to adopt the point where the jet first strikes fully against the bucket as the one by which the entrance angle α, is fixed.

Details of Construction

Buckets.—All tangential impulse wheel buckets have the "splitter" or backbone, and the differences are, principally, in the form of working surface. In the original "Pelton" bucket, the curves were circular arcs, which is also true of all other buck-

[1] R. L. Daugherty: "Hydraulic Turbines."

ets. The original Pelton bucket is semi-cylindrical in form, so that a section cut through it at any point parallel to the backbone, is a straight line.

The Doble bucket differs from the Pelton in that it is curved in both directions. A section transverse to the backbone is a circular arc, while a section parallel to the backbone is also curved, being of an elliptical form, so that the working surface is more nearly normal to the jet at every point in its rotation within the field of the jet. The Doble bucket has part of its outer edge cut away near the middle, so that the jet can continue to strike against a bucket after it has travelled to a considerable distance towards the point tangent to the jet, without being intercepted by the next succeeding bucket, the jet passing through the trench made in the bucket. This does not mean that a bucket of this kind is acted on by the jet any greater length of time than any other form of vane, for though it can travel a greater distance toward the rear without interruption, it does not come within the path of the jet as early as the uncut buckets. The object of this form of bucket is to have the jet move it while it is in a position most nearly perpendicular to the jet and moving over the path parallel to the axis of the jet—in other words, to reduce the angle of entry, α, to zero, or as nearly zero as is physically possible.

The buckets are made of cast iron, cast steel or bronze, ground or machined to give a smooth working surface, and a sharp "splitter." The under portions which abut against the wheel rim are machined to fit in place, and the buckets are fastened in position by bolts passing sidewise—or parallel to the axis—through the rim.

The number of buckets varies, 12 being the minimum for any size of wheel. As many as 24 buckets have been used on large wheels, but 18 to 20 seems the usual practice. Of course, this is partly dependent on the size of the jet, one of large diameter usually requiring a greater number of buckets than one of small diameter. There is, however, no direct proportion between the number of buckets and the wheel diameter.

For the best efficiency, the area of the jet should not exceed 0.1 times the projected area of the bucket, and the diameter of the jet should not be greater than 0.3 times the width of the bucket. The best ratio of diameter of jet to width of bucket seems to lie between 0.13 and 0.15.

Size of Jet.—The size of the jet is fixed by the head and power to be delivered. The quantity of water available is known. The pipe line losses from the source of supply to the nozzle can be computed. Then the velocity is

$V_1 = 0.96\sqrt{2gh}$, in which h is the net head at the nozzle.

$$Q = V_1F \text{ and } F = \frac{Q}{V_1} = \frac{\pi d^2}{4}$$

Whence, the diameter of the jet is

$$d = \sqrt{\frac{4Q}{\pi V_1}} \text{ ft., or } d_1 = 4.88\sqrt{\frac{Q}{\sqrt{h}}} \text{ inches.} \tag{265}$$

Also, since horsepower $Hp. = \frac{Qh}{11}$ (approx., for eff'y = 80 per cent.)

$$d_1 = 16.2\sqrt{\frac{Hp.}{h^{3/2}}} \text{ in.} \tag{266}$$

h is taken in feet for all formulæ.

The largest size of nozzle, at present in use, is 10½ in. in diameter.

Diameter of Wheel.—By "diameter of wheel" is meant the diameter of the circle to which the axis of the jet is a tangent.

The diameter is limited by the fact that the most efficient velocity of the bucket (midpoint) is from 0.45 to 0.47 times the velocity of the jet, or, taking 0.46 as the average, the linear velocity of the bucket is $U = 0.96\sqrt{2gh} \times 0.46 = \frac{\pi Dn}{60}$, n being the number of revolutions per minute, and D the diameter, in feet. Hence

$$D = \frac{67.5\sqrt{h}}{n} \text{ feet} \tag{267}$$

The minimum diameter is fixed by the condition that it must be at least ten times as great as the diameter of the jet. Where this condition conflicts with that for best efficiency, the stream must be divided into two or more jets.

Figure 218 is a chart from which all the factors of size of jet, its velocity, size of wheel and its speed can be taken by inspection.

The lines, beginning at A, bottom of chart, show the manner in which it is used to solve a problem.

Given a head of 440 ft., and the power to be developed, 220 hp.

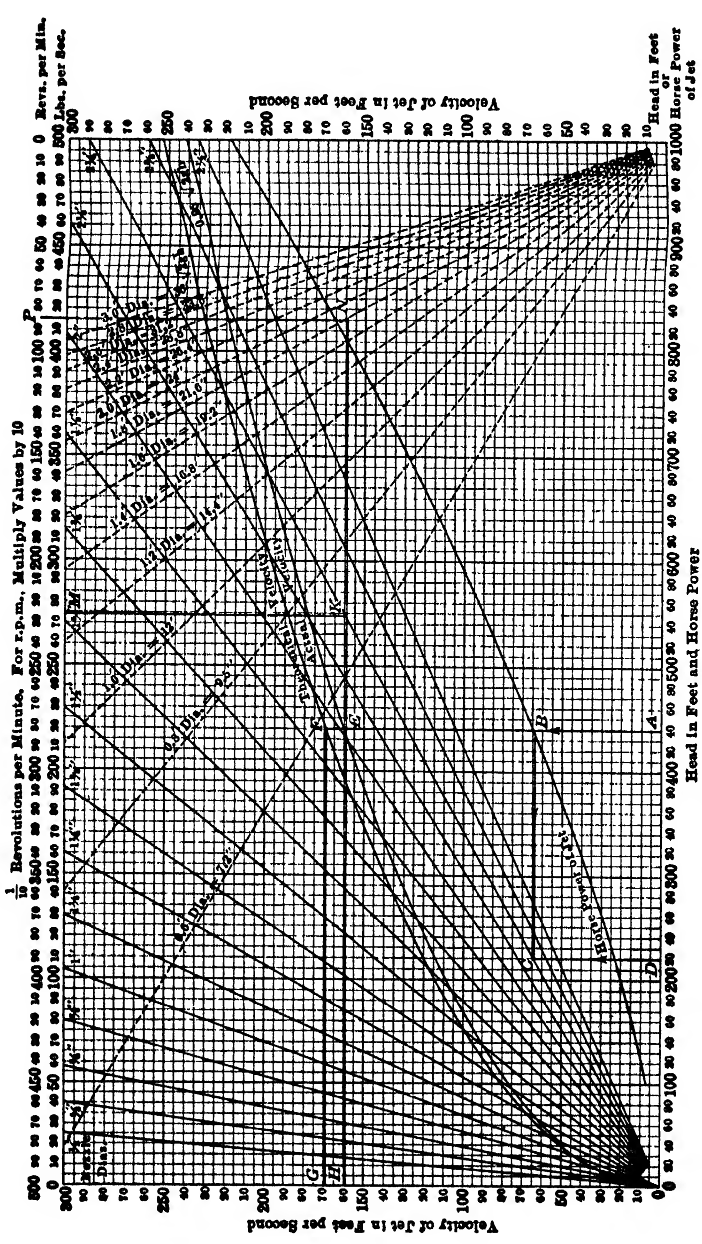


Fig. 218.—Chart for computation of impulse wheels.

From the bottom line of the chart at 440, marked A, follow vertically to the curve marked "Horsepower of Jet" (point B). Then move horizontally toward the left an indefinite distance.

From a point at the bottom of the chart, at 220 (point D), follow vertically upwards until this vertical intersects the indefinite horizontal line (point C). This intersection will fall on, or near, one of the diagonals giving the diameter of the jet—in this case 2¼ in. If the intersection does not fall on a diagonal, the size of jet is found by interpolation.

Going back to the starting point, A, (head), continue the vertical line until it intersects the velocity curves, one of which is the theoretical, the other, the actual, being corrected by the coefficient $C = 0.95$. Horizontal lines drawn from these points out to the scale on the left will give, directly, the value of the velocity of the jet. The head taken for the chart is the net head at the nozzle, and not the total head.

From the point of intersection of the vertical at A with the actual velocity curve (point E), follow horizontally to the right an indefinite distance. From the scale of wheel speeds, at top of chart take a vertical downward, beginning at a point where the desired speed is located—in this case 850 r.p.m. The intersection of this vertical, with the horizontal from the jet velocity curve, will fall on, or near, one of the diagonals representing wheel diameters. In this case the intersection is at N and it falls on a diameter of 1.8 ft. Where the intersections do not fall on the diagonals, interpolation can usually be made, with sufficient accuracy, by the eye.

The quantity of water is found by taking a vertical line to the scale at the upper edge, marked "lb. per sec.", the vertical starting at the point where the horizontal line, drawn at the elevation of the actual velocity, intersects the diagonal line which corresponds with the size of nozzle. For this case, the point of intersection is marked K, and the vertical from K to M cuts the scale of "lb. per sec." at 274 lb., or 4.38 cu. ft. per second.

This chart is useful for finding the values of V and h, where either is given to find the other.

In order to make the diagram applicable to all conditions, the values are all theoretical, and must be corrected for efficiency. For instance, the size of nozzle and quantity of water are based on the formula: Horsepower $= \frac{QH}{8.8}$, which is correct only when

there are no losses, and the efficiency is 100 per cent. Also, the diameter of the wheel is based on the velocity of its periphery being one-half the velocity of the jet, when the actual velocity of the wheel rim should be from 0.45 to 0.47 that of the jet. Hence, this chart gives the theoretical and uncorrected factors only.

Casing.—The wheel chamber, or casing, for an impulse wheel is not an essential part of the machine, and its form and character have no bearing on the efficiency of the wheel. A tangential jet wheel works just as well without any casing as with one. The casing is simply a covering for the wheel—usually of thin steel plate—to prevent splashing of the discharge from the buckets. Figs. 219 and 220 show, respectively, the longitudinal

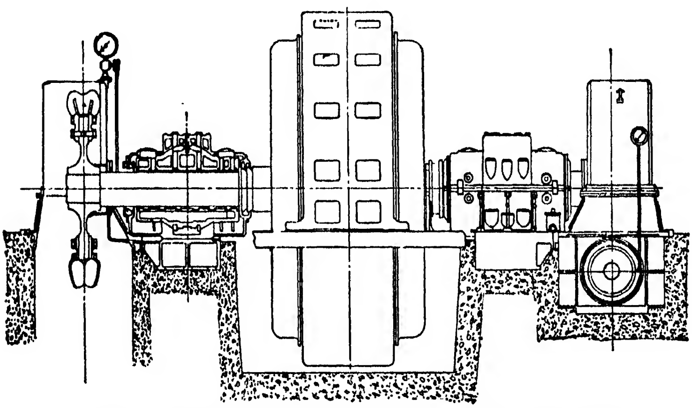


FIG. 219.—Double overhung impulse wheel, direct-connected to generator.

and transverse elevations of two impulse wheels. Fig. 219 shows a pair of "overhung" wheels driving a generator on the shaft between the wheels.

Regulation.—The regulation can be effected only by varying either the position of the nozzle, or the area of the jet. If the first means is employed, the flow of water is constant for a variable power output and is, at all times, equal to the quantity of water required for the maximum power. This is too wasteful for any hydro-electric plant and is applicable only to certain conditions where the water supply available is in excess of the amount needed for power.

To vary the area of the jet, and, at the same time, leave

its axis and discharge coefficient both constant the needle nozzle was devised. A cross-section of this nozzle is shown in Fig. 221, and Fig. 222 is the picture of a jet issuing from it. The principle is obvious. The "needle," is an interior stem,

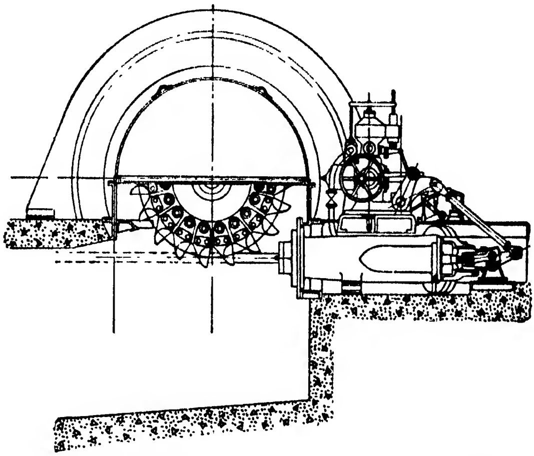


FIG. 220.—Impulse wheel with governor controlling needle nozzle.

circular in cross-section at any point in its length and having, at one end, a gradually increasing diameter, which, on attaining its maximum value, is then gradually decreased until a minimum diameter is reached which is very nearly a point. This

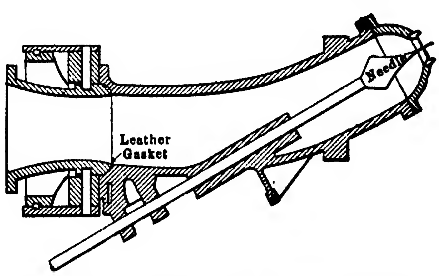


FIG. 221.—Section through needle nozzle.

stem, or "needle," is set centrally, inside the nozzle, and arranged to be moved longitudinally, so that a greater, or less, length of it may protrude outside the nozzle and thus vary the area of the jet. The needle may be moved either manually, or by an automatic speed governor.

Where constant speed is required, and the changes in the power delivered to the wheel must be quickly effected, a combination of the needle nozzle and the jet deflection is sometimes used. If a sudden diminution in load should occur, the needle can not instantly be thrust forward to decrease the area of the jet, because the result would be an increase in velocity through the diminished area, which would, for a few seconds, give more energy to the wheel, instead of less.

The pressure set up in the pipe line leading to the nozzle, by a sudden reduction of the nozzle area, due to the stored

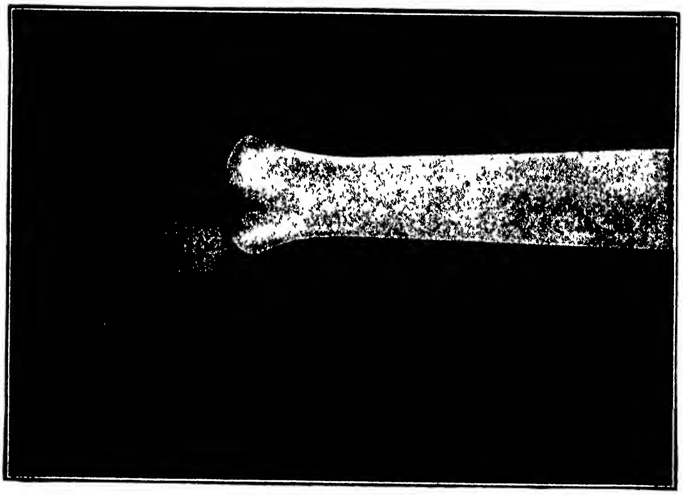


FIG. 222.—Jet issuing from needle nozzle.

energy in the mass of water moving in the pipe, may produce dangerous bursting stresses, and will certainly increase the velocity through the reduced area of opening, the velocity becoming $V = \sqrt{2g(h + h_a)}$ in which h_a is the head that is equivalent to the pressure set up, and equal to $\frac{p}{62.5}$.

The nozzle may be deflected instantly, so that only a portion of the jet strikes the wheel buckets. This gives the necessary quick change in the energy imparted to the wheel. Then the needle may be moved, slowly, to reduce the area of the jet, the rate of motion being so slow that the velocity of the water in the pipe is gradually retarded and no appreciable pressure is

produced. The mechanism which moves the needle, moves the nozzle also, at the same rate, so that as the jet area decreases, the proportion of the total jet which strikes the buckets becomes greater and greater, the final position being the normal one for the jet with its area diminished to correspond with the change in load.

There can be, however, no converse operation of this combined system. An increase in load is not met by any movement of the nozzle, as no other position can be given it that will help the condition. The only possible movement is that of the needle which travels back to increase the nozzle area, but the quantity of water flowing through the nozzle cannot be increased until the whole column of water in the pipe line has become accelerated.

Instead of a combination of deflecting and needle control, a bypass valve is sometimes used. The bypass valve for high heads is usually made in the form of a needle nozzle, similar to the main working nozzle. When the needle in the main nozzle moves to reduce the flow of water through it, the bypass nozzle, which is normally kept closed, opens at the same rate as that with which the main nozzle is closed. The total area of opening for the passage of water remains constant during the period of governing. After movement of the two needles is completed, the bypass nozzle returns slowly to its normal, closed position. The two needles are usually operated by the water-wheel governor.

This subject is treated more fully in the chapter on "Speed Regulation of Water Wheels."

A modification of the deflecting nozzle is the so-called "rotating nozzle" of the Allis-Chalmers Co. This is a deflecting nozzle which has its moving portion and center of oscillation near the discharge end of the pipe. Also, the nozzle pipe is rigid and may be permanently fastened to the wheel housing.

Figure 223 shows the parts of this nozzle. A dome-shaped cover is bolted to the stationary nozzle pipe by a flanged joint. At two diametrically opposite points of the joint, a bearing is made and bushed with bronze. A deflecting hood, which is provided with a pair of trunions, is placed inside the dome, its trunions resting in the two bearings. The deflecting hood is a heavy steel casting, having cast integral with it the trunions and, also, the bottom webs which form a guide for a needle nozzle

stem. The front portion of the hood is spherical and fits into the concave portion of the dome cover, both surfaces being machined. The joint between them is made tight by means of a leather packing ring interposed between the hood and the dome. This ring is held in place by a cover ring, bolted to the front end of the dome and concentric with the hole through the dome, the hole through the cover plate being of considerably greater diameter than the maximum size of the jet issuing from the hole in the hood.

If a projecting end of one of the hood trunions be rotated, the jet will be deflected.

The needle stem is provided with a universal joint so that the area of the jet can be altered in any position of the hood, and the discharge nozzle is always concentric with the needle.

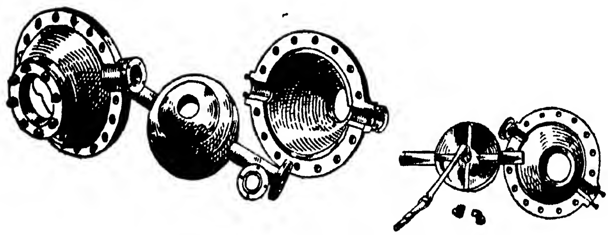


FIG. 223.—Rotating nozzle.

The cast-steel hood, together with the needle, constitute the only parts to be deflected, and the angular motion of the trunnions is considerably greater than that of the standard deflecting nozzles. Direct connection between the trunions and the governor is, therefore, possible, together with a more thorough lubrication of the surface of the trunions. The leather packing is relatively small and can be renewed without dismantling any other parts of the nozzle. The packing is located in a position where it is not affected by sand, as is the case with the standard deflecting nozzle, where a large leather ring is placed at the joint between the deflecting nozzle and the stationary swivel head of the penstock. Since full penstock pressure is exerted between the dome and the hood, except over the small area covered by the leather packing ring, the nozzle is nearly in hydraulic balance and can be easily moved.

The needle stem is moved by hand, or automatically, as may be desired.

CHAPTER XI

SPEED REGULATION OF WATER WHEELS AND ABNORMAL PENSTOCK PRESSURES

Speed Regulation of Water Wheels.—The problem of speed regulation of water wheels is a complex one when the wheel is fed by a pipe line, and is in an enclosed pressure chamber, because any load changes produce changes in: speed; flywheel energy; gate opening; pressure in pipe line and wheel chamber, and all these factors are interdependent.

For wheels set in open penstocks the question is simply a flywheel problem and the solution easy and direct.

The different factors will first be discussed separately and then combined to derive the complete results and formulæ.

(1) **Energy Delivered by Flywheels.**—A rotating body having a moment of inertia I[1] and making n_1 revolutions per second, has stored in it an amount of energy equal to

$$E_1 = \frac{2\pi^2 I n_1^2}{g} \text{ ft.-lb.}$$

If the speed be changed to some other value, as n_2, the energy in the mass will be

$$E_2 = \frac{2\pi^2 I n_2^2}{g} \text{ ft.-lb.}$$

The difference between these two values of E_1 and E_2 is

$$E_1 - E_2 = \frac{2\pi^2 I(n_1^2 - n_2^2)}{g} \text{ ft.-lb.} \qquad (268)$$

This represents the energy added to or subtracted from the wheel to cause the change in speed. If n_1 is greater than n_2 the total quantity is positive, showing a reduction in

[1] The definition of moment of inertia is sometimes given in *weight units* and sometimes in *mass units*, and confusion arises, at times because of these two different definitions. In this discussion, I is always taken in *weight units.*

speed and the energy was absorbed from the flywheel in doing external work.

If $E_1 - E_2$ is negative, showing increase in speed, the energy was added to the wheel from an external source, the additional energy being stored in the wheel at the higher speed.

Since 1 hp. = 550 ft.-lb. per second, the horsepower stored in, or given up, by a flywheel is

$$Hp. = \frac{2\pi^2 I(n_1^2 - n_2^2)}{550\,g} = 0.001114\ I(n_1^2 - n_2^2) \qquad (269)$$

if $(n_1 - n_2)$ is the changed value of the speed which occurs in 1 sec. If the change in speed from n_1 to n_2, takes place in T sec., then the rate of work, per second, is

$$Hp. = \frac{0.001114\ I(n_1^2 - n_2^2)}{T} \qquad (270)$$

When the load on the water wheel is changed from $(Hp.)_1$ to $(Hp.)_2$, without an instantaneous change in the power of the water wheel, the flywheel effect of the moving parts must supply energy to carry the increased load over the period of time required for the governor to act and increase the power delivered to the wheel to conform with the new load.

At the instant when this occurs, the load carried by the flywheel is

$$(Hp.)_1 - (Hp.)_2$$

As the power of the water wheel gradually increases, more and more of the load increase is carried by it, the power absorbed from the flywheel gradually diminishing until, at the end of the time required for the power of the water wheel to become equal to the new load, there is no power absorbed from the flywheel.

Hence, the *average* rate at which power is delivered by the flywheel over the time during which it gives out energy

$$\frac{(Hp.)_1 - (Hp.)_2}{2}$$

Put $\delta P = (Hp)_1 - (Hp)_2$

Equating this to the horsepower delivered, or absorbed, by a flywheel, in terms of its constants and speed,

$$\frac{\delta P}{2} = \frac{0.001114 I(n_1^2 - n_2^2)}{T} \qquad (271)$$

From this equation, the following are derived by transposition and combination of constants.

$$I = \frac{450\delta PT}{(n_1^2 - n_2^2)} \tag{272}$$

$$T = \frac{(n_1^2 - n_2^2)I}{450\delta P} \tag{273}$$

$$\delta P = \frac{I(n_1^2 - n_2^2)}{450T} \tag{274}$$

$$n_2 = \sqrt{n_1^2 - \frac{450\delta PT}{I}} \tag{275}$$

Note that n_1 and n_2 are revolutions *per second* $= \frac{N}{60}$ when $N =$ revolutions per minute.

For *reduction* in load,

$$n_2 = \sqrt{n_1^2 + \frac{450\delta PT}{I}} \tag{275a}$$

The moment of inertia of generators is usually given by the manufacturers. It may be computed for any rotating body as follows:

(*a*) I for an annular ring rotating about its center, *i.e.*, flywheel rim, is

$$I_1 = W_1 \left[\frac{r_1^2 + r_2^2}{2} + r^2\right] \tag{276}$$

W_1 = weight of ring, in pounds.
r_1 = inner radius of ring, in feet.
r_2 = outer radius, in feet.
r = mean radius $= \frac{r_1 + r_2}{2}$, in feet.

(*b*) I for straight rod revolving about one end, *i.e.*, spokes of wheel, is

$$I_2 = W_2\left[\frac{l_1^2}{12} + r_3^2\right] \tag{277}$$

W_2 = weight of all the spokes.
l_1 = length of spoke (or rod), in feet.
r_3 = radius from center of rotation to middle of spoke (or rod), in feet.

If spokes are of variable cross-section, the measurement for r_3

should be from the center of rotation to the center of gravity of the spoke. This gives a nearer approximation.

For a flywheel, the total moment of inertia is $I_1 + I_2$, *i.e.*, the sum of the moments of inertia of the rim and of the spokes.

The flywheel effect for hydro-electric generating units is furnished by the revolving field of the generator. In this case,

$$I = W_1\left[\frac{r_1^2 + r_2^2}{2} + r^2\right] + W_2\left[\frac{l_1^2}{12} + r_3^2\right] + W_3\left[\frac{l_2^2}{12} + r_4^2\right] \quad (278)$$

W_3 = total weight of all magnets.
l_2 = length (radial) of magnet.
r_4 = radius from center of wheel to middle point of magnet.

The last term of the equation is the moment of inertia of the magnets on the outside of the rim of the rotor. Obviously, the second and third terms of the equation are similar, both being for rods or spokes revolving around their ends.

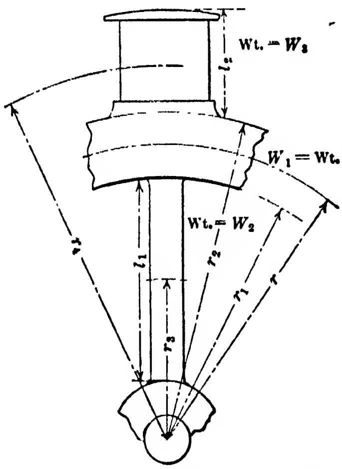


FIG. 224.—Diagram showing W and r for moment of inertia formulæ.

Figure 224 shows the several elements used in the formulæ.

If the moment of inertia of a generator rotor is to be computed, the weight of the field magnets is found by unbolting one from the rotor rim and weighing it.

The weight of the rim is then computed from the formula

$$W_1 = 480t\pi(r_2^2 - r_1^2) \text{ lb.} \quad (279)$$

if the measurements are in feet, or

$$W_1 = 0.27t\pi(r_2^2 - r_1^2) \text{ lb.} \quad (279a)$$

if the measurements are in inches.

t = width of rim face.

The weight of the hub is then computed from the same formula, the inner and outer hub radii being taken. Call this W_h.

Then the weight of the spokes, W_2, will be

$W_2 = W - (W_1 + W_h + W_3)$, in which W is the total weight of the rotor.

The flywheel effect of the shaft and hub are usually neglected, as is also that of the water wheel, though the water wheel has an appreciable value of I. Omitting it from the computations produces a factor of safety in the results derived for regulation.

Example of the Use of the Foregoing Formulæ.—Consider a water-power unit, the water wheel set in an open penstock.

Let time in which governor operates = 3 sec.

Maximum load change from 1000 to 3000 hp.

Normal speed 240 r.p.m.

Find what the moment of inertia of the rotating parts must be to keep the speed within 5 per cent. of normal, for the maximum change in load.

$$(Hp.)_1 - (Hp.)_2 = \delta P = 1000 - 3000 = -2000$$

$$n_1 = \frac{240}{60} = 4 \text{ revs. per sec.}$$

$$n_2 = 4 \text{ less 5 per cent.} = 3.8 \text{ revs. per sec.}$$

$$I = \frac{450 \times (-2000) \times 3}{[(4)^2 - (3.8)^2]} = -1{,}730{,}770 \text{ lb.-ft.}^2$$

The negative sign has no significance in computing I.

This result is not strictly correct, as the time which elapses between the instant of load change and that when the governor begins to move is neglected. The computations must be corrected for this as will presently be shown.

Consider another case, in which the maximum instantaneous change in power is from zero to 4000 hp.; normal r.p.m. = 300. I = 700,000 lb.-ft.² Required change in speed if governor acts in 2.5 sec.

$$n_1 = \frac{300}{60} = 5 \text{ revs. per sec.}$$

$$\delta P = 4000$$

From formula 275

$$n_2 = \sqrt{(5)^2 - \frac{450 \times 4000 \times 2.5}{700{,}000}} = 4.309 \text{ r.p.s.} = 258.5 \text{ r.p.m.}$$

Per cent. change in speed is

$$100\left(\frac{5 - 4.309}{5}\right), \text{ or } 100\left(\frac{300 - 258.5}{300}\right) = 13.83 \text{ per cent.}$$

As a quick approximation, the regulation can be computed on a slide rule from the equation.

$$\text{per cent. regulation} = \frac{\delta P \times 81 \times 10^6 T}{IN^2}.$$

δP = *change* in horsepower = $(Hp.)_1 - (Hp.)_2$
N = revolutions per *minute*, normal.
I = moment of inertia of the rotating parts.

This equation is not sufficiently accurate where the regulation exceeds 6 per cent.

It is to be noted that the flywheel effect of power stations is not limited to that of the generating units. Every motor driven from the plant, and all machinery driven by the motors, add their flywheel effects to that of the generating units.

This explains the reason why the speed regulation of a plant is so much better when supplying current to a commercial load, than when under test and the current used up in rheostats.

Acceleration of Moving Column of Water.—The foregoing examples were conditioned on the water wheels being set in open penstocks, in which case, the quantity of water will, almost instantaneously, change with change of gate opening, provided the velocities in the wheel chamber are less than 4 ft. per sec.

If a water wheel be set in an enclosed pressure chamber and fed through a pipe line, a sudden change in gate opening is not accompanied by an immediate change in the power delivered to the wheel. The velocity of the moving mass of water in the pipe can not be changed instantly, except with the application of infinite forces. If a given quantity of water is passing through the water-wheel gates, the velocity through the pipe corresponding with the delivery of this quantity, and the gates be suddenly opened due to a sudden increase in load, the immediate consequence is that the quantity of water is unchanged and, as the gate opening is greater, the velocity of entry, v_1, to the wheel is diminished, so that instead of an increase in power to accord with the increase in load, the power is actually decreased.

The column of water begins to accelerate, and within a short time, its velocity through the pipe increases, and the greater quantity of water required for the increased load is supplied to the water wheel, the velocity through the wheel gates rising, until the normal value is again reached.

Conversely, if the load be suddenly diminished, and the governor quickly moves the turbine gates toward partial closure, the quantity of water is not instantly reduced, but immediately after the gate openings have been reduced the same quantity of water flows through them at a higher velocity than normal, so that the power of the wheel, for a short time, is increased. Under certain conditions which will be discussed, excessive pressures may be produced in the pipe line by very quick closing of the gates.

The quick opening of the water-wheel gates may produce a vacuum in the pipe line and consequent collapse of the pipes, while quick closing of the gates may produce the so-called "water hammer."

The force required for the acceleration of any mass is:

Force = mass $\times$ acceleration.

Let P = force in pounds, averaged over the whole time during which it acts $= \frac{P_{max}}{2}$ for uniform acceleration, P_{max}, being the initial force set up which gradually and uniformly reduces to zero.

W = weight of water in pipe line in pounds.

V_1 = velocity of water in pipe line before change in gate opening.

V_2 = final velocity after gate opening.

T = time in seconds required for velocity to change from V_1 to V_2.

Then

$$P = \frac{W}{g} \times \frac{V_1 - V_2}{T} \text{ lb.} \tag{281}$$

$$W = 62.5\pi R^2 L$$

R = radius of pipe, in feet

L = length of pipe, in feet

Then

$$P = \frac{62.5\pi R^2 L}{g} \times \frac{(V_1 - V_2)}{T} \text{ lb.} \tag{282}$$

P is the total pressure (average) required to accelerate the column of water.

The pressure per square foot of cross-section of the column is

$$p = \frac{P}{\pi R^2}$$

$$P = p\pi R^2 = \frac{62.5\pi R^2 L}{g} \times \frac{V_1 - V_2}{T}$$

Whence

$$p = \frac{62.5}{g} \times \frac{(V_1 - V_2)L}{T} \tag{283}$$

All the water pressures must be referred to water head. The head corresponding to p is

$$h_a = \frac{p}{62.5}$$

$$62.5h_a = p = \frac{62.5}{g} \times \frac{(V_1 - V_2)L}{T}$$

Whence

$$h_a = \frac{(V_1 - V_2)L}{gT} = \frac{0.0311L(V_1 - V_2)}{T} \tag{284}$$

$$T = \frac{0.0311L(V_1 - V_2)}{h_a} \tag{285}$$

$$V_a = \frac{h_a gT}{L} \text{ ft. per second} \tag{286}$$

h_a is the average of the head required for accelerating the column, L ft. long, T the time in seconds required to produce the acceleration from V_1 to V_2 with the force h_a acting on the water column, and V_a is the velocity, in feet per second, produced by the accelerating head h_a.

The net head acting on the water wheel, when equilibrium is established, is

$$h_n = H - h_f \tag{287}$$

h_n = net head.
H = total, or gross, head.
h_f = loss of head due to entry, friction and discharge velocity.

In order to eliminate the factors of mechanical friction in the unit, and efficiency of the water wheel at different loads, the velocities of flow in the pipe will not be assumed as proportional to the load, but the actual quantities of water required at different loads are used to fix the change in velocity.

Thus, for a given generating unit, if the quantity of water for 1000 hp. = Q_1 and that for 3000 hp. = Q_2, then the velocities corresponding to Q_1 and Q_2 give, directly, the values of V_1 and V_2, including efficiency and mechanical losses. Q_1 and Q_2 are always known from the combined efficiency curve of water wheel and generator.

To determine h_a and the time of acceleration, T, it is necessary

to compare the velocities through the wheel gates for different areas of opening. Before increasing the opening of the gates, and while all the forces are in equilibrium, the head acting to discharge water through the gates is h_{n1} and the velocity with which the water passes through the gates is $v_1 = c\sqrt{2gh_{n1}}$. On the instant of opening the gates wider, the velocity through them falls, due to the greater opening for the same quantity of water, so that the head necessary to pass the water through the gates at the diminished velocity is correspondingly reduced.

To illustrate this, if the gates are opened enough to admit 100 cu. ft. per second under a head of 100 ft. and there is a friction-head loss of $h_{f1} = 0.2$ ft. in the pipe, for this value of Q_1 the net head h_{n1} is practically 100 ft., and the velocity through the gates is, $v_1 = c\sqrt{2g \times 100} = 68$ ft. per second for $c = 0.85$. Consequently, the area of the gate opening is $\frac{100}{68} = 1.47$ sq. ft. If the gates be opened to admit 600 cu. ft. sec., the friction loss will be, practically, $h_{f2} = \frac{0.2 \times (600)^2}{(100)_2} = 7.2$ ft., and the net head, h_{n2}, ultimately becomes 92.8 ft. Hence, the area of gate opening must be $\frac{600}{0.85\sqrt{2g \times 92.8}} = 9.75$ sq. ft.

Velocity through the gates, at the instant after opening, is $v_e = \frac{100}{9.75} = 10.23$ ft. per second. Head to produce velocity v_e through the gates is $h_e = \frac{(10.23)^2}{2gc^2} = 2.25$ ft., or less than 3 per cent. of the total head.

Obviously, the remaining part of the total head, or 97.75 ft. in this case, is the initial accelerating head, which gradually reduces to zero as the velocity of the water in the pipe increases, so that the *average* accelerating head for uniform acceleration, is,

$$h_a = \frac{h_{n1} - h_e}{2} \tag{288}$$

These considerations are modified by the fact that no governor opens the gates instantly, a length of time of 2 to 3 sec. being necessary. Hence, the acceleration of the water begins before the final position of the gates is reached.

In practice, however, no appreciable error is introduced by assuming that the gates are opened instantly, and the time of acceleration is that due to instant change in conditions, begin-

ning with motion of the governor. It is understood that the time of governor action is considerably less than the time required for accelerating the water. Of course, if the time period of action of the governor is longer than the normal time for accelerating the water, the actual time of acceleration will be increased. The time of governor action should not be longer than 30 per cent. of the time of natural acceleration, for these approximate formulæ to hold.

Except for the one condition of complete closing of the gate, these statements all apply equally to reduction in gate area. With reduction in area, a pressure is produced in the penstock which retards the water and reduces its velocity. For this condition, all the formulæ apply and the computations are the same, except that in some of them, the signs are changed. This, however, is so obvious that in only one or two cases have they been repeated to show this change in sign.

The assumption that the gate area is instantly reduced is also adopted for computation of speed regulation, and the results are accurate enough for any practical purpose. It is understood, however, that actual, instantaneous gate closure will produce "water hammer," as elsewhere discussed.

As close, practical formulæ, the following are given:

h_{n_1} = net head on wheel before change in gate opening $= H - h_{f1}$

h_{n_2} = net head on wheel after change in gate opening $= H - h_{f2}$

H = total head.

h_{f_1} and h_{f_2} are friction-head losses before and after gates are opened, respectively, and when equilibrium is established.

v_1 = velocity through guide vanes of wheel before change in gate opening.

$$v_1 = c\sqrt{2gh_{n_1}} \tag{289}$$

c = constant of discharge, = 0.85 to 0.90

v_e = velocity through guide vanes immediately after gate opening—gates assumed moved to new position instantly.

$$v_e = \frac{v_1 A_1}{A_2} \tag{290}$$

$$h_e = \frac{v_e^2}{2gc^2} \tag{291}$$

A_1 = area of opening of gates prior to change.

A_2 = area of opening of gates after change $= c\dfrac{Q_2}{\sqrt{2gh_{n_2}}}$ (292)

Q_1 = cubic feet, per second, of water delivered to turbine before change in gate opening.

Q_2 = cubic feet, per second, delivered after change in gate opening.

$$h_a = \tfrac{1}{2}\left(h_{n_1} - \frac{v_e^2}{2gc^2}\right) \tag{293}$$

V_1 = velocity of water in the penstock, in feet, per second, before the change in gate opening.

V_2 = velocity in penstock after gate opening and when equilibrium has been established.

A = area of penstock, in square feet.

The foregoing formulæ do not apply to the special case of complete closure of the turbine gate, which is separately treated in the paragraph on "Water Hammer."

Effect of Change in Head on Water Wheels.—Under ordinary conditions of operation, the power of a water wheel varies with the $\frac{3}{2}$ power of the head. This law holds only for the condition that the speed of the wheel changes as the square root of the head, which does not obtain for changes of head due to acceleration or deceleration of the velocity of the mass of water in a penstock, because the speed is maintained substantially constant by the flywheel effect during the time the water velocity is changing.

In order to determine the actual power delivered by the wheel, where the head of water and speed of the turbine are known, the formulæ for computing the horsepower of water wheels, which are given in a previous chapter, may be used. This is not necessary, however, as the power can be approximated with sufficient accuracy by taking it as equal to 92 per cent. of the power produced by the changed head and unchanged quantity of water, the power being taken as proportional to the head. Expressed algebraically,

$$H_1p_1 = \frac{0.92\,Hp_1h_e}{h_{n_1}} \tag{294}$$

H_1p_1 is the minimum power delivered by the wheel at the instant of changing the gate opening, Hp_1 is the power delivered by the wheel before change in gate opening, h_e, the head at the wheel at the instant after change in gate opening and h_{n1}, the head at the wheel before change in gate opening.

Thus, if the power at 100-ft. head (net) were 1000 hp. and the

gates were suddenly opened, so that the drop in head (*i.e.*, the accelerating head) were 70 ft., the remaining net head, for the instant, would be 30 ft. and the approximate power, at the instant, would be $\frac{30 \times 1000 \times 0.92}{100} = 276$ hp.

In making computations for determination of speed control, accuracy is not obtained by any exact calculation of the change in power of the wheel, because the moments of inertia of some of the parts are not considered, and, in any case, there are always some partially indeterminate factors. For this reason the foregoing approximation is sufficiently close to fulfil every requirement of the problem.

Time Period of Governor.—Whenever a change in load occurs, the governor does not instantly respond. All governors, now on the market, are speed-controlled devices, and a change in speed must take place before the governor begins to move the gates. The time required to move the gates is nearly constant, whether for short or long movements of the gate shaft. The governor, therefore, has two time periods; one, that which elapses from the time of load change until the mechanism begins to act, the other, the time of gate movement.

Most governors are adjusted to move the gates before the change in speed has reached 1½ per cent. of normal. Some of the best governors will operate on a speed change of 0.5 or 0.6 per cent. of normal. The time element for this speed change is variable, because the change in speed with time varies with the change in load. Since, however, the percentage of speed change before operation begins is known, this time element need not be known, as it can be carried in the problem as a fixed change. From the changed speed at the time gate moving begins, as a starting point, the other factors may be computed.

For instance, if the normal speed of a wheel is 300 r.p.m. or 5 r.p.s. under a given load, and the governor begins to operate on a 1.5 per cent. speed change, and the load be suddenly increased, the speed will drop to 5 − 1.5 per cent. = 4.925 r.p.s. before motion of the gates begins to take place. Taking this as the starting point, n_1, as used in the formulæ becomes 4.925 instead of 5. With this value, the computations are made and the final result referred to 5 in determining the total change in speed.

The application of the formulæ is best shown by a practical example.

Consider a plant in which the following conditions obtain for each generating unit:

Head	100 ft.
Max. load on unit	5000 hp.
δP = max. load change	3000 hp.

n_1 = normal speed = 240 r.p.m. = 4 r.p.s.
L = length of pipe line = 1200 ft.
Diameter of pipe = 10 ft.
A = area of pipe = 78.54 sq. ft.

Water-wheel efficiency:

¼ gate	60 per cent.
½ gate	70 per cent.
¾ gate	82 per cent.
Full gate	80 per cent

Governor to operate on a change of 1 per cent. in speed.
Area of draft tube at discharge end = 150 sq. ft.
Consider a change of load from 1000 to 4000 hp.
At 1000 hp., quantity of water is

$$Q_1 = \frac{1000 \times 8.8}{100 \times 0.60} = 146.6 \text{ cu. ft. sec.}$$

$$V_1 = \text{velocity in pipe} = \frac{146.6}{78.54} = 1.865 \text{ ft. per second.}$$

$$V_3 = \text{discharge velocity from draft tube} = \frac{146.6}{150} = 0.977 \text{ ft. per second.}$$

Friction loss in pipe (from tables) $= 0.1260$ ft.

Entry loss for pipe $= \dfrac{(1.856)^2}{2 \times 2g} = 0.0267$ ft.

Discharge loss from draft tube $= \dfrac{(0.977)^2}{2g} = 0.0148$ ft.

h_{f1} = total conduit losses $= 0.1675$ ft.
$h_{n_1} = 100 - 0.1675 = 99.8325$. Call this = H, or 100 ft.

When the load is changed to 4000 hp. the quantity of water becomes

$$Q_2 = \frac{4000 \times 8.8}{100 \times 0.82} = 430 \text{ cu. ft. sec.}$$

V_2 = velocity in pipe = 5.46 ft. per second.

V_3 = discharge velocity from draft tube $= \frac{430}{150} = 2.866$ ft. per second.

Friction loss in pipe	= 0.936 ft.
Entry loss for pipe $= \frac{(5.46)^2}{2 \times 2g}$	= 0.233 ft.
Discharge loss from draft tube $= \frac{(2.866)^2}{2g}$	= 0.127 ft.
h_{f2} = total conduit losses	= 1.296 ft.

$h_{n_2} = 100 - 1.296 = 98.704$

It is to be noted that the quantity of water required for the given horsepower is, really,

$$Q_2 = \frac{4000 \times 8.8}{98.7 \times 0.82} = 435.8 \text{ cu. ft. sec.}$$

The difference between the actual and the approximate values is so small that it is seldom necessary to make this correction in Q_2.

v_1 = initial velocity through gate $= \sqrt{2g \times 100} = 80.2$ ft. per second.

$$A_1 = \frac{Q_1}{v_1} = \frac{146.6}{80.2} = 1.825 \text{ sq. ft.}$$

$$A_2 = \frac{Q_2}{\sqrt{2gh_{n_2}}} = \frac{430}{\sqrt{2g \times 98.7}} = 5.44 \text{ sq. ft.}$$

$$v_e = \frac{80.2 \times 1.825}{5.44} = 26.8 \text{ ft. per second.}$$

$$h_e = \frac{(26.8)^2}{2g} = 11.1 \text{ ft.}$$

$$h_a = \frac{h_{n_1} - h_e}{2} = \frac{100 - 11.1}{2} = 44.45 \text{ ft.}$$

T = time required to accelerate the water column,

$$= \frac{0.0311\, L\,(V_2 - V_1)}{h_a} = \frac{0.0311 \times 1200 \times (5.46 - 1.865)}{44.45}$$

$$= 3.02 \text{ sec.}$$

The power of the wheel drops, for the instant of opening, from 1000 hp. to

$$\frac{0.92 \times 1000 \times 11.1}{100} = 102 \text{ hp.}$$

Total load change $= 4000 - 102 = 3898$ hp.
At the time the governor begins to move the gates the speed has become reduced by 1 per cent.
Hence

$$n'_1 = 4 - 1 \text{ per cent.} = 3.96 \text{ r.p.s.}$$

To determine:

(a) Moment of inertia of rotating parts to give a regulation of 4 per cent., *i.e.*, drop in speed $= 0.04 \times 4 = 0.16$ r.p.s., so that $n_2 = 4 - 0.16 = 3.86$ r.p.s.

$$I = \frac{450 \times 3898 \times 3.02}{(3.96) - (3.86)} = 6{,}774{,}000 \text{ lb.-ft.}^2$$

(b) What speed regulation is obtained if the moment of inertia be 1,000,000 lb.-ft.^{2}

$$n_2 = \sqrt{n_1^2 - \frac{450\ \delta PT}{I}} = \sqrt{(3.96)^2 - \frac{450 \times 3898 \times 3.02}{1{,}000{,}000}}$$
$$= 3.222 \text{ r.p.s.}$$

$$\text{Per cent. regulation} = 100\left(\frac{4.00 - 3.222}{4}\right) = 19.45 \text{ per cent.}$$

Water Hammer.—The foregoing formulæ for changes of pressure in pipes do not apply if the rapidity of gate closing exceeds a certain limiting value and the gate be moved from an open to a *completely closed* position.

If the gate were closed instantaneously, the rise in pressure would be infinitely great if it were not for the ductility of the conducting pipe and the elasticity of the water. Due to these factors, the pressure which is set up by instantaneous closure of the valve is not infinite but, on the contrary, it is small compared with its theoretical possibilities. This pressure, however, may reach a value in excess of the strength of the pipe and turbine chamber.

Church's formula for the increase in pressure at the end of a pipe line, when the flow of water is instantly arrested, is

$$p = v\sqrt{\frac{\omega EE't}{g(tE' + 2RE)}} \text{ lb. per square } inch \qquad (294)$$

in which p = increase in internal pressure, per square *inch*.

v = initial velocity of flow, in feet per second.

ω = weight of prism of water, 1 ft. long and 1 sq. in. in cross-section = 0.43416 lb.

E = modulus of compressibility of water = 294,000 lb. per square inch.

E' = modulus of elasticity of plate steel = 28,000,000 lb. per square inch.
t = thickness of pipe wall in inches.
g = acceleration due to gravity = 32.2.
R = internal radius of the pipe, in inches.

As an example, consider a pipe 5 ft. = 60 in. in diameter, having a thickness of wall = 0.25 in., and with water flowing in it at a velocity of 6 ft. per second. Then the increase in internal pressure due to instantly arresting the movement of the water will be

$$p = 6\sqrt{\frac{0.434 \times 294 \times 10^3 \times 28 \times 10^6 \times 0.25}{32.2[0.25 \times 28 \times 10^6 + 2 \times 30 \times 294 \times 10^3]}} = 200 \text{ lb.}$$

per square inch.

From which it is seen that the pressure produced by *instantaneous* closure is independent of the length of the pipe. An appreciable time, however, is required to close any valve, and, with the introduction of the time element, the length of the pipe also enters as a factor into the problem.

The theory, in general, of the phenomena which take place on instantaneous gate closure, is that the kinetic energy of the moving mass of water changes to potential energy, distending the pipe and compressing the water. This compression of the water continues for an instant only, as immediately after compression it begins to extend itself; this act of expansion again sets up the pressure and causes compression. The cycle is repeated, and this continues until the friction of the water in the pipe and the internal molecular friction of the steel and the water decrease the amplitude to nearly zero. The whole occurrence is an oscillatory one and resembles somewhat the phenomenon of "surging" in electrical transmission lines carrying alternating currents. The velocity of wave propagation in the pipe is the same as the velocity of sound in water, and this velocity varies with varying conditions of thickness of pipe shell, modulus of material for shell, and its internal radius. Uhl[1] gives a formula for velocity of wave propagation which is

$$C = \frac{22{,}720}{\sqrt{23.5 + K\frac{D}{t}}} \text{ ft. per second} \tag{295}$$

[1] *Trans.* Am. Soc. Mech. Eng., February, 1911.

in which

D = diameter of pipe in inches.
t = thickness of pipe in inches.
K = constant.
for steel plate pipe = 0.232.
for cast-iron pipe = 0.464.
for wood-stave pipe = 41.50.

Knowing the velocity of the propagation of the wave, the excess pressure set up by instantaneous closure is computed from the following simple formulæ:

$$p = \frac{Cvw}{g} \text{ lb. per square inch} = \frac{CvW}{g} \text{ lb. per square foot} \quad (296)$$

$$h'_a = \frac{Cv}{g} = \text{pressure in feet head} \quad (297)$$

in which

C = velocity of wave propagation in feet per second.
w = weight of prism of water, 1 ft. long 1 sq. in. in cross-section = 0.434 lb.
W = weight of 1 cu. ft. of water = 62.5 lb.
v = velocity of water in pipe before closure in feet per second.
g = 32.2.

For the preceding example and values of v, D and t.

$$C = \frac{22{,}725}{\sqrt{23.5 + 0.232\left(\frac{60}{0.25}\right)}} = 2552 \text{ ft. per second.}$$

The internal pressure set up in the pipe, by formula 296, is $p = \frac{6 \times 2552 \times 0.434}{32.2} = 206$ lb. per square inch, which is within 3 per cent. of the value obtained by the use of formula (294).

Also

$$P = \frac{6 \times 2552 \times 62.5}{32.2} = 29{,}751 \text{ lb. per square foot.}$$

and

$$h'_a = \frac{6 \times 2552}{32.2} = 475.5 \text{ ft. head.}$$

The pressure wave travels from the valve, along the pipe to its

end, where it enters the forebay or other source of water supply, and is reflected back to the valve, passing again over the length of pipe in an opposite direction.

The time required for the wave to make the round trip, from the valve to the other end back again, is

$$T = \frac{2L}{C} \text{ sec.} \tag{298}$$

L being the length of the pipe, in feet.

This time period is called the "critical time" for any pipe line.

The experiments of Joukovsky, and others, have shown that if a valve be completely closed within a period of time $= \frac{2L}{C}$, or the critical time, the excess pressure set up in the pipe is the same as if the valve had been closed instantaneously. Also, the laws previously given for change in the internal pressure in a penstock with change in gate opening do not hold for complete valve closure. The pressure rise for this condition is

$$P_m = \frac{P_c T_c}{T_m} \tag{299}$$

in which

P_c = increase in internal pressure in pounds per square foot for closure within critical time.

T_c = critical time $= \frac{2L}{C}$

T_m = time within which valve is actually closed.

P_m = pressure set up by closure in pounds per square foot.

P_c and P_m are transformed to equivalent heads by dividing by 62.5.

As an example, consider the pipe having the constants before given, namely, D = 5 ft. t = 0.25 in. Velocity of moving water = 6 ft-sec., length = 1500 ft., and time of valve closure = 4 sec. Previous computations show that

P_c = 29,751 lb. per square foot.

C = 2552 ft. per second.

Then

$$T_c = \frac{2 \times 1500}{2552} = 1.172 \text{ sec.}$$

$$P_m = \frac{29{,}751 \times 1.172}{4} = 8717 \text{ lb. per square foot} = 139 \text{ ft. head.}$$

These increases in head are independent of the working head acting, and are additional to it. Hence, the excess pressures set up by gate closure are dangerous, or only moderate, according to whether the normal head is small or great. Thus, if the normal head were 40 ft. and a sudden excess of 140 lb. were imposed, this would make a pressure proportional to 180-ft. head, acting on a pipe, turbine chamber and wheel, that are designed for less than one-fourth this pressure.

If, however, the normal head were 400 ft., the increase to 540 ft. would be an increase of only 35 per cent. above normal.

There are two practical methods of reducing the pressure changes; one, being the application of relief valves, the other, the use of surge tanks.

Relief Valves.—Relief valves are of many kinds. One form has been described in the chapter on "Pipe Lines and Penstocks," and this is illustrative of the general type of relief valve which operates when the pressure in the penstock increases some predetermined amount above its normal value.

The details of these valves, as made by different manufacturers, vary, but they are practically all based on the principle of a relay which is operated by increase in penstock pressure and is sensitive to comparatively small pressure changes and subject to adjustment. The main valve itself is normally under the action of opposing forces of which one predominates and acts to keep the valve closed. When the relay moves, it opens an auxiliary valve and thereby changes one, or both, of the pressures acting on the main valve, so that the force which normally predominates becomes the smaller, and the main valve, therefore, quickly opens. After the pressure on the penstock is relieved, the pressure-operated relay valve, returns to its normal position, and thereby causes the forces acting on the main valve to take their original condition of unbalance in the direction to close. The value of the difference in these opposing pressures is such that closure takes place slowly and thereby prevents the production of any appreciable excess pressure in the penstock. Obviously, the forces which open and close the main valve are produced by the water pressures in the penstock and no external source of power is required.

The mechanical application of these principles can be worked out in different ways, and in the selection of a valve of this kind, the engineer should consider the simplicity and reliability of the

various parts, and in particular, the dependability of the relay and the permanence of its adjustment. In general, the relay which is required to exert the least force is the most reliable in its action.

Another form of relay valve, sometimes termed "pressure regulator," is one which is similar in principle to the foregoing, but the relay is operated by the governor instead of by change in penstock pressure. The fundamental difference in the action of the two is that an actual increase of penstock pressure must occur in order that the pressure-operated valve may move, while in the governor-operated type the action is coincident with the change in conditions which tends to produce a rise in penstock pressure, but this tendency is checked in its inception by the action of the valve, due to the fact that its operation is coincident with the change in the gate opening of the water wheel. A number of different manufacturers supply relief valves of this kind which differ in details of construction, but they are all based on the foregoing principles. Fig. 225 shows one form of governor-operated relief valve[1] and Fig. 226 shows a section through it. The details are clear from the cuts.

When the size of the water-wheel gate is small, as, for instance, the needle nozzle for impulse wheels, the bypass may be directly operated by the governor. The bypass for a needle nozzle is usually a similar needle nozzle and, as the main needle is moved outward to reduce the area and the quantity of water discharged, the bypass needle is drawn in, allowing some predetermined amount of water equal, or proportional, to the reduction in the quantity of water in the main nozzle, to flow through it. When the needle in the main nozzle has been moved to its proper position for the new load, the needle in the bypass nozzle returns slowly to its normal position of complete closure.

All governor-operated relief valves, whether worked through the intervention of a relay, or directly by the governor mechanism, must be arranged to return slowly to their normal position of closure, this return motion being usually effected by an unbalanced pressure acting on the valve tending to close it, which pressure is produced by the water in the penstock. There are one or two types which depend on the application of some external force to produce a slow rate of closure.

Another essential feature is that when governor action

[1] Wellman-Seaver-Morgan Co.

takes place on increase in load to open the wheel gates, and the gate movement is quickly produced and if this action takes place while the relay valve is still open, the valve must be closed by the governor action at the same rate of speed as that with which the turbine gates are opened. Hence, the valve must be so designed that its long time period of closure can be neutralized and it can be driven by the action of the governor to close quickly.

FIG. 225.—Governor-operated relief valve.

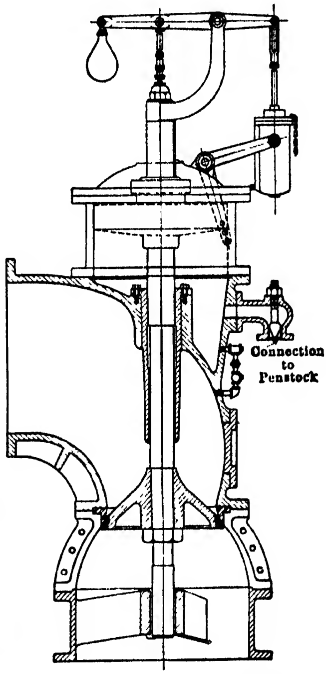


FIG. 226.—Governor-operated relief valve. Section.

It is not commercially practicable to make valves of such a size as will entirely prevent increase in the internal penstock pressure, nor is an absolutely constant pressure necessary for good service.

Uhl[1] states that the following relationships will hold between relative areas of relief valve and turbine gate, and the maximum change in head on the turbine.

[1] *Trans. Am. Soc. Mech. Eng.*, Feb., 1911.

If the sensitiveness of the pressure regulator is 10 per cent., that is, if the operating mechanism is so adjusted that it will open if a pressure increase amounting to 10 per cent. of the normal pressure occurs, the pressure rise in the penstock will be as follows:

Discharge capacity of relief valve, 100 per cent. of turbine discharge, pressure rise 10 per cent.

Discharge capacity, 75 per cent. of turbine discharge, pressure rise 20 per cent.

Discharge capacity, 50 per cent. of turbine discharge, pressure rise 30 per cent.

These values apply to pressure-operated relief valves. For governor-operated valves, the increase in head will be reduced 10 per cent. below these figures. In practice, the area of relief valves is usually made 40 to 50 per cent. of the area of the turbine gates, and seldom are such valves installed having an area greater than 75 per cent. of that of the turbine gates.

As has been pointed out, in the discusion on "Water Hammer," the greater the head, the less is the operation of the plant affected by changes in the internal pressure in the penstock, and it naturally follows that the area of relief valves selected should bear some inverse relationship to the head under which the plant operates.

Obviously, relief valves can compensate for increase in internal pressure only. Under normal conditions of operation, where the relief valve is closed, and the velocity in the penstock has a value corresponding to the load on the water wheel, any increase of load which causes a greater opening of the water-wheel gates will produce a diminution in the penstock pressure. This can not be compensated for in any way except by using surge tanks.

Surge Tanks.—Surge tanks, or reservoirs, comprise auxiliary sources of water supply close to the turbine, and they act, also, as openings in the penstock near the water wheel to allow outflow of water against a static head, whenever an excessive internal pressure is set up. The effect is to reduce the length of the column of water to be accelerated quickly, and to supply, or take up, water during a change of load while the flow of the water is being accelerated or retarded in that portion of the penstock beyond the reservoir, or standpipe. In small plants, the equalizing reservoir is seldom used, as the same effect can be produced more cheaply with a standpipe. However, in large plants under medium heads, where the quantity of water used is considerable, the reservoir will usually prove more economical in first cost and

better adapted to reduce pressure variations, and hence, will produce better speed regulation. The larger the surface area of a reservoir or standpipe, the smaller will be the pressure variation.

The topography of the country surrounding a power site usually influences the decision whether a reservoir or standpipe should be used. If artificial reservoirs prove very expensive, the standpipe is resorted to. It will be occasionally found advantageous to change the shortest or most economical route of the pipe line to one more circuitous, in order to use a natural reservoir site, or to bring the reservoir closer to the power house. If the standpipe is of suitable diameter and close to the turbine,

FIG. 227.—Surge tank.

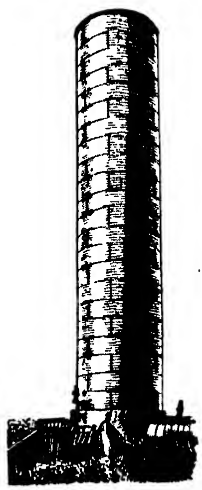


FIG. 228.—Surge tank.

the speed regulation will approach that obtainable with an open flume. Otherwise, the problem becomes that of a plant with a closed penstock of a length equal to that of the draft tube, plus the length of the penstock from the turbine to the standpipe, plus the height of the standpipe itself. To approach more nearly the effect of regulating reservoirs, high standpipes should have their upper part enlarged, in the shape of a tank, as indicated in Fig. 227. This tank may be supported on structural-steel

columns. The pipe leading to this tank should be of a diameter not less than that of the penstock. Where a power house is located near a gently sloping hillside, the standpipe may take an inclined instead of a vertical, position and laid up this hillside and supported by it, instead of being supported by columns or otherwise. Standpipes for heads of 1000 ft. have been constructed in this manner.[1]

The standpipe should be located as near the turbine as possible, as has been previously stated. If it is arranged with an overflow, the pressure rise can be practically eliminated, and the pressure drop will depend directly upon the size or capacity.

Standpipes are sometimes built high enough to prevent the water from overflowing, even with the maximum load change. In this case, the change in head with reduction in load may be considerable, unless the area of the tank or reservoir is large. Better regulation is obtainable with tanks that overflow. The main difficulty with overflow tanks is to protect the ground on which they rest and adjacent to them from the action of water falling from a considerable height. In cases where the tank is placed on a rocky hillside or near a creek bed, this condition can be easily and cheaply provided for. In most cases, however, it costs less to build the tank high enough to take all upward surges without overflow than to provide against the erosive effects of the overflow water.

In northern climates, where there is danger of freezing, the entire standpipe should be well lagged and sometimes it must be provided with steam-pipe coils, supplied with steam from a boiler installed at the foot of the standpipe.

In order to reduce the height of the standpipe as much as possible, the slope of the pipe line should be as little as is consistent with the velocity head and friction head required, because the top of the standpipe, must in any case, be higher than the level of the water in the forebay to meet the conditions of a shutdown with pipe line full.

If the load be suddenly thrown on, a certain time will elapse before the water accelerates in that part of the pipe between the standpipe and the turbine, another and longer time elapses before the water in the part of the pipe beyond the standpipe accelerates to the velocity required by the new load. During the latter time, the standpipe must supply the turbine with a quantity of

[1] Uhl: *Trans. Am. Soc. Mech. Eng.*, February, 1911.

water which is the difference between that used by the turbine at the new load, and that supplied by the penstock beyond the standpipe at the reduced velocity during acceleration.

Computations for Surge Tanks.—The development of the mathematical formulæ for surge tanks is somewhat complex, and will not be given here. The formulæ and their methods of application are sufficient for the designer, and these are set forth in this discussion. For their derivation the reader is referred to the treatises and discussions of Pressl,[1] Prasil,[2] Parker,[3] Johnson,[4] Harza,[5] Larner[6] and others.

Johnson gives the following formula for the drop in level in a surge tank when the gate opening is increased in a short time (3 or 4 sec.):

$$Y = \sqrt{\frac{AL}{Fg}[V_2 - V_1]^2 + k^2[V_2^2 - V_1^2]^2} \tag{300}$$

From which

$$F = \frac{AL(V_2 - V_1)^2}{g[Y^2 - k^2(V_2^2 - V_1^2)^2]} \tag{301}$$

In which

Y = drop in water level, in surge tank or reservoir, *i.e.*, number of feet the water surface sinks below its normal level, existing just before the change in load. This is arbitrarily assumed by the engineer, and is a compromise between the cost of the tank and the desired regulation. Usually, Y should be kept within 10 per cent. of the head for the maximum predicted load change.

A = area of penstock in square feet.
If several penstocks in parallel supply water to the plant, A = area of all penstocks connected to surge tank.

L = length of penstock, in feet.

F = area of reservoir or surge tank, in square feet.

V_1 = velocity of water in penstock prior to change in turbine gate opening, in feet per second.

V_2 = final velocity of water in the penstock after change in turbine gate opening, and after the water has accelerated

[1] *Schweitzerische Bauzeitung*, January, 1909.
[2] *Schweitzerische Bauzeitung*, January, 1908.
[3] PHILIP A. MORLEY PARKER: "Control of Water."
[4] *Trans.* Am. Soc. Mech. Eng., vols. 30 and 31.
[5] *Trans.* Am. Soc. Mech. Eng., vols. 30 and 31.
[6] *Trans.* Am. Soc. Mech. Eng., vols. 30 and 31.

in the penstock to its proper value for the new gate opening.

g = 32.2

k = friction factor $= \frac{h_f}{V_1^2}$

h_f = loss in head at velocity V_1, due to friction in pipe and at entry.

It is to be noted that the rate of acceleration of the water in the penstock is much less when a surge tank is connected with it, than if it be without a surge tank. The physical reasons are obvious. With a surge tank, any increase in turbine gate opening is met by a comparatively quick inflow of water from the tank, which diminishes the drop in head at the water wheel. Since the drop in head (h_a) causes acceleration of the water in the penstock, any diminution in this force correspondingly reduces the rate of acceleration; therefore, the time required to bring the velocity of the water in the penstock up to its proper value for the new load, is much greater than the time of acceleration when no surge tank is provided. Hence, the formula, $T = \frac{0.0311 \times 2\,(V_2 - V_1)}{h_a}$, does not hold for a penstock having a surge tank connected with it.

Harza gives the following formula for the value of T,

$$T = \pi\sqrt{\frac{FL}{Ag}} \text{ sec.} \tag{302}$$

T = time required for return to normal head after change in load, *i.e.*, time required to accelerate the water in the penstock under the retarding influence of a standpipe.

F = area of horizontal cross-section of tank or reservoir in square feet.

L = length of penstock, in feet.

A = area of the penstock, in square feet.

g = 32.2.

Also, his formula for Y, or drop in water level in a surge tank, is

$$Y = H - \sqrt{H^2 - \frac{2A\Delta}{F}} \tag{303}$$

and

$$\Delta = \frac{L}{2g}(V_2^2 - V_1^2) + \frac{HT}{\pi}(V_2 - V_1) \tag{304}$$

H = gross head.

Design of Surge Tanks.—Data given:

Horsepower of turbines	9000 hp.
H = head (gross)	150 ft.
L = penstock length	4000 ft.
penstock diameter	11 ft.
A = penstock area	95 sq. ft.

Maximum change in load from 4000 to 9000 hp.

Turbine efficiency for 4000 hp	72 per cent.
Turbine efficiency for 9000 hp	82 per cent.

$$Q_1 = \text{Water required for 4000 hp.} = \frac{4000 \times 8.8}{150 \times 0.72} = 336 \text{ cu. ft. sec.}$$

$$Q_2 = \text{Water required for 9000 hp.} = \frac{9000 \times 8.8}{150 \times 0.82} = 644 \text{ cu. ft. sec.}$$

$$V_1 = \text{velocity of water in pipe for 4000 hp.} = \frac{336}{95} = 3.53 \text{ ft. sec.}$$

$$V_2 = \text{velocity of water in pipe for 9000 hp.} = \frac{644}{95} = 6.77 \text{ ft. sec.}$$

Loss of head for $V_1' = 3.53$ ft. per second = 0.32 ft. per 1000-ft. length of pipe (see tables).

$$h_{f_1} = \text{total loss of head at 3.53 ft. sec.} = 4 \times 0.32 = 1.28 \text{ ft.}$$

$$k = \frac{h_f}{V_1^2} = \frac{1.28}{(3.53)^2} = 0.103.$$

Assume that a 10 per cent. drop in the level of the surge tank water is admissible, or $Y = 15$ ft. Then, taking Johnsons formula, the required area of the tank is

$$F = \frac{95 \times 4000\,(6.77 - 3.53)^2}{32.2[(15)^2 - (0.103)^2 \times (6.77^2 - 3.53^2)^2]} = 581 \text{ sq. ft.}$$

Its diameter is $2\sqrt{\frac{581}{\pi}} = 27.2$ ft.

The time required to accelerate the water in the penstock is

$$T = \pi\sqrt{\frac{581 \times 4000}{95 \times 32.2}} = 87.2 \text{ sec.}$$

Checking back from this, the drop Y, as computed by **Harza's** formula, is

$$Y = 150 - \sqrt{(150) - \frac{2 \times 95\Delta}{581}}$$

$$\Delta = \frac{4000}{64.4}[(6.77)^2 - (3.53)^2] + \left(\frac{150 \times 87.2}{\pi}\right)(6.77 - 3.53)$$

$$= 15{,}410$$

$$Y = 150 - \sqrt{22{,}500 - \frac{190}{581} \times (15{,}410)} = 17.8 \text{ ft.}$$

which value is 2.8 ft., or nearly 20 per cent., greater than the drop in the water level as given by Johnson's formula.

No formula in common use and simple enough for practical purposes, can be more than an approximation. A complete analysis would require that several factors be included which are commonly omitted. For instance, the distance of the tank, or reservoir, from the turbine is evidently a factor. If it were not, the forebay itself would form a surge reservoir.

Another factor is the form of the tank. If the cross-section be constant, from penstock to the top, like the tank shown in Fig. 228, the velocity that must be set up in the tank itself is less than it would be in the case of a tank of just sufficient height to cover the limits of the upward and downward surges with a connecting pipe of smaller cross-section from the tank to the penstock, such as is shown in Fig. 227.

The factor which is least accurate is the assumption of the maximum load change for which the tank is to be designed. This is only an intelligent guess, except for the one condition of the total change from zero to maximum load, and it is unwise to expend money for a tank large enough to compensate for this extreme load change if there is no probability of its occurrence in ordinary operation.

Hence, the discrepancy between the formulæ is not so serious as it at first appears.

The formula for maximum height of surge fixes the upper limit of the tank height for such as are designed to avoid overflow.

This is

$$Y = \sqrt{\frac{AV_1^2}{F}\left(\frac{L}{g} - \frac{kTV_1}{3}\right)} \qquad (305)$$

For the previous problem, and assuming a quick shut down from full load,

$$Y = \sqrt{\frac{95}{581} \times (3.53)^2 \left(\frac{4000}{32.2} - \frac{0.103 \times 87.2 \times 3.53}{3}\right)} = 15.4 \text{ ft.}$$

which is the height of the top of the tank above the forebay level.

Differential Surge Tanks.—The differential surge tank, devised by Johnson, consists of an elevated tank surrounding, and concentric with, an inner tank of considerably smaller diameter. The inner tank is connected with the penstock. The outer tank does not communicate with the penstock, except through ports or openings made through the wall of the inner tank, as indicated in Fig. 229.

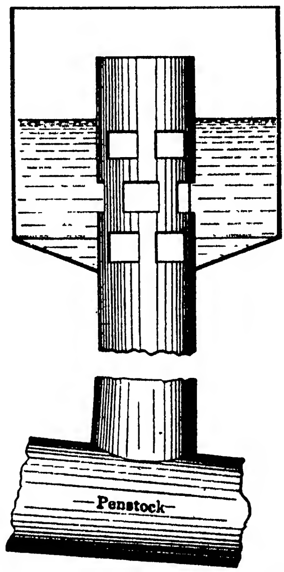


FIG. 229.—Differential surge tank.

The object of this arrangement is to diminish the size and cost of the tank for a given maximum change in level, and to "damp out" the oscillations set up by surges.

If the water-wheel gates be suddenly opened, the water from an ordinary form of plain surge tank, rushes to the turbine and prevents rapid acceleration of water in the penstock. The change in pressure and velocity in the penstock does not take place at a uniform rate, however. The curve of pressure change, plotted with time as abscissæ, is practically a sine curve. This shows that the rate of drop in level of the water in an ordinary tank is very slow at first, increasing rapidly. The level sinks to the lowest point within half the time period of a complete cycle, as given by previous formulæ. Then the level begins to rise rapidly, gradually slowing down in its rate of rise until, toward the end of the cycle, the rate of rise of the water level, is very slow.

The rate of acceleration of the water in the penstock depends on the drop in pressure at the water wheel and, therefore, at every instant it is proportional to the pressure drop, or accelerating head, h_a.

Obviously, the rate of acceleration begins and ends gradually, and, as the maximum drop in pressure (and tank level) lasts over a very short period of time, the total time required to bring the

velocity of the water in the penstock from V_1 and V_2 is considerable.

If the tank could be arranged to limit the maximum drop in pressure to the same allowable value as in the plain surge tank but keep this drop practically constant over the period of time required for a complete cycle, the water in the penstock would be accelerated much more rapidly, and the time of a complete cycle, diminished. Since the quantity of water supplied by the tank to the water wheel, to make up the deficiency of penstock supply, is proportional to the time this deficiency exists, the volume of a surge tank is reduced if the time period be reduced, other conditions remaining the same.

This the differential surge tank accomplishes, as is evident from a consideration of its action. A sudden opening of the water wheel gate is followed, almost immediately, by a rush of water from the inner tank, its level quickly sinking to the lowest point which is attained for the particular change in load. Additional water pours through the ports from the outer tank, and at a rate just sufficient to keep the water level in the inner tank from sinking any further. The water supplied to the turbine by the tank is enough to make up the deficiency of penstock supply while the drop in head continues, until the penstock begins to deliver the full required quantity of water, after which the level in the inner tank quickly regains its normal height, and the water pours back through the ports into the outer tank until equilibrium is reëstablished.

The formulæ for size of outer tank, area of ports, and drop in water level are as follows:

F = area of outer tank (includes inner tank).
A = area of penstock, in square feet.
L = length in penstock, in feet.
g = 32.2.
$k = \dfrac{h_{f_1}}{V_1^2}$.
h_{f_1} = loss of head in pipe line due to friction at velocity V_1.
V_1 = initial velocity before gate is opened.
V_2 = final velocity for new load after conditions of equilibrium are established.
Y = drop in level in tank, in feet.

The area of the inner tank is the same as that of the penstock or slightly greater—10 to 25 per cent. more.

For a predetermined maximum drop in water level in inner tank

$$F = \frac{AL(V_2 - V_1)^2}{2g[Y^2 - k^2(V_2^2 - V_1^2)^2]} \tag{306}$$

Comparing this with formula (301), it is clear that the area of a differential surge tank for a given drop in water level is just half that of a plain tank.

The time required for a complete cycle, that is, from the opening of the turbine gate until complete acceleration of the water in the penstock, is

$$T = \frac{2.3L}{2g\sqrt{kY + k^2V_1^2}} \log \frac{(Z - V_1)(Z + V_2)}{(Z + V_1)(Z - V_2)} \tag{307}$$

in which

$$Z = \sqrt{\frac{Y}{k} + V_1^2} \tag{308}$$

The area of the ports in the sides of the inner tank is fixed by the equation

$$a = \frac{A(V_2 - V_1)}{c\sqrt{2gY}} \tag{309}$$

a = total area of ports in sq. ft.

c = coefficient of discharge through ports, generally taken as 0.6.

To illustrate the methods of computation for differential surge tanks, and also, to compare the resulting dimensions with those of a plain surge tank designed to fulfil the same conditions, take the constants of the plant before used as an example in this discussion.

$$\begin{aligned} L &= 4000 \text{ ft.} \\ A &= 95 \text{ sq. ft.} \\ V_1 &= 3.53 \text{ ft. per second.} \\ V_2 &= 6.77 \text{ ft. per second.} \\ k &= 0.103 \\ Y &= 15 \text{ ft.} \end{aligned}$$

The area of tank required to limit the drop in water level to 15 ft. with a change in penstock velocity from 3.53 to 6.77 ft. per second will be

$$F = \frac{95 \times 4000 \times (6.77 - 3.53)^2}{64.4[(15)^2 - (0.103)^2 \times (6.77^2 - 3.53^2)^2]} = 290.7 \text{ sq. ft.}$$

The time required for a complete cycle is

$$T = \frac{2.3 \times 4000}{64.4\sqrt{0.103 \times 15 + 0.103^2 \times 3.53^2}} \log \frac{(Z-3.53)(Z+6.77)}{(Z+3.53)(Z-6.77)}$$

$$Z = \sqrt{\frac{15}{0.103} + 3.53^2} = 12.44$$

$$T = \frac{9200}{83.4} \times \log 1.89 = 30.49 \text{ sec.}$$

Compare these results with the corresponding values obtained for the plain surge tank in which the tank area was 581 sq. ft. or exactly twice as much as for the differential tank and the time of the cycle is 87.2 sec. as against 30.49 sec. for the differential tank.

The total port area must be

$$a = \frac{95(6.77 - 3.53)}{0.6\sqrt{2g} \times 15} = 16.55 \text{ sq. ft.}$$

Note that the ports must be submerged by the water in the outer tank at all stages or fluctuations in water level.

Pipe Vents.—If a long pipe has in it a vertical bend, at the end of a long section that is horizontal, or has a small slope, as indicated in Fig. 230, a sudden opening of the water-wheel gates will produce an accelerating force due to the diminution in pressure at the lower end of the pipe while the total head acting remains unchanged. This accelerating force is divided through the mass of water in proportion to the head acting on any arbitrarily assumed element of its length. The mass of water in the pipe from E to F is greater than that portion from F to K because the length EF is greater than the length FK (constant cross-section of pipe and draft tube being assumed). The force acting to accelerate the mass of water from E to F is proportional to the head on the pipe at F, which is h_1 in the figure. The force acting to accelerate the lesser mass in the portion FK is proportional to h_2 in the figure. Hence, the water in FK must accelerate faster than that in EF, and if the water-wheel gates be opened wide and quickly, the column of water will separate a short distance from F, back towards the forebay, a vacuum will form and the pipe will be subjected to a collapsing stress.

To provide against this occurrence, two devices are applicable; one being a surge tank, the other an automatic air inlet valve.

The capacity and height of the surge tank should be such that with maximum acceleration of the water in the downward inclined portion and the slower acceleration of the horizontal section, the water supplied by the surge tank must be sufficient to make up the difference between that delivered to the water wheel by the more rapidly accelerated column of water and that supplied by the more slowly accelerated body. This is, however, not an absolute requirement. If the standpipe be large enough to admit a sufficient quantity of air to prevent collapse after all the water has discharged from it, its essential function will be fulfilled.

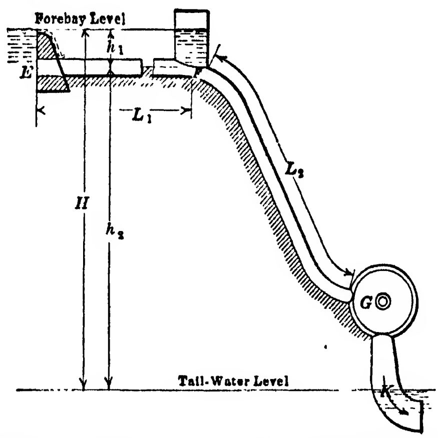


FIG. 230.—Diagram showing quantities for computing collapsing stress on pipes.

The area of air opening necessary and, hence, the minimum allowable cross-section of the standpipe is given in the succeeding discussion of vent valves.

Vent or air valves are simply large check valves set in the wall of the penstock and located on its upper surface. The valve flap moves inward to open, so that the water pressure inside the penstock keeps it closed. A comparatively weak spring is also provided which will hold the valve closed when the pressure in the pipe is a few ounces less (per square inch) than the atmospheric pressure.

If the pressure inside the pipe should fall to less than atmos-

pheric, the external air pressure will open the valve and admit air to the interior of the pipe.

Compute the rate of acceleration from E to F with the gates at the bottom of the penstock wide open.

In equations 281 to 284 it is shown that if the acceleration of a column of water is produced by an accelerating head h_a, then

$$h_a = \frac{(V_2 - V_1)L}{gT} \tag{284}$$

h_a = total accelerating head acting on the entire column of water,
V_1 = initial velocity of water in pipe,
V_2 = velocity in pipe at the end of T sec. after the application of the accelerating force,
If, H = total head on pipe,
h_1 = head on section L_1,
h_2 = head on section L_2,

the proportion of the total accelerating head acting on the portion of the water column L_1 is $\frac{h_1}{H}$ and the actual accelerating head is $\frac{h_a h_1}{H}$ feet.

The change in velocity, $V_2 - V_1$, in one second, is obtained from (284) by putting $T = 1$, *i.e,*

$$V_2 - V_1 = \frac{gh_a}{L} \text{ feet per second.} \tag{310}$$

Since for the section L_1, the actual accelerating head is $\frac{h_a h_1}{H}$, the velocity change for the section L_1 is

$$V_2{}^1 - V_1 = g\frac{h_a h_1}{HL_1} \tag{310a}$$

Likewise, for the section L_2,

$$V_2{}'' - V_1 = g\frac{h_a h_2}{HL_2} \tag{311}$$

Hence, the difference in the velocity changes in L_1 and L_2 is

$$u = \frac{gh_a}{H}\left[\frac{h_2}{L_2} - \frac{h_1}{L_1}\right] \text{ ft. per second,} \tag{312}$$

which is the rapidity with which the two ends of the water column would separate, friction being neglected.

If A = area of the pipe, then the quantity of air which would have to enter the pipe to fill the space between the ends of the water columns is

$$Au = \frac{gh_aA}{H}\left(\frac{h_2}{L_2} - \frac{h_1}{L_1}\right) \text{ cu. ft. per second} \tag{313}$$

Knowing Au, the area of the air valve opening may be computed.

The following discussion is from Enger and Seeley.[1]

The area of valve opening required for the entrance of the volume of air, Au, into the pipe may be determined from the equations for the flow of air through orifices, provided the coefficient of discharge for the valve opening is known. If the difference of pressure causing the flow of air is small (not more than 3 lb. per square inch) the flow may be treated as if the air were incompressible.

The theoretical velocity v of flow through the valve opening, for small pressure differences, is $\sqrt{2gh}$, h being in *feet of air.* Then

$$Q = cF\sqrt{\frac{2g(P_2 - P_1)}{w}} \tag{314}$$

where Q = quantity of air discharged through the valve, in cubic feet per second; c = coefficient of discharge; F = area of valve in square feet; g acceleration due to gravity = 32.2; P_2 = the absolute pressure of the air surrounding the pipe, in pounds per square foot; P_1 = absolute pressure within the pipe, in pounds per square fooot; and w the weight of 1 cu. ft. of air at pressure P_2 and temperature T_2 (0.0764 lb. at 60°F. and atmospheric pressure).

Since the quantity of air flowing through the valve must be equal to Au,

$$Q = Au = cF\sqrt{\frac{2g\,(P_2 - P_1)}{w}} \text{ cu. ft. per sec.}$$

and

$$\frac{F}{A} = \frac{u}{c}\sqrt{\frac{w}{2g\,(P_2 - P_1)}} \tag{315}$$

From this equation the ratio of the area of the valve opening to the area of the cross-section of the pipe may be determined for any value of u, when the pressure difference is small.

When the flow of air is due to pressure differences which are not

[1] *Eng. Record*, May 23, 1914.

small as compared with the absolute pressure of the air, the expansion of the air should be considered in calculating the flow.

The time required for air at pressure P_2 to expand through the valve is very short. The expansion may therefore be considered adiabatic. The temperature change may be very great, as is shown by the pressure-temperature curve in Fig. 231. As soon as the air enters the pipe, its temperature gradually rises toward that of the pipe and water. The time required for this reheating is very long compared with the time of expanding through the valve, and since the collapse of the pipe would be rapid if the pressure difference became sufficient to start the failure, the rise of temperature of the air after entrance in the pipe, with the corresponding increase in pressure, should be neglected. The effect of neglecting the rise in temperature is to give results on the safe side.

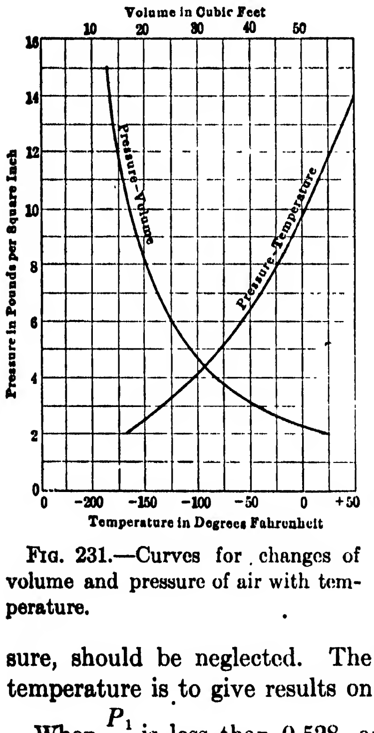


FIG. 231.—Curves for changes of volume and pressure of air with temperature.

When $\frac{P_1}{P_2}$ is less than 0.528, and assuming adiabatic expansion,

$$W = 15.01\ cF\sqrt{\frac{P_2}{\omega_2}\left[\left(\frac{P_1}{P_2}\right)^{1.43} - \left(\frac{P_1}{P_2}\right)^{1.71}\right]} \tag{316}$$

where W is the weight of air discharged through the valve per second. ω_2 = volume of 1 lb. air at atmospheric pressure and 60°F. = 13 cu. ft. If volume ω_1 of air at any pressure P_1, is known, the volume at any other pressure P_2, is $\omega_2 = \frac{\omega_1 P_1}{P_2}$ cu. ft., if the temperature remains constant.

If the expansion is adiabatic, the change in volume in passing from a pressure P_1 to a pressure P_2, is $\omega_1 = \omega_2\left(\frac{P_2}{P_1}\right)^{0.715}$ cu. ft., or if ω_2 be taken at 60°F. and atmospheric pressure, $\omega_1 = 13\left(\frac{P_2}{P_1}\right)^{0.715}$ cu. ft.

Therefore, $Q = W\omega_1$

$$= 195\ cF\left(\frac{P_1}{P_2}\right)^{0.715}\sqrt{\frac{P_2}{\omega_2}\left[\left(\frac{P_1}{P_2}\right)^{1.41} - \left(\frac{P_1}{P_2}\right)^{1.71}\right]}\ \text{cu. ft. per sec.} \tag{317}$$

and

$$\omega_2 = \omega_1\left(\frac{P_1}{P_2}\right)^{0.715}\ \text{cu. ft.} \tag{318}$$

where ω_1 and ω_2 are the volumes of air, in cubic feet per pound, corresponding to the pressures P_1 and P_2, and all other symbols have the same meaning as in previous equations.

Since Q must equal Au,

$$\frac{F}{A} = \frac{u}{195c}\left(\frac{P_1}{P_2}\right)^{0.715} \div \sqrt{\frac{P_2}{\omega_2}\left[\left(\frac{P_1}{P_2}\right)^{1.41} - \left(\frac{P_1}{P_2}\right)^{1.71}\right]} \tag{319}$$

When $\frac{P_1}{P_2} = 0.528$.

$$Q = Au = 50\ cF\left(\frac{P_2}{P_1}\right)^{0.715}\sqrt{\frac{P_2}{\omega_2}}\ \text{cu. ft. per sec.} \tag{320}$$

and

$$\frac{F}{A} = \frac{u}{50c}\left(\frac{P_1}{P_2}\right)^{0.715}\sqrt{\frac{\omega_2}{P_2}} \tag{321}$$

The value of c to be used in the above equations probably varies for each type of valve, and should be determined. Experiments on the flow of air through orifices, short tubes and annular valve openings are not numerous. From a few experiments, however, made under widely varying conditions, a reasonable basis is offered for the selection of the value of the coefficient of discharge.

From a study of the effect of sharp edges, approach to openings, length of tube, circuitous passages, etc., it is believed that $c = 0.85$ should be the maximum assumed value for the coefficient of discharge of an air-inlet valve. It may be considerably less. It will probably lie between 0.60 and 0.95. Unwin, in his treatise on hydraulics, page 46, gives values of 0.64 for sharp-edge circular orifices, 0.81 to 0.83 for short cylindrical mouthpieces without rounding at inner edge, 0.97 for conoidal mouthpieces and 0.86 for coned blast nozzles.

The pressure-temperature curve for adiabatic expansion, shown in Fig. 231, is obtained from the equation

$$T_1 = T_2\left(\frac{P_1}{P_2}\right)^{0.286}\ \text{deg. F.} \tag{322}$$

where T_1 = absolute temperature of the air when the pressure becomes P_1 and T_2 = absolute temperature of the air at pressure P_2 – taken as atmospheric pressure and 521°F.

The values obtained from equations (313) and (315) are shown in Fig. 232 in graphic form, using a value of $c = 0.85$. From this diagram the value of $\frac{F}{A}$ may be taken directly.

As an example, consider a pipe line having the following constants:

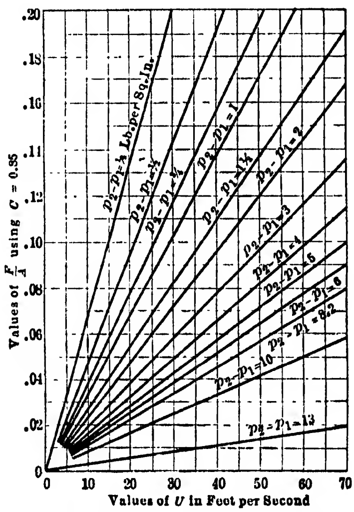


FIG. 232.—Chart for determination of sizes of air valves.

L_1 = length of pipe from forebay to downward bend = 3500 ft.
L_2 = length of pipe, plus draft tube, from bend to tail water = 500 ft.
h_1 = head on pipe from forebay to bend = 50 ft.
H = total head on pipe = 250 ft.
h_2 = head from bend to tail water = 200 ft.
D = diameter of pipe = 3.5 ft.
A = area of pipe = 9.62 sq. ft.
V_1 = initial velocity of water in pipe = 1 ft. per second.

If the water-wheel gates be quickly opened to give an area such that the final velocity, V_2, in the pipe will be 8 ft. per second, the velocity through the water wheel will drop to (approximately)

$\frac{1}{8}$ of its normal value. Head on the water wheel, prior to gate opening, is 250 ft. $= H = \frac{V_1^2}{2g}$ (friction neglected). Since the head at the wheel varies as the square of the velocity through the gates, $H : V_1^2 : : H^1 : V_2^2$, or $H^1 = H\left(\frac{V_2}{V_1}\right)^2$, H^1 being the head on the wheel the instant after the change in gate opening. For this case, the head on the wheel after gate opening, will be $\frac{H}{(8)^2} = \frac{250}{64} =$ 3.91 ft. Hence, the initial accelerating head h_a is $250 - 3.9 =$ 246.1 ft. Of this, $\frac{50 \times 246.1}{250} = 49.2$ ft. act on the section 3500 ft. long while the remainder $= 196.9$ ft. acts on the section 500 ft. long.

Then $Au = g \times 9.62\left(\frac{196.9}{500} - \frac{49.2}{3500}\right) 118$ cu. ft. = per second for the first instant. This ultimately falls to zero when equilibrium is established so that the average rate of separation of the two portions of the water column is $\frac{118}{2} = 59$ cu. ft. per second. This value, however, does not enter into the problem, as the object of the air valve or other pipe opening is to prevent the formation of any appreciable vacuum in the tube.

From the foregoing data, determine the area F of the air valve required to prevent the internal pressure from falling below 1 lb. per square inch, or 144 lb. per square foot, below atmospheric pressure.

From equation (315)

$$F = \frac{Au}{c\sqrt{\frac{2g(P_2 - P_1)}{\omega}}} \text{ sq. ft.}$$

Take $\omega = 0.0764$ lb. per cubic foot of air, and

$c = 0.85$

Then

$$F = \frac{118}{0.85\sqrt{\frac{2g \times 144}{0.0764}}} = 0.4 \text{ sq. ft.}$$

$$D = \text{diameter} = \sqrt{\frac{0.4}{0.7854}} = 0.714 \text{ ft.} = 8.5 \text{ in.}$$

The diameter of the valves should always be computed for a

movement of the water-wheel gate from a position of no load, or entirely closed, to maximum opening.

In using the diagram, Fig. 232, to determine, directly, the values of $\frac{F}{A}$ for a given value of u, when u is small the results will be more accurate if u be multiplied by 10 and the corresponding value of $\frac{F}{A}$ found and afterwards divided by 10. For instance in the preceding example $Au = 118$ and if A were 50.26 sq. ft., then $u = \frac{118}{50.26} = 2.3$, $10u = 23$. For $u = 23$ and $P_2 - P_1 = 1$ lb. per square inch, the corresponding value of $\frac{F}{A}$ in the diagram is 0.08. Actual value of $\frac{F}{A}$ for the example is $\frac{0.08}{10} = 0.008$. $F = 0.008 \times 50.26 = 0.40$ sq. ft.

Water-wheel Governors.—Water-wheel governors are mechanisms which are set in operation by a small change in speed of the water wheel and which operate to move the wheel gates to some new position of opening, to accord with the load change which caused the change in speed. Obviously, that portion of the machine which is operated by speed change must be a comparatively small and highly sensitive device and, therefore, must be required to perform very little work and to overcome small forces only, in order to move its operative parts. The amount of work which must be done, and the forces which must be overcome to move the wheel gates rapidly, are considerable and, in some instances, very great. Therefore, it is impossible that the speed-sensitive device can be made to move the gates directly. Hence, a water-wheel governor of any type consists, essentially, of a gate-moving mechanism which is rugged and powerful, and controlled by a fly-ball governor that sets in motion the gate-moving mechanism, power being supplied to the latter from some external source. In the larger sizes of governors there is usually an intermediate mechanism between the speed governor and the gate-moving mechanism, because the last-named apparatus may be too heavy for the fly-ball governor to even set it in motion by any direct action. It, therefore, first starts the movement of an intermediate device, which is small and light enough to be easily set in motion, and this intermediate mechanism starts the main gate-moving machine.

A condition of operation must be that after the gate-moving mechanism is started, it is brought to rest and movement of the gates must cease before the speed of the water wheel has returned to its normal value. In the analysis of speed control given in preceding portions of this chapter, it was shown that the time required for the acceleration of a column of water is, usually, considerably longer than the time used in moving water-wheel gates. The speed of the water wheel does not return to its normal value until the column of water in the penstock has been fully accelerated to deliver the quantity of water required by the new load, the accelerating head disappearing, and the net head on the water wheel being nearly the same as that which existed prior to load change. It is clear, therefore, that unless the movement of the governor and of the gates is arrested before the column of water in the penstock is fully accelerated or retarded, and the speed of the water wheel is brought back to its normal value, the gates will be moved a distance considerably beyond the point which would correspond with the change in load. This, in turn, would bring about a condition of speed change in the opposite direction, the wheel speed becoming greater or less than the increase, or diminution, required. The governor would have to move to change the gate again in the opposite direction, and in doing so, would again overtravel and, in this way, it would "hunt" back and forth without ever attaining any stable position. Hence, one of the most important features of a water-wheel governor is the so-called "compensating" device which brings the governor to rest before the speed has returned to its normal value.

Another essential feature of a water-wheel governor is a reliable source of external power for operating the gates and a proper mechanical method of applying it for this purpose. Other details of construction which are necessary are: (*a*) A means of adjustment of the fly-ball governor controller, for fixing the speed of the water wheel at the desired value. (*b*) An automatic means for varying the adjustment of the fly-ball controller with change in load and position of the gates so that the speed of the unit at full load is less than the speed at no load by some predetermined amount, usually 2 to 2½ per cent. This is necessary in order that the generators driven by the water wheels may divide the load among them in proportion to their respective capacities. (*c*) A means for varying the speed of the water wheel

by hand manipulation in order to synchronize generators when first starting them. (*d*) A hand-operated means for opening the wheel gates and moving them to any desired position for adjustment or starting. This manual movement of the gates must not interfere with the governor, nor must the governor interfere with it.

The general characteristics of the parts must be, as follows: The fly-ball, speed-controlling device must be as sensitive as it is mechanically possible to make it and, at the same time, have it reliable and able to perform the work imposed on it. The compensating device must, likewise, be sensitive, and easy to adjust for any desired point of stoppage of governor operation, as related to displacement of the speed-controlling device. The gate-moving mechanism must be powerful and amply strong to move the gates quickly and positively, and the power supply to cause operation must be always ready to take the load instantaneously.

Obviously, there can be many forms of mechanisms which fulfil the requirements set forth, and many have been devised. The generic difference between the several types offered on the market in America is that one class uses power from the water-wheel shaft to effect the gate movement and these are termed "mechanical governors." The other class uses oil (or water) under pressure, which is admitted to one end or the other of a cylinder, thereby forcing the piston to move the wheel gates, and the machines of this class are termed "hydraulic governors."

Mechanical governors have a nearly constant speed of movement, and the time required to move the gates, after operation begins, is practically proportional to the degree of gate movement. If 2 sec. are required to completely open the gates, then they will be opened halfway in 1 sec. Hydraulic governors, however, have almost a constant time of gate movement, regardless of the distance moved, except in the case of very small changes in gate opening, say within 15 per cent. of the total stroke. The time required to fully open the gates differs very little from that required to open them halfway.

While excellent mechanical governors have been constructed, they, necessarily, have parts which are more subject to wear and to probability of getting out of adjustment than the hydraulic governors. Another advantage of the hydraulic governor is that the operative cylinder being solidly filled with oil, the piston

and, therefore, the gates, are maintained in a fixed position regardless of the continual force acting on the gate shaft tending to close the gates and produced by the water pressure against them. There is a further advantage in that the oil pressure may be applied to either end of the piston with comparatively little movement of the speed-control mechanism and without the intervention of operating mechanisms between the speed-control device and the gate-moving apparatus. Due to these advantages and other minor ones and, also, to the fact that the hydraulic governor was the first one brought to a satisfactory stage of commercial development, engineers prefer this type except for the governing of small units.

From the discussions which have preceded it is obvious that no manufacturer of water-wheel governors can predict or guarantee any definite speed regulation. The regulation depends on factors quite outside the control of the governor. The only reasonable requirements that the engineer can impose on the manufacturer of governors are, the change in speed within which the governor will begin operation and the length of time the governor will require to move the gates from completely open to completely closed position, or *vice versa*. The manufacturer can not even guarantee the speed with which the gates will be moved unless he is, at the same time, informed as to the resistance to motion of the gates and their distance of travel, in other words, the total number of foot-pounds required to move the gates over their entire range. A guarantee of speed regulation by a manufacturer of water-wheel governors is open to question and should be regarded with suspicion. It can mean nothing without a thorough analysis of all the accessory factors, and these can not be entirely predicted, because the amount of fly-wheel effect which external rotating apparatus may furnish can not be known.

There are several excellent hydraulic governors manufactured by various firms, among which are the Lombard, Sturgess, Allis-Chalmers, I. P. Morris, Pelton and Woodward. This last-named company also makes mechanical governors adapted for gate movements requiring 3000 ft.-lb. or less.

A hydraulic governor requires a source of oil supply, a pressure chamber and a pump, to force the oil into the pressure chamber and at all times maintain the oil pressure above some limiting value. It is customary to make each governor a complete

unit, including the oil pump and pressure tank, except in large installations, where the number of units is five or more, in which case an oil-pressure system is provided, which comprises two pressure pumps, each of sufficient capacity to supply all the governors in the station, two oil-pressure chambers, each, likewise, large enough for all the governors; and a system of high-pressure piping from the pressure chamber to the governors.

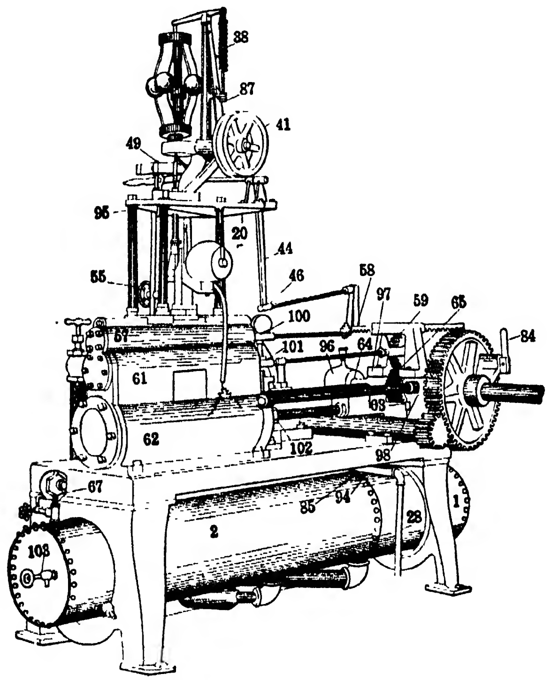


FIG. 233.—Lombard governor.

It is not within the scope of this text to describe all of the governors available; hence, only one will be described in detail, and this, more for the purpose of fixing clearly in the mind of the engineer the various interdependent features and devices which are equally applicable to any successful governor mechanism, rather than to describe any specific machine. For an example of the type of hydraulic governor, the Lombard is selected for the reason that more of these are used than any other governor in this country.

Figure 233 is a picture of one of the several designs manufactured by the Lombard Co., and Fig. 234 is a partial cross-section through the controller and actuating mechanism of the governor. Refer first to Fig. 233. It is seen that this machine has a main cylinder (62) in which is the actuating piston. The piston rod terminates in a toothed rack, turned with the teeth upward, and this rack engages with a pinion (65) on the main shaft. This pinion is not fixed on the shaft but is free to rotate on it, unless clutched to it by the pin clutch, the handle of which is numbered (84) and fixed to the gear wheel (98). When this pin clutch is thrown in, the gate shaft will rotate with movement of the piston in or out. The small pinion which meshes with the gear wheel (98) is attached to a shaft which passes across the machine frame to a hand wheel. When the pin clutch is open, movement of the hand wheel will rotate the gate shaft, so that the gates may be moved by hand. When the governor is at work, the hand-wheel pinion is disengaged from the main gear by simply sliding the hand-wheel shaft, axially, until they no longer mesh together. Immediately above the main cylinder is the cylinder (61), in which work the piston valves which admit oil to one side or the other of the main cylinder, while they allow the oil to be exhausted from the cylinder on the side of the piston opposite to that on which pressure is applied. Above the valve cylinder (61) is the relay cylinder (57), in which works a small piston that may have pressure applied on either side as the requirements of regulation may dictate. The valve rod which is connected with the valves in cylinder (61), has its other end connected with a so-called floating gear (59) which lies on top of, and meshes with, the main-shaft pinion (65). The relay piston is connected with a rod which terminates, at its outer end, in a toothed rack which is placed with the teeth downward and meshes with the teeth of the floating gear (59), from which it is seen that if the relay piston should move outward, the rack connected to it will push the floating gear (59) forward, causing, in turn, a movement of the valve rod outward, and with it, the valve. This will open the port in the rear end of the main cylinder (62) and the piston will move outward causing rotation of the gate shaft. The movement of the relay piston is controlled by the controlling valve (14), which is moved by the speed governor. All of this is clear from the cross-section, Fig. 234. With this understanding of the operation, the action of the speed control can be ex-

plained. The governor head comprises four fly balls mounted on leaf springs, and they are rotated by means of the bevel gears and governor pulley, as indicated. The springs are fixed to a projecting collar at the bottom of the governor spindle and to a sliding collar at the top of the spindle. With change of speed, the balls will change their positions radially and thereby cause vertical, axial movement of the upper collar. The stem of the controlling valve, with the several intermediate connections, later to be explained, passes through the governor spindle and has a collar near its upper end, which presses against the under side of the upper collar to which the leaf springs are attached. Obviously, the sliding action of the upper collar will be communicated to the valve stem. A jointed continuation of the valve stem projects above the governor spindle, and is attached to a horizontal cross-lever which is fulcrumed on a support, and a tension spring (38) is attached to the outer end of the lever so that the pull of this spring provides a counterforce tending to lift the valve stem and to oppose radially outward movement of the balls. When the system is properly adjusted and the speed of the governor balls is 480 r.p.m., the centrifugal force of the balls, the moment of the force produced by the ball-supporting leaf springs, and the tension spring (38), are all in equilibrium; the controlling valve is in its central position, and the gate-moving mechanism is at rest. An increase in speed will increase the centrifugal force of the balls, so that they move outward from the spindle, depressing the upper collar and moving the valve stem and controller valve downward. This admits oil through the upper portion of the control valve to the right-hand side of the relay piston, which moves outward, moving with it the main valve by the action of the upper rack and the floating gear, and this admits oil to the left-hand end of the main piston, causing it to move inward and thereby reduce the gate opening. The next action which the governor must accomplish is to stop the movement of the main piston before the speed of the water wheel has returned to normal. By referring to Fig. 234 it is seen that the valve stem carries on it a small pinion (47) which has a very wide face. Engaging with this pinion is a rack, the teeth of which are not visible in the drawing. Movement of this rack rotates the valve stem. Inside the governor spindle, and about at the level of the balls, is a threaded socket into which the upper end of the valve stem screws. The threads in the socket

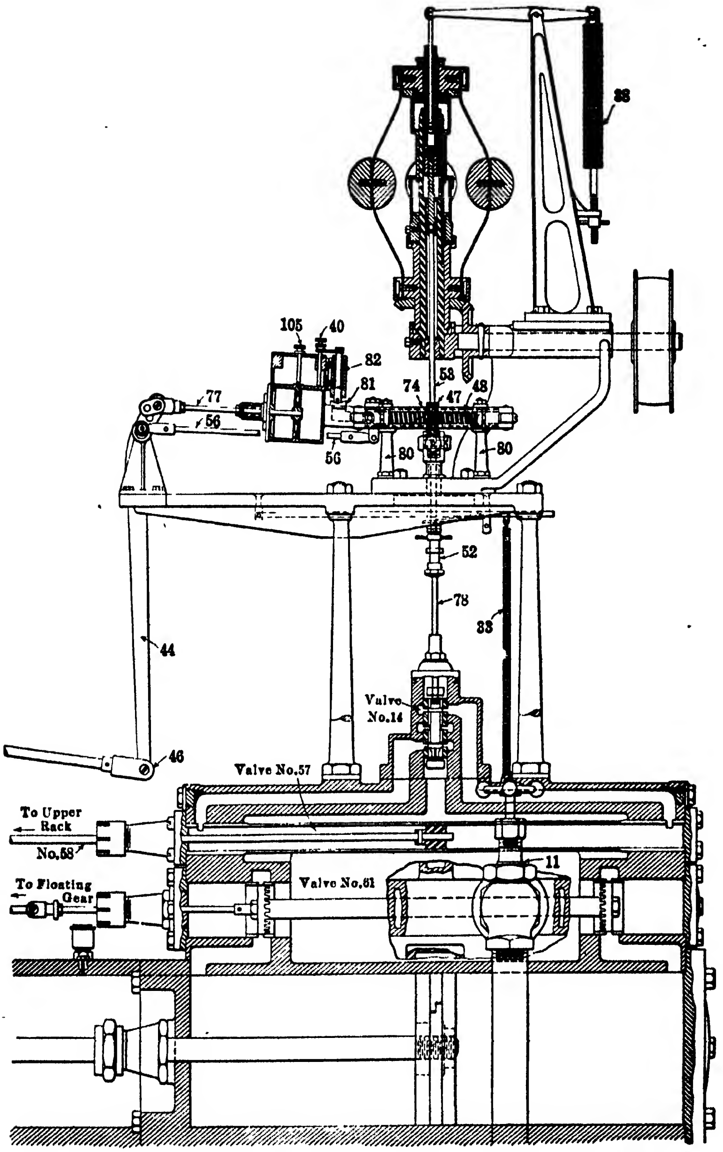


Fig. 234.—Lombard governor. Section through governor head and operating cylinders.

and on the stem have a very steep pitch, so that a small degree of rotation will cause a comparatively great change in the length of the valve stem. If the rack be moved in a direction to rotate the valve stem so that it shortens, the control valve will be moved back to its neutral position while the governor balls are still depressing the upper sliding collar. This arrests the motion of the relay piston. The floating gear then moves to push the main valve to its neutral position, because the upper rack has become stationary, and movement of the main-shaft pinion, which meshes with the floating gear, will cause the latter to work backward along the upper rack, so that the governor comes to rest. It is now necessary that the small rack, which coöperates with pinion (47) on the valve stem, shall rotate back and restore the length of the stem to its normal value. The small rack slides in grooves made in the transverse piece that is supported on the vertical studs (80). If it moves in either direction, it extends the horizontal springs (48). The rack is not connected directly to the gate-moving mechanism, but through two yielding devices, one, a dashpot, and the other, a spring plunger (82) fastened on the right-hand end of the dashpot and pressing into a "V"-shaped groove (81) in a piece of metal, which latter is connected with the rack. The dashpot piston rod (77) is connected through a bell crank and lever (44) to an upwardly projecting piece on the relay valve rod. Therefore, motion of the relay valve moves the piston in the dashpot positively. If the relay piston moves outward, the dashpot piston moves inward and pushes with it the dashpot and spring plunger, and these move the rack which rotates the pinion on the valve stem. The motion of the rack is not strictly proportional to the motion of the relay piston, because the dashpot piston moves, to some extent, in the dashpot, which amount of motion becomes greater and greater as the extension of spring (48) increases. After a certain point is reached the spring pressure will arrest the motion of the rack, in which case the lower end of the spring plunger (82) will begin to ride upward on one side of the "V"-notch (81). When the governor comes to rest, the spring shoves back the rack (74) and with it the dashpot, the rate of motion being fixed by the degree of opening of the valve (40) in one end of the dashpot. After the rack has reached its normal position, a further outward movement of the dashpot takes place, due to the spring plunger pushing downward against

the side of the "V"-notch. The degree of slippage of the piston in the dashpot is fixed by the degree of opening of valve (40), and this is adjusted after the governor is set up and in operation to conform with the specific conditions under which it works.

The small rod (56), which has a slight motion whenever the gate-moving mechanism moves, is connected solidly to the framework which rests on the studs (80). This framework is moved by movement of the rod (56). The effect of moving the whole framework is to change the neutral position of the valve stem and rack, and this means that it changes, by a slight amount, the length of the valve stem, which, in turn, gives a different speed to the water wheel. This is the so-called "paralleling rod," and it is adjusted to cause a change in speed between no load and full load, giving about 2 per cent. drop in speed at full load.

The sleeve (52) on the valve stem is threaded and rotatable. This is for adjusting the position of the controller valve to give proper speed to the water wheel. Above this sleeve, on the valve stem, is a pair of nuts, and just above them, a trigger. The trigger holds up a lever which is shown dotted in the figure, a heavy coil spring (33) tending to pull the lever down. This is a safety device and operates to shut down the water wheel in case the governor balls should quit rotating, whether caused by a broken belt, or for any other reason. Its operation is obvious. If rotation stops, the valve stem will move violently upward tending to cause opening of the gate. The adjustable nuts will strike and release the trigger, the coil spring will pull down the lever, which in turn, will press against the valve-stem nuts and overpower the other springs acting, move the controller valve downward and cause a movement of the main piston to shut down the wheel.

The pump is not shown in either of these figures but located on the opposite side from that shown in Fig. 233. The oil-pressure tank (2) is underneath the machine, as shown, and this is kept approximately half full of oil at all times.

The latest form of the Lombard hydraulic governor is shown in Fig. 235. This is known under the trade designation of "Type T." The governor balls are enclosed . The compensating spring is suspended vertically and is clearly seen in both the elevations of the machine. The oil-pressure pump and tank are separate and placed in any convenient position. The

operating piston shown on the right-hand side elevation can be connected either for vertical or for horizontal gate shaft, by merely

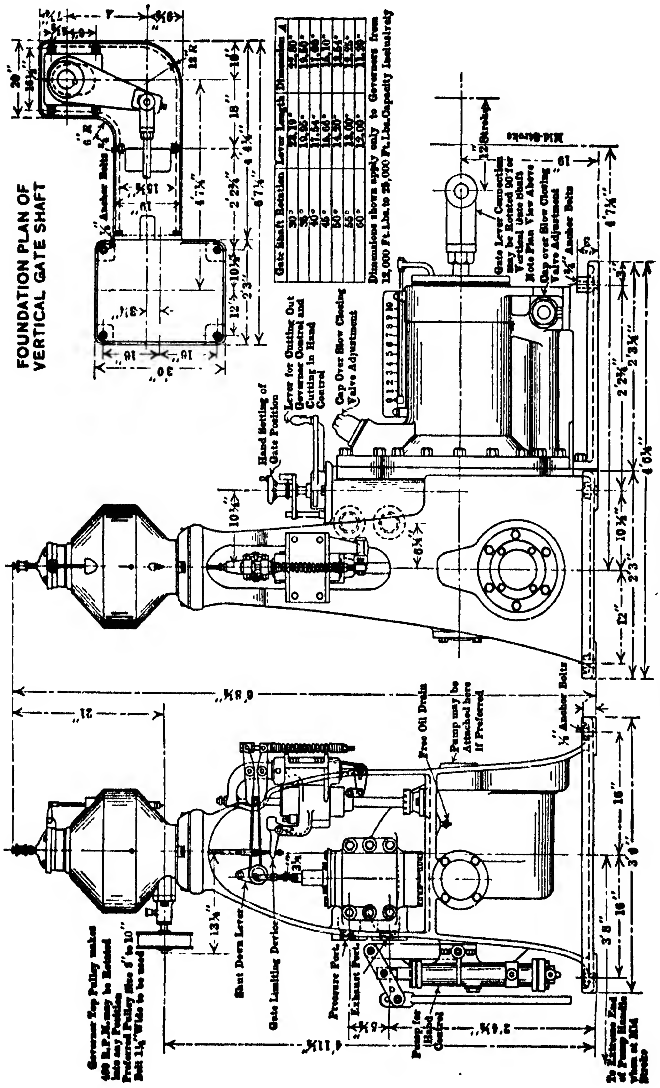


FIG. 235.—Lombard governor—Type T.

changing the bed plate, as indicated in the upper right-hand corner of the figure. There is no hand wheel for opening the gate but, instead, a small valve, marked "Hand Setting of Gate

Position," is provided that has on its stem a marker and, parallel with its stem, a gauge. By opening this valve and bringing the marker to any desired point on the gauge, the gates will open by oil pressure and move to a point of opening corresponding to the position shown by the indicator. In case no oil pressure is available to cause this movement, a small hand pump is used to supply the needed pressure, as shown on the left-hand side of the end elevation. By this means, even the heaviest gates are easily moved for starting or adjusting. One new and very desirable feature of this machine is that no matter how rapidly the wheel gates may be moved by it, the velocity of motion is reduced near the end of the travel toward gate closure, so that in complete closing of the gates the final portion of the movement takes place slowly. The discussion of "Water Hammer" indicates the reason for this arrangement. Another convenient feature of this machine is the ability to adjust it to open the gates on either inward, or outward, motion of the piston. This allows a greater freedom in power-station design in the location of the governors, and in many instances avoids the necessity of an intermediate gear.

MATHEMATICAL TABLES

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MATHEMATICAL TABLES

Notes on Use of Logarithms

Decimal Fractions.—The characteristic of the log is a negative number whose numerical value is equal to 1+ number of zeros to the right of the decimal point. Of the whole logarithm, the characteristic only, is negative, the mantissa being positive, and this is indicated by placing the minus sign *above* the characteristic.

Thus, the log of 42 has the mantissa 6232 and, therefore,

$$\begin{aligned} \log 0.42 &= \bar{1}.6232 \\ \log 0.042 &= \bar{2}.6232 \\ \log 0.0042 &= \bar{3}.6232 \end{aligned}$$

$\bar{3}.6232$, for instance, stands for $-3 + 0.6232$.

In order to carry out numerical operations with logs having negative characteristics, the characteristics must be separated from the mantissa and the two quantities worked independently, and finally recombined at the end of the computations.

For instance, to raise 0.0042 to the 1.842 power, the operation is as follows:

$$\text{Log } (0.0042)^{1.842} = 1.842 \times \log 0.0042 = 1.842 \times \bar{3}.6232$$

$$\begin{aligned} 1.842 \times 0.6232 &= 1.1479 \\ 1.842 \times (-3) &= -5.526 \\ \hline \log\ (0.0042)^{1.842} &= -4.3781 \end{aligned}$$

The whole log is now negative, so that the number corresponding to $(0.0042)^{1.842}$ is

$$\frac{1}{\text{No. whose log is } 4.3781} = \frac{1}{23{,}890} = 0.00004187$$

The result may be also obtained by taking a characteristic having a value greater by one than the negative characteristic of the log, and subtracting the log from it, thus:

$$\begin{aligned} -4.3781 &= -5 + (5 - 4.3781) \\ &= -5 + 0.6219 = \bar{5}.6219 \end{aligned}$$

The number corresponding to the mantissa 6219 is 4187 and the number of zeros to the right of the decimal point is equal to

the numerical value of the characteristic less unity, which for this case = 4 zeros.

Hence, number corresponding to log $\bar{5}.6219$, is 0.00004187.

Proportional Parts.—The table of logarithms which follows, is computed for three places only. If the log of a four digit number is required, the accompanying table of proportional parts must be used.

To find the proportional part, take the log of the number nearest to the number of which the log is required. Then, take the P.P. in the column of P.P. which corresponds, numerically to the fourth figure of the number of which the log is sought, and in the same horizontal line as that in which the nearest number is found.

If the nearest number found in the table is greater than the number of which the log is sought, the P.P. is to be subtracted from the log of the nearest number: if the nearest number is less than the given number, the P.P. is to be added to the log.

The position of the P.P. in the addition or subtraction, is such that the last figure of the P.P. is under the last figure of the log.

As an example, find the log of 53.84. Nearest number to this in the table, is 53.8. Log 538 is 7308, and for 53.8 is 1.7308.

The last figure of the given number is 4. In column 4 of P.P., and opposite to nearest number 538, is found the figure 3 which is the P.P. required. This must be added to the log of the nearest number. $1.7308 + 3 = 1.7311$, which is the log of 53.84.

If the number corresponding to a given log is to be found, and the log is not found in the table, the process is reversed.

For example, to find the number corresponding to the log 2.3813. Log in the table nearest to this log is 3820.

Subtracting this log from the given log,

$$\text{Diff.} = 0.3820 - 0.3813 = 0.0007.$$

On the line containing the mantissa 3820, look for the number 7. This number lies in the P.P. column marked 4. This means that a number difference of 4 corresponds to a log difference of 0.0007.

The mantissa 3820 corresponds to the number 241. Since 0.3820 is greater than 0.3813, the number difference must be subtracted from 241, so that the number corresponding to the mantissa 3813 is $2410 - 4 = 2406$. As the characteristic of the given log is 2, the number corresponding to the given log is 240.6.

* TABLE 38.—LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	PROPORTIONAL PARTS 1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7

*** From Bottomley's Four Figure Mathematical Tables, by courtesy of The Macmillan Company.**

Table 38.—Logarithms. Proportional Parts

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

TABLE 39.—NATURAL SINES AND COSINES

NOTE.—For cosines use right-hand column of degrees and lower line of tenths.

Deg.	°0.0	°0.1	°0.2	°0.3	°0.4	°0.5	°0.6	°0.7	°0.8	°0.9	
0°	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157	89
1	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332	88
2	0.0349	0.0366	0.0384	0.0401	0.0419	0.0436	0.0454	0.0471	0.0488	0.0506	87
3	0.0523	0.0541	0.0558	0.0576	0.0593	0.0610	0.0628	0.0645	0.0663	0.0680	86
4	0.0698	0.0715	0.0732	0.0750	0.0767	0.0785	0.0802	0.0819	0.0837	0.0854	85
5	0.0872	0.0889	0.0906	0.0924	0.0941	0.0958	0.0976	0.0993	0.1011	0.1028	84
6	0.1045	0.1063	0.1080	0.1097	0.1115	0.1132	0.1149	0.1167	0.1184	0.1201	83
7	0.1219	0.1236	0.1253	0.1271	0.1288	0.1305	0.1323	0.1340	0.1357	0.1374	82
8	0.1392	0.1409	0.1426	0.1444	0.1461	0.1478	0.1495	0.1513	0.1530	0.1547	81
9	0.1564	0.1582	0.1599	0.1616	0.1633	0.1650	0.1668	0.1685	0.1702	0.1719	80°
10°	0.1736	0.1754	0.1771	0.1788	0.1805	0.1822	0.1840	0.1857	0.1874	0.1891	79
11	0.1908	0.1925	0.1942	0.1959	0.1977	0.1994	0.2011	0.2028	0.2045	0.2062	78
12	0.2079	0.2096	0.2113	0.2130	0.2147	0.2164	0.2181	0.2198	0.2215	0.2232	77
13	0.2250	0.2267	0.2284	0.2300	0.2317	0.2334	0.2351	0.2368	0.2385	0.2402	76
14	0.2419	0.2436	0.2453	0.2470	0.2487	0.2504	0.2521	0.2538	0.2554	0.2571	75
15	0.2588	0.2605	0.2622	0.2639	0.2656	0.2672	0.2689	0.2706	0.2723	0.2740	74
16	0.2756	0.2773	0.2790	0.2807	0.2823	0.2840	0.2857	0.2874	0.2890	0.2907	73
17	0.2924	0.2940	0.2957	0.2974	0.2990	0.3007	0.3024	0.3040	0.3057	0.3074	72
18	0.3090	0.3107	0.3123	0.3140	0.3156	0.3173	0.3190	0.3206	0.3223	0.3239	71
19	0.3256	0.3272	0.3289	0.3305	0.3322	0.3338	0.3355	0.3371	0.3387	0.3404	70°
20°	0.3420	0.3437	0.3453	0.3469	0.3486	0.3502	0.3518	0.3535	0.3551	0.3567	69
21	0.3584	0.3600	0.3616	0.3633	0.3649	0.3665	0.3681	0.3697	0.3714	0.3730	68
22	0.3746	0.3762	0.3778	0.3795	0.3811	0.3827	0.3843	0.3859	0.3875	0.3891	67
23	0.3907	0.3923	0.3939	0.3955	0.3971	0.3987	0.4003	0.4019	0.4035	0.4051	66
24	0.4067	0.4083	0.4099	0.4115	0.4131	0.4147	0.4163	0.4179	0.4195	0.4210	65
25	0.4226	0.4242	0.4258	0.4274	0.4289	0.4305	0.4321	0.4337	0.4352	0.4368	64
26	0.4384	0.4399	0.4415	0.4431	0.4446	0.4462	0.4478	0.4493	0.4509	0.4524	63
27	0.4540	0.4555	0.4571	0.4586	0.4602	0.4617	0.4633	0.4648	0.4664	0.4679	62
28	0.4695	0.4710	0.4726	0.4741	0.4756	0.4772	0.4787	0.4802	0.4818	0.4833	61
29	0.4848	0.4863	0.4879	0.4894	0.4909	0.4924	0.4939	0.4955	0.4970	0.4985	60°
30°	0.5000	0.5015	0.5030	0.5045	0.5060	0.5075	0.5090	0.5105	0.5120	0.5135	59
31	0.5150	0.5165	0.5180	0.5195	0.5210	0.5225	0.5240	0.5255	0.5270	0.5284	58
32	0.5299	0.5314	0.5329	0.5344	0.5358	0.5373	0.5388	0.5402	0.5417	0.5432	57
33	0.5446	0.5461	0.5476	0.5490	0.5505	0.5519	0.5534	0.5548	0.5563	0.5577	56
34	0.5592	0.5606	0.5621	0.5635	0.5650	0.5664	0.5678	0.5693	0.5707	0.5721	55
35	0.5736	0.5750	0.5764	0.5779	0.5793	0.5807	0.5821	0.5835	0.5850	0.5864	54
36	0.5878	0.5892	0.5906	0.5920	0.5934	0.5948	0.5962	0.5976	0.5990	0.6004	53
37	0.6018	0.6032	0.6046	0.6060	0.6074	0.6088	0.6101	0.6115	0.6129	0.6143	52
38	0.6157	0.6170	0.6184	0.6198	0.6211	0.6225	0.6239	0.6252	0.6266	0.6280	51
39	0.6293	0.6307	0.6320	0.6334	0.6347	0.6361	0.6374	0.6388	0.6401	0.6414	50°
40°	0.6428	0.6441	0.6455	0.6468	0.6481	0.6494	0.6508	0.6521	0.6534	0.6547	49
41	0.6561	0.6574	0.6587	0.6600	0.6613	0.6626	0.6639	0.6652	0.6665	0.6678	48
42	0.6691	0.6704	0.6717	0.6730	0.6743	0.6756	0.6769	0.6782	0.6794	0.6807	47
43	0.6820	0.6833	0.6845	0.6858	0.6871	0.6884	0.6896	0.6909	0.6921	0.6934	46
44	0.6947	0.6959	0.6972	0.6984	0.6997	0.7009	0.7022	0.7034	0.7046	0.7059	45
	°1.0	°0.9	°0.8	°0.7	°0.6	°0.5	°0.4	°0.3	°0.2	°0.1	Deg.

TABLE 39.—NATURAL SINES AND COSINES.—*Concluded*

Deg.	°0.0	°0.1	°0.2	°0.3	°0.4	°0.5	°0.6	°0.7	°0.8	°0.9	
45	0.7071	0.7083	0.7096	0.7108	0.7120	0.7133	0.7145	0.7157	0.7169	0.7181	44
46	0.7193	0.7206	0.7218	0.7230	0.7242	0.7254	0.7266	0.7278	0.7290	0.7302	43
47	0.7314	0.7325	0.7337	0.7349	0.7361	0.7373	0.7385	0.7396	0.7408	0.7420	42
48	0.7431	0.7443	0.7455	0.7466	0.7478	0.7490	0.7501	0.7513	0.7524	0.7536	41
49	0.7547	0.7559	0.7570	0.7581	0.7593	0.7604	0.7615	0.7627	0.7638	0.7649	40°
50°	0.7660	0.7672	0.7683	0.7694	0.7705	0.7716	0.7727	0.7738	0.7749	0.7760	39
51	0.7771	0.7782	0.7793	0.7804	0.7815	0.7826	0.7837	0.7848	0.7859	0.7869	38
52	0.7880	0.7891	0.7902	0.7912	0.7923	0.7934	0.7944	0.7955	0.7965	0.7976	37
53	0.7986	0.7997	0.8007	0.8018	0.8028	0.8039	0.8049	0.8059	0.8070	0.8080	36
54	0.8090	0.8100	0.8111	0.8121	0.8131	0.8141	0.8151	0.8161	0.8171	0.8181	35
55	0.8192	0.8202	0.8211	0.8221	0.8231	0.8241	0.8251	0.8261	0.8271	0.8281	34
56	0.8290	0.8300	0.8310	0.8320	0.8329	0.8339	0.8348	0.8358	0.8368	0.8377	33
57	0.8387	0.8396	0.8406	0.8415	0.8425	0.8434	0.8443	0.8453	0.8462	0.8471	32
58	0.8480	0.8490	0.8499	0.8508	0.8517	0.8526	0.8536	0.8545	0.8554	0.8563	31
59	0.8572	0.8581	0.8590	0.8599	0.8607	0.8616	0.8625	0.8634	0.8643	0.8652	30°
60°	0.8660	0.8669	0.8678	0.8686	0.8695	0.8704	0.8712	0.8721	0.8729	0.8738	29
61	0.8746	0.8755	0.8763	0.8771	0.8780	0.8788	0.8796	0.8805	0.8813	0.8821	28
62	0.8829	0.8838	0.8846	0.8854	0.8862	0.8870	0.8878	0.8886	0.8894	0.8902	27
63	0.8910	0.8918	0.8926	0.8934	0.8942	0.8949	0.8957	0.8965	0.8973	0.8980	26
64	0.8988	0.8996	0.9003	0.9011	0.9018	0.9026	0.9033	0.9041	0.9048	0.9056	25
65	0.9063	0.9070	0.9078	0.9085	0.9092	0.9100	0.9107	0.9114	0.9121	0.9128	24
66	0.9135	0.9143	0.9150	0.9157	0.9164	0.9171	0.9178	0.9184	0.9191	0.9198	23
67	0.9205	0.9212	0.9219	0.9225	0.9232	0.9239	0.9245	0.9252	0.9259	0.9265	22
68	0.9272	0.9278	0.9285	0.9291	0.9298	0.9304	0.9311	0.9317	0.9323	0.9330	21
69	0.9336	0.9342	0.9348	0.9354	0.9361	0.9367	0.9373	0.9379	0.9385	0.9391	20°
70°	0.9397	0.9403	0.9409	0.9415	0.9421	0.9426	0.9432	0.9438	0.9444	0.9449	19
71	0.9455	0.9461	0.9466	0.9472	0.9478	0.9483	0.9489	0.9494	0.9500	9.9505	18
72	0.9511	0.9516	0.9521	0.9527	0.9532	0.9537	0.9542	0.9548	0.9553	0.9558	17
73	0.9563	0.9568	0.9573	0.9578	0.9583	0.9588	0.9593	0.9598	0.9603	0.9608	16
74	0.9613	0.9617	0.9622	0.9627	0.9632	0.9636	0.9641	0.9646	0.9650	0.9655	15
75	0.9659	0.9664	0.9668	0.9673	0.9677	0.9681	0.9686	0.9690	0.9694	0.9699	14
76	0.9703	0.9707	0.9711	0.9715	0.9720	0.9724	0.9728	0.9732	0.9736	0.9740	13
77	0.9744	0.9748	0.9751	0.9755	0.9759	0.9673	0.9767	0.9770	0.9774	0.9778	12
78	0.9781	0.9785	0.9789	0.9792	0.9796	0.9799	0.9803	0.9806	0.9810	0.9813	11
79	0.9816	0.9820	0.9823	0.9826	0.9829	0.9833	0.9836	0.9839	0.9842	0.9845	10°
80°	0.9848	0.9851	0.9854	0.9857	0.9860	0.9863	0.9866	0.9869	0.9871	0.9874	9
81	0.9877	0.9880	0.9882	0.9885	0.9888	0.9890	0.9893	0.9895	0.9898	0.9900	8
82	0.9903	0.9905	0.9907	0.9910	0.9912	0.9914	0.9917	0.9919	0.9921	0.9923	7
83	0.9925	0.9928	0.9930	0.9932	0.9934	0.9936	0.9938	0.9940	0.9942	0.9943	6
84	0.9945	0.9947	0.9949	0.9951	0.9952	0.9954	0.9956	0.9957	0.9959	0.9960	5
85	0.9962	0.9963	0.9965	0.9966	0.9968	0.9969	0.9971	0.9972	0.9973	0.9974	4
86	0.9976	0.9977	0.9978	0.9979	0.9980	0.9981	0.9982	0.9983	0.9984	0.9985	3
87	0.9986	0.9987	0.9988	0.9989	0.9990	0,9990	0.9991	0.9992	0.9993	0.9993	2
88	0.9994	0.9995	0.9995	0.9996	0.9996	0.9997	0.9997	0.9997	0.9998	0.9998	1
89	0.9998	0.9999	0.9999	0.9999	0.9999	1.000	1.000	1.000	1.000	1.000	°
	°1.0	°0.9	°0.8	°0.7	°0.6	°0.5	°0.4	°0.3	°0.2	°0.1	Deg.

TABLE 40.—NATURAL TANGENTS AND COTANGENTS

NOTE.—For cotangents use right-hand column of degrees and lower line of tenths

Deg.	°0.0	°0.1	°0.2	°0.3	°0.4	°0.5	°0.6	°0.7	°0.8	°0.9	
0°	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157	89
1	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332	88
2	0.0349	0.0367	0.0384	0.0402	0.0419	0.0437	0.0454	0.0472	0.0489	0.0507	87
3	0.0524	0.0542	0.0559	0.0577	0.0594	0.0612	0.0629	0.0647	0.0664	0.0682	86
4	0.0699	0.0717	0.0734	0.0752	0.0769	0.0787	0.0805	0.0822	0.0840	0.0857	85
5	0.0875	0.0892	0.0910	0.0928	0.0945	0.0963	0.0981	0.0998	0.1016	0.1033	84
6	0.1051	0.1069	0.1086	0.1104	0.1122	0.1139	0.1157	0.1175	0.1192	0.1210	83
7	0.1228	0.1246	0.1263	0.1281	0.1299	0.1317	0.1334	0.1352	0.1370	0.1388	82
8	0.1405	0.1423	0.1441	0.1459	0.1477	0.1495	0.1512	0.1530	0.1548	0.1566	81
9	0.1584	0.1602	0.1620	0.1638	0.1655	0.1673	0.1691	0.1709	0.1727	0.1745	80°
10°	0.1763	0.1781	0.1799	0.1817	0.1835	0.1853	0.1871	0.1890	0.1908	0.1926	79
11	0.1944	0.1962	0.1980	0.1998	0.2016	0.2035	0.2053	0.2071	0.2089	0.2107	78
12	0.2126	0.2144	0.2162	0.2180	0.2199	0.2217	0.2235	0.2254	0.2272	0.2290	77
13	0.2309	0.2327	0.2345	0.2364	0.2382	0.2401	0.2419	0.2438	0.2456	0.2475	76
14	0.2493	0.2512	0.2530	0.2549	0.2568	0.2586	0.2605	0.2623	0.2642	0.2661	75
15	0.2679	0.2698	0.2717	0.2736	0.2754	0.2773	0.2792	0.2811	0.2830	0.2849	74
16	0.2867	0.2886	0.2905	0.2924	0.2943	0.2962	0.2981	0.3000	0.3019	0.3038	73
17	0.3057	0.3076	0.3096	0.3115	0.3134	0.3153	0.3172	0.3191	0.3211	0.3230	72
18	0.3249	0.3269	0.3288	0.3307	0.3327	0.3346	0.3365	0.3385	0.3404	0.3424	71
19	0.3443	0.3463	0.3482	0.3502	0.3522	0.3541	0.3561	0.3581	0.3600	0.3620	70°
20°	0.3640	0.3659	0.3679	0.3699	0.3719	0.3739	0.3759	0.3779	0.3799	0.3819	69
21	0.3839	0.3859	0.3879	0.3899	0.3919	0.3939	0.3959	0.3979	0.4000	0.4020	68
22	0.4040	0.4061	0.4081	0.4101	0.4122	0.4142	0.4163	0.4183	0.4204	0.4224	67
23	0.4245	0.4265	0.4286	0.4307	0.4327	0.4348	0.4369	0.4390	0.4411	0.4431	66
24	0.4452	0.4473	0.4494	0.4515	0.4536	0.4557	0.4578	0.4599	0.4621	0.4642	65
25	0.4663	0.4684	0.4706	0.4727	0.4748	0.4770	0.4791	0.4813	0.4834	0.4856	64
26	0.4877	0.4899	0.4921	0.4942	0.4964	0.4986	0.5008	0.5029	0.5051	0.5073	63
27	0.5095	0.5117	0.5139	0.5161	0.5184	0.5206	0.5228	0.5250	0.5272	0.5295	62
28	0.5317	0.5340	0.5362	0.5384	0.5407	0.5430	0.5452	0.5475	0.5498	0.5520	61
29	0.5543	0.5566	0.5589	0.5612	0.5635	0.5658	0.5681	0.5704	0.5727	0.5750	60°
30°	0.5774	0.5797	0.5280	0.5844	0.5867	0.5890	0.5914	0.5938	0.5961	0.5985	59
31	0.6009	0.6032	0.6056	0.6080	0.6104	0.6128	0.6152	0.6176	0.6200	0.6224	58
32	0.6249	0.6273	0.6297	0.6322	0.6346	0.6371	0.6395	0.6420	0.6445	0.6469	57
33	0.6494	0.6519	0.6544	0.6569	0.6594	0.6619	0.6644	0.6669	0.6694	0.6720	56
34	0.6745	0.6771	0.6796	0.6822	0.6847	0.6873	0.6899	0.6924	0.6950	0.6976	55
35	0.7002	0.7028	0.7054	0.7080	0.7107	0.7133	0.7159	0.7186	0.7212	0.7239	54
36	0.7265	0.7292	0.7319	0.7346	0.7373	0.7400	0.7427	0.7454	0.7481	0.7508	53
37	0.7536	0.7563	0.7590	0.7618	0.7646	0.7673	0.7701	0.7729	0.7757	0.7785	52
38	0.7813	0.7841	0.7869	0.7898	0.7926	0.7954	0.7983	0.8012	0.8040	0.8069	51
39	0.8098	0.8127	0.8156	0.8185	0.8214	0.8243	0.8273	0.8302	0.8332	0.8361	50°
40°	0.8391	0.8421	0.8451	0.8481	0.8511	0.8541	0.8571	0.8601	0.8632	0.8662	49
41	0.8693	0.8724	0.8754	0.8785	0.8816	0.8847	0.8878	0.8910	0.8941	0.8972	48
42	0.9004	0.9036	0.9067	0.9099	0.9131	0.9163	0.9195	0.9228	0.9260	0.9293	47
43	0.9325	0.9358	0.9391	0.9424	0.9457	0.9490	0.9523	0.9556	0.9590	0.9623	46
44	0.9657	0.9691	0.9725	0.9759	0.9793	0.9827	0.9861	0.9896	0.9930	0.9965	45
	°1.0	°0.9	°0.8	°0.7	°0.6	°0.5	°0.4	°0.3	°0.2	°0.1	Deg.

TABLE 40.—NATURAL TANGENTS AND COTANGENTS.—*Concluded*

Deg.	°0.0	°0.1	°0.2	°0.3	°0.4	°0.5	°0.6	°0.7	°0.8	°0.9	
45	1.0000	1.0035	1.0070	1.0105	1.0141	1.0176	1.0212	1.0247	1.0283	1.0319	44
46	1.0355	1.0392	1.0428	1.0464	1.0501	1.0538	1.0575	1.0612	1.0649	1.0686	43
47	1.0724	1.0761	1.0799	1.0837	1.0875	1.0913	1.0951	1.0990	1.1028	1.1067	42
48	1.1106	1.1145	1.1184	1.1224	1.1263	1.1303	1.1343	1.1383	1.1423	1.1463	41
49	1.1504	1.1544	1.1585	1.1626	1.1667	1.1708	1.1750	1.1792	1.1833	1.1875	40°
50°	1.1918	1.1960	1.2002	1.2045	1.2088	1.2131	1.2174	1.2218	1.2261	1.2305	39
51	1.2349	1.2393	1.2437	1.2482	1.2527	1.2572	1.2617	1.2662	1.2708	1.2753	38
52	1.2799	1.2846	1.2892	1.2938	1.2985	1.3032	1.3079	1.3127	1.3175	1.3222	37
53	1.3270	1.3319	1.3367	1.3416	1.3465	1.3514	1.3564	1.3613	1.3663	1.3713	36
54	1.3764	1.3814	1.3865	1.3916	1.3968	1.4019	1.4071	1.4124	1.4176	1.4229	35
55	1.4281	1.4335	1.4388	1.4442	1.4496	1.4550	1.4605	1.4659	1.4715	1.4770	34
56	1.4826	1.4882	1.4938	1.4994	1.5051	1.5108	1.5166	1.5224	1.5282	1.5340	33
57	1.5399	1.5458	1.5517	1.5577	1.5637	1.5697	1.5757	1.5818	1.5880	1.5941	32
58	1.6003	1.6066	1.6128	1.6191	1.6255	1.6319	1.6383	1.6447	1.6512	1.6577	31
59	1.6643	1.6709	1.6775	1.6842	1.6909	1.6977	1.7045	1.7113	1.7182	1.7251	30°
60°	1.7321	1.7391	1.7461	1.7532	1.7603	1.7675	1.7747	1.7820	1.7893	1.7966	29
61	1.8040	1.8115	1.8190	1.8265	1.8341	1.8418	1.8495	1.8572	1.8650	1.8728	28
62	1.8807	1.8887	1.8967	1.9047	1.9128	1.9210	1.9292	1.9375	1.9458	1.9542	27
63	1.9626	1.9711	1.9797	1.9883	1.9970	2.0057	2.0145	2.0233	2.0323	2.0413	26
64	2.0503	2.4059	2.0686	2.0778	2.0872	2.0965	2.1060	2.1155	2.1251	2.1348	25
65	2.1445	2.1543	2.1642	2.1742	2.1842	2.1943	2.2045	2.2148	2.2251	2.2355	24
66	2.2460	2.2566	2.2673	2.2781	3.2889	2.2998	2.3109	2.3220	2.3332	2.3445	23
67	2.3559	2.3673	2.3789	2.3906	2.4023	2.4142	2.4262	2.4383	2.4504	2.4627	22
68	2.4751	2.4876	2.5002	2.5129	2.5257	2.5386	2.5517	2.5649	2.5782	2.5916	21
69	2.6051	2.6187	2.6325	2.6464	2.6605	2.6746	2.6889	2.7034	2.7179	2.7326	20°
70°	2.7475	2.7625	2.7776	2.7929	2.8083	2.8239	2.8397	2.8556	2.8716	2.8878	19
71	2.9042	2.9208	2.9375	2.9544	2.9714	2.9887	3.0061	3.0237	3.0415	3.0595	18
72	3.0777	3.0961	3.1146	3.1334	3.1524	2.1716	3.1910	3.2106	3.2305	3.2506	17
73	3.2709	3.2914	3.3122	3.3332	3.3544	3.3759	3.3977	3.4197	3.4420	3.4646	16
74	3.4874	3.5105	1.5339	3.5576	3.5816	3.6059	3.6305	3.6554	3.6806	3.7062	15
75	3.7321	3.7583	3.7848	3.8118	3.8391	3.8667	3.8947	3.9232	3.9520	3.9812	14
76	4.0108	4.0408	4.0713	4.1022	4.1335	4.1653	4.1976	4.2303	4.2635	4.2972	13
77	4.3315	4.3662	4.4015	4.4374	4.4737	4.5107	4.5483	4.5864	4.6252	4.6646	12
78	4.7046	4.7453	4.7867	4.8288	2.8716	4.9152	4.9594	5.0045	5.0504	5.0970	11
79	5.1446	5.1929	5.2422	5.2924	5.3435	5.3955	5.4486	5.5026	5.5578	5.6140	10°
80°	5.6713	5.7297	5.7894	5.8502	5.9124	5.9758	6.0405	6.1066	6.1742	6.2432	9
81	6.3138	6.3859	6.4596	6.5350	6.6122	6.6912	6.7720	6.8548	6.9395	7.0264	8
82	7.1154	7.2066	7.3002	7.3962	7.4947	7.5958	7.6996	7.8062	7.9158	8.0285	7
83	8.1443	8.2636	8.3863	8.5126	8.6427	8.7769	8.9152	9.0579	9.2052	9.3572	6
84	9.5144	9.677	9.845	10.02	10.20	10.39	10.58	10.78	10.99	11.20	5
85	11.43	11.66	11.91	12.16	12.43	12.71	13.00	13.30	13.62	13.95	4
86	14.30	14.67	15.06	15.46	15.89	16.35	16.83	17.34	17.89	18.46	3
87	19.08	19.74	20.45	21.20	22.02	22.90	23.86	24.90	26.03	27.27	2
88	28.64	30.14	31.82	33.69	35.80	38.19	40.92	44.07	47.74	52.08	1
89	57.29	63.66	71.62	81.85	95.49	114.6	143.2	191.0	286.5	573.0	0°
	°1.0	°0.9	°0.8	°0.7	°0.6	°0.5	°0.4	°0.3	°0.2	°0.1	Deg.

TABLE 41.—SQUARES, CUBES, SQUARE AND CUBE ROOTS OF NUMBERS FROM 1 TO 500

No.	Square	Cube	Square root	Cube root	No.	Square	Cube	Square root	Cube root
1	1	1	1.0000	1.0000	51	2601	132651	7.1414	3.7084
2	4	8	1.4142	1.2599	52	2704	140608	7.2111	3.7325
3	9	27	1.7321	1.4422	53	2809	148877	7.2801	3.7563
4	16	64	2.0000	1.5874	54	2916	157464	7.3485	3.7798
5	25	125	2.2361	1.7100	55	3025	166375	7.4162	3.8030
6	36	216	2.4495	1.8171	56	3136	175616	7.4833	3.8259
7	49	343	2.6458	1.9129	57	3249	185193	7.5498	3.8485
8	64	512	2.8284	2.0000	58	3364	195112	7.6158	3.8709
9	81	729	3.0000	2.0801	59	3481	205379	7.6811	3.8930
10	100	1000	3.1623	2.1544	60	3600	216000	7.7460	3.9149
11	121	1331	3.3166	2.2240	61	3721	226981	7.8102	3.9365
12	144	1728	3.4641	2.2894	62	3844	238328	7.8740	3.9579
13	169	2197	3.6056	2.3513	63	3969	250047	7.9373	3.9791
14	196	2744	3.7417	2.4101	64	4096	262144	8.0000	4.0000
15	225	3375	3.8730	2.4662	65	4225	274625	8.0623	4.0207
16	256	4096	4.0000	2.5198	66	4356	287496	8.1240	4.0412
17	289	4913	4.1231	2.5713	67	4489	300763	8.1854	4.0615
18	324	5832	4.2426	2.6207	68	4624	314432	8.2462	4.0817
19	361	6859	4.3589	2.6684	69	4761	328509	8.3066	4.1016
20	400	8000	4.4721	2.7144	70	4900	343000	8.3666	4.1213
21	441	9261	4.5826	2.7589	71	5041	357911	8.4261	4.1408
22	484	10648	4.6904	2.8020	72	5184	373248	8.4853	4.1602
23	529	12167	4.7958	2.8439	73	5329	389017	8.5440	4.1793
24	576	13824	4.8990	2.8845	74	5476	405224	8.6023	4.1983
25	625	15625	5.0000	2.9240	75	5625	421875	8.6603	4.2172
26	676	17576	5.0990	2.9625	76	5776	438976	8.7178	4.2358
27	729	19683	5.1962	3.0000	77	5929	456533	8.7750	4.2543
28	784	21952	5.2915	3.0366	78	6084	474552	8.8318	4.2727
29	841	24389	5.3852	3.0723	79	6241	493039	8.8882	4.2908
30	900	27000	5.4772	3.1072	80	6400	512000	8.9443	4.3089
31	961	29791	5.5678	3.1414	81	6561	531441	9.0000	4.3267
32	1024	32768	5.6569	3.1748	82	6724	551368	9.0554	4.3445
33	1089	35937	5.7446	3.2075	83	6889	571787	9.1104	4.3621
34	1156	39304	5.8310	3.2396	84	7056	592704	9.1652	4.3795
35	1225	42875	5.9161	3.2711	85	7225	614125	9.2195	4.3968
36	1296	46656	6.0000	3.3019	86	7396	636056	9.2736	4.4140
37	1369	50653	6.0828	3.3322	87	7569	658503	9.3276	4.4310
38	1444	54872	6.1644	3.3620	88	7744	681472	9.3808	4.4480
39	1521	59319	6.2450	3.3912	89	7921	704969	9.4340	4.4647
40	1600	64000	6.3246	3.4200	90	8100	729000	9.4868	4.4814
41	1681	68921	6.4031	3.4482	91	8281	753571	9.5394	4.4979
42	1764	74088	6.4807	3.4760	92	8464	778688	9.5917	4.5144
43	1849	79507	6.5574	3.5034	93	8649	804357	9.6437	4.5307
44	1936	85184	6.6332	3.5303	94	8836	830584	9.6954	4.5468
45	2025	91125	6.7082	3.5569	95	9025	857375	9.7468	4.5629
46	2116	97336	6.7823	3.5830	96	9216	884736	9.7980	4.5789
47	2209	103823	6.8557	3.6088	97	9409	912673	9.8489	4.5947
48	2304	110592	6.9282	3.6342	98	9604	941192	9.8995	4.6104
49	2401	117649	7.0000	3.6593	99	9801	970299	9.9499	4.6261
50	2500	125000	7.0711	3.6840	100	10000	1000000	10.0000	4.6416

TABLE 41.—SQUARES, CUBES, SQUARE AND CUBE ROOTS OF NUMBERS FROM 1 TO 500.—*Continued*

No.	Square	Cube	Square root	Cube root	No.	Square	Cube	Square root	Cube root
101	10201	1030301	10.0499	4.6570	151	22801	3442951	12.2882	5.3251
102	10404	1061208	10.0995	4.6723	152	23104	3511808	12.3288	5.3368
103	10609	1092727	10.1489	4.6875	153	23409	3581577	12.3693	5.3485
104	10816	1124864	10.1980	4.7027	154	23716	3652264	12.4097	5.3601
105	11025	1157625	10.2470	4.7177	155	24025	3723875	12.4499	5.3717
106	11236	1191016	10.2956	4.7326	156	24336	3796416	12.4900	5.3832
107	11449	1225043	10.3441	4.7475	157	24649	3869893	12.5300	5.3947
108	11664	1259712	10.3923	4.7622	158	24964	3944312	12.5698	5.4061
109	11881	1295029	10.4403	4.7769	159	25281	4019679	12.6095	5.4175
110	12100	1331000	10.4881	4.7914	160	25600	4096000	12.6491	5.4288
111	12321	1367631	10.5357	4.8059	161	25921	4173281	12.6886	5.4401
112	12544	1404928	10.5830	4.8203	162	26244	4251528	12.7279	5.4514
113	12769	1442897	10.6301	4.8346	163	26569	4330747	12.7671	5.4626
114	12996	1481544	10.6771	4.8488	164	26896	4410944	12.8062	5.4737
115	13225	1520875	10.7238	4.8629	165	27225	4492125	12.8452	5.4848
116	13456	1560896	10.7703	4.8770	166	27556	4574296	12.8841	5.4959
117	13689	1601613	10.8167	4.8910	167	27889	4657463	12.9228	5.5069
118	13924	1643032	10.8628	4.9049	168	28224	4741632	12.9615	5.5178
119	14161	1685159	10.9087	4.9187	169	28561	4826809	13.0000	5.5288
120	14400	1728000	10.9545	4.9324	170	28900	4913000	13.0384	5.5397
121	14641	1771561	11.0000	4.9461	171	29241	5000211	13.0767	5.5505
122	14884	1815848	11.0454	4.9597	172	29584	5088448	13.1149	5.5613
123	15129	1860867	11.0905	4.9732	173	29929	5177717	13.1529	5.5721
124	15376	1906624	11.1355	4.9866	174	30276	5268024	13.1909	5.5828
125	15625	1953125	11.1803	5.0000	175	30625	5359375	13.2288	5.5934
126	15876	2000376	11.2250	5.0133	176	30976	5451776	13.2665	5.6041
127	16129	2048383	11.2694	5.0265	177	31329	5545233	13.3041	5.6147
128	16384	2097152	11.3137	5.0397	178	31684	5639752	13.3417	5.6252
129	16641	2146689	11.3578	5.0528	179	32041	5735339	13.3791	5.6357
130	16900	2197000	11.4018	5.0658	180	32400	5832000	13.4164	5.6462
131	17161	2248091	11.4455	5.0788	181	32761	5929741	13.4536	5.6567
132	17424	2299968	11.4891	5.0916	182	33124	6028568	13.4907	5.6671
133	17689	2352637	11.5326	5.1045	183	33489	6128487	13.5277	5.6774
134	17956	2406104	11.5758	5.1172	184	33856	6229504	13.5647	5.6877
135	18225	2460375	11.6190	5.1299	185	34225	6331625	13.6015	5.6980
136	18496	2515456	11.6619	5.1426	186	34596	6434856	13.6382	5.7083
137	18769	2571353	11.7047	5.1551	187	34969	6539203	13.6748	5.7185
138	19044	2628072	11.7473	5.1676	188	35344	6644672	13.7113	5.7287
139	19321	2685619	11.7898	5.1801	189	35721	6751269	13.7477	5.7388
140	19600	2744000	11.8322	5.1925	190	36100	6859000	13.7840	5.7489
141	19881	2803221	11.8743	5.2048	191	36481	6967871	13.8203	5.7590
142	20164	2863288	11.9164	5.2171	192	36864	7077888	13.8564	5.7690
143	20449	2924207	11.9583	5.2293	193	37249	7189057	13.8924	5.7790
144	20736	2985984	12.0000	5.2415	194	37636	7301384	13.9284	5.7890
145	21025	3048625	12.0416	5.2536	195	38025	7414875	13.9642	5.7989
146	21316	3112136	12.0830	5.2656	196	38416	7529536	14.0000	5.8088
147	21609	3176523	12.1244	5.2776	197	38809	7645373	14.0357	5.8186
148	21904	3241792	12.1655	5.2896	198	39204	7762392	14.0712	5.8285
149	22201	3307949	12.2066	5.3015	199	39601	7880599	14.1067	5.8383
150	22500	3375000	12.2474	5.3133	200	40000	8000000	14.1421	5,8480

TABLE 41.—SQUARES, CUBES, SQUARE AND CUBE ROOTS OF NUMBERS FROM 1 TO 500.—*Continued*

No.	Square	Cube	Square root	Cube root	No.	Square	Cube	Square root	Cube root
201	40401	8120601	14.1774	5.8578	251	63001	15813251	15.8430	6.3080
202	40804	8242408	14.2127	5.8675	252	63504	16003008	15.8745	6.3164
203	41209	8365427	14.2478	5.8771	253	64009	16194277	15.9060	6.3247
204	41616	8489664	14.2829	5.8868	254	64516	16387064	15.9374	6.3330
205	42025	8615125	14.3178	5.8964	255	65025	16581375	15.9687	6.3413
206	42436	8741816	14.3527	5.9059	256	65536	16777216	16.0000	6.3496
207	42849	8869743	14.3875	5.9155	257	66049	16974593	16.0312	6.3579
208	43264	8998912	14.4222	5.9250	258	66564	17173512	16.0624	6.3661
209	43681	9129329	14.4568	5.9345	259	67081	17373979	16.0935	6.3743
210	44100	9261000	14.4914	5.9439	260	67600	17576000	16.1245	6.3825
211	44521	9393931	14.5258	5.9533	261	68121	17779581	16.1555	6.3907
212	44944	9528128	14.5602	5.9627	262	68644	17984728	16.1864	6.3988
213	45369	9663597	14.5945	5.9721	263	69169	18191447	16.2173	6.4070
214	45796	9800344	14.6287	5.9814	264	69696	18399744	16.2481	6.4151
215	46225	9938375	14.6629	5.9907	265	70225	18609625	16.2788	6.4232
216	46656	10077696	14.6969	6.0000	266	70756	18821096	16.3095	6.4312
217	47089	10218313	14.7309	6.0092	267	71289	19034163	16.3401	6.4393
218	47524	10360232	14.7648	6.0185	268	71824	19248832	16.3707	6.4473
219	47961	10503459	14.7986	6.0277	269	72361	19465109	16.4012	6.4553
220	48400	10648000	14.8324	6.0368	270	72900	19683000	16.4317	6.4633
221	48841	10793861	14.8661	6.0459	271	73441	19902511	16.4621	6.4713
222	49284	10941048	14.8997	6.0550	272	73984	20123648	16.4924	6.4792
223	49729	11089567	14.9332	6.0641	273	74529	20346417	16.5227	6.4872
224	50176	11239424	14.9666	6.0732	274	75076	20570824	16.5529	6.4951
225	50625	11390625	15.0000	6.0822	275	75625	20796875	16.5831	6.5030
226	51076	11543176	15.0333	6.0912	276	76176	21024576	16.6132	6.5108
227	51529	11697083	15.0665	6.1002	277	76729	21253933	16.6433	6.5187
228	51984	11852352	15.0997	6.1091	278	77284	21484952	16.6733	6.5265
229	52441	12008989	15.1327	6.1180	279	77841	21717639	16.7033	6.5343
230	52900	12167000	15.1658	6.1269	280	78400	21952000	16.7332	6.5421
231	53361	12326391	15.1987	6.1358	281	78961	22188041	16.7631	6.5499
232	53824	12487168	15.2315	6.1446	282	79524	22425768	16.7929	6.5577
233	54289	12649337	15.2643	6.1534	283	80089	22665187	16.8226	6.5654
234	54756	12812904	15.2971	6.1622	284	80656	22906304	16.8523	6.5731
235	55225	12977875	15.3297	6.1710	285	81225	23149125	16.8819	6.5808
236	55696	13144256	15.3623	6.1797	286	81796	23393656	16.9115	6.5885
237	56169	13312053	15.3948	6.1885	287	82369	23639903	16.9411	6.5962
238	56644	13481272	15.4272	6.1972	288	82944	23887872	16.9706	6.6039
239	57121	13651919	15.4596	6.2058	289	83521	24137569	17.0000	6.6115
240	57600	13824000	15.4919	6.2145	290	84100	24389000	17.0294	6.6191
241	58081	13997521	15.5242	6.2231	291	84681	24642171	17.0587	6.6267
242	58564	14172488	15.5563	6.2317	292	85264	24897088	17.0880	6.6343
243	59049	14348907	15.5885	6.2403	293	85849	25153757	17.1172	6.6419
244	59536	14526784	15.6205	6.2488	294	86436	25412184	17.1464	6.6494
245	60025	14706125	15.6525	6.2573	295	87025	25672375	17.1756	6.6569
246	60516	14886936	15.6844	6.2658	296	87616	25934336	17.2047	6.6644
247	61009	15069223	15.7162	6.2743	297	88209	26198073	17.2337	6.6719
248	61504	15252992	15.7480	6.2828	298	88804	26463592	17.2627	6.6794
249	62001	15438249	15.7797	6.2912	299	89401	26730899	17.2916	6.6869
250	62500	15625000	15.8114	6.2996	300	90000	27000000	17.3205	6.6943

Table 41.—Squares, Cubes, Square and Cube Roots of Numbers from 1 to 500.—*Continued*

No.	Square	Cube	Square root	Cube root	No.	Square	Cube	Square root	Cube root
301	90601	27270901	17.3494	6.7018	351	123201	43243551	18.7350	7.0540
302	91204	27543608	17.3781	6.7092	352	123904	43614208	18.7617	7.0607
303	91809	27818127	17.4069	6.7166	353	124609	43986977	18.7883	7.0674
304	92416	28094464	17.4356	6.7240	354	125316	44361864	18.8149	7.0740
305	93025	28372625	17.4642	6.7313	355	126025	44738875	18.8414	7.0807
306	93636	28652616	17.4929	6.7387	356	126736	45118016	18.8680	7.0873
307	94249	28934443	17.5214	6.7460	357	127449	45499293	18.8944	7.0940
308	94864	29218112	17.5499	6.7533	358	128164	45882712	18.9209	7.1006
309	95481	29503629	17.5784	6.7606	359	128881	46268279	18.9473	7.1072
310	96100	29791000	17.6068	6.7679	360	129600	46656000	18.9737	7.1138
311	96721	30080231	17.6352	6.7752	361	130321	47045881	19.0000	7.1204
312	97344	30371328	17.6635	6.7824	362	131044	47437928	19.0263	7.1269
313	97969	30664297	17.6918	6.7897	363	131769	47832147	19.0526	7.1335
314	98596	30959144	17.7200	6.7969	364	132496	48228544	19.0788	7.1400
315	99225	31255875	17.7482	6.8041	365	133225	48627125	19.1050	7.1466
316	99856	31554496	17.7764	6.8113	366	133956	49027896	19.1311	7.1531
317	100489	31855013	17.8045	6.8185	367	134689	49430863	19.1572	7.1596
318	101124	32157432	17.8326	6.8256	368	135424	49836032	19.1833	7.1661
319	101761	32461759	17.8606	6.8328	369	136161	50243409	19.2094	7.1726
320	102400	32768000	17.8885	6.8399	370	136900	50653000	19.2354	7.1791
321	103041	33076161	17.9165	6.8470	371	137641	51064811	19.2614	7.1855
322	103684	33386248	17.9444	6.8541	372	138384	51478848	19.2873	7.1920
223	104329	33698267	17.9722	6.8612	373	139129	51895117	19.3132	7.1984
324	104976	34012224	18.0000	6.8683	374	139876	52313624	19.3391	7.2048
325	105625	34328125	18.0278	6.8753	375	140625	52734375	19.3649	7.2112
326	106276	34645976	18.0555	6.8824	376	141376	53157376	19.3907	7.2177
327	106929	34965783	18.0831	6.8894	377	142129	53582633	19.4165	7.2240
328	107584	35287552	18.1108	6.8964	378	142884	54010152	19.4422	7.2304
329	108241	35611289	18.1384	6.9034	379	143641	54439939	19.4679	7.2368
330	108900	35937000	18.1659	6.9104	380	144400	54872000	19.4936	7.2432
331	109561	36264691	18.1934	6.9174	381	145161	55306341	19.5192	7.2495
332	110224	36594368	18.2209	6.9244	382	145924	55742968	19.5448	7.2558
333	110889	36926037	18.2483	6.9313	383	146689	56181887	19.5704	7.2622
334	111556	37259704	18.2757	6.9382	384	147456	56623104	19.5959	7.2685
335	112225	37595375	18.3030	6.9451	385	148225	57066625	19.6214	7.2748
336	112896	37933056	18.3303	6.9521	386	148996	57512456	19.6469	7.2811
337	113569	38272753	18.3576	6.9589	387	149769	57960603	19.6723	7.2874
338	114244	38614472	18.3848	6.9658	388	150544	58411072	19.6977	7.2936
339	114921	38958219	18.4120	6.9727	389	151321	58863869	19.7231	7.2999
340	115600	39304000	18.4391	6.9795	390	152100	59319000	19.7484	7.3061
341	116281	39651821	18.4662	6.9864	391	152881	59776471	19.7737	7.3124
342	116964	40001688	18.4932	6.9932	392	153664	60236288	19.7990	7.3186
343	117649	40353607	18.5203	7.0000	393	154449	60698457	19.8242	7.3248
344	118336	40707584	18.5472	7.0068	394	155236	61162984	19.8494	7.3310
345	119025	41063625	18.5742	7.0136	395	156025	61629875	19.8746	7.3372
346	119716	41421736	18.6011	7.0203	396	156816	62099136	19.8997	7.3434
347	120409	41781923	18.6279	7.0271	397	157609	62570773	19.9249	7.3496
348	121104	42144192	18.6548	7.0338	398	158404	63044792	19.9499	7.3558
349	121801	42508549	18.6815	7.0406	399	159201	63521199	19.9750	7.3619
350	122500	42875000	18.7083	7.0473	400	160000	64000000	20.0000	7.3681

TABLE 41.—SQUARES, CUBES, SQUARE AND CUBE ROOTS OF NUMBERS FROM 1 TO 500.—*Concluded*

No.	Square	Cube	Square root	Cube root	No.	Square	Cube	Square root	Cube root
401	160801	64481201	20.0250	7.3742	451	203401	91733851	21.2368	7.6688
402	161604	64964808	20.0499	7.3803	452	204304	92345408	21.2603	7.6744
403	162409	65450827	20.0749	7.3864	453	205209	92959677	21.2838	7.6800
404	163216	65939264	20.0998	7.3925	454	206116	93576664	21.3073	7.6857
405	164025	66430125	20.1246	7.3986	455	207025	94196375	21.3307	7.6914
406	164836	66923416	20.1494	7.4047	456	207936	94818816	21.3542	7.6970
407	165649	67419143	20.1742	7.4108	457	208849	95443993	21.3776	7.7026
408	166464	67917312	20.1990	7.4169	458	209764	96071912	21.4009	7.7082
409	167281	68417929	20.2237	7.4229	459	210681	96702579	21.4243	7.7138
410	168100	68921000	20.2485	7.4290	460	211600	97336000	21.4476	7.7194
411	168921	69426531	20.2731	7.4350	461	212521	97972181	21.4709	7.7250
412	169744	69934528	20.2978	7.4410	462	213444	98611128	21.4942	7.7306
413	170569	70444997	20.3224	7.4470	463	214369	99252847	21.5174	7.7362
414	171396	70957944	20.3470	7.4530	464	215296	99897344	21.5407	7.7418
415	172225	71473375	20.3715	7.4590	465	216225	100544625	21.5630	7.7473
416	173056	71991296	20.3961	7.4650	466	217156	101194696	21.5870	7.7529
417	173889	72511713	20.4206	7.4710	467	218089	101847563	21.6102	7.7584
418	174724	73034632	20.4450	7.4770	468	219024	102503232	21.6333	7.7639
419	175561	73560059	20.4695	7.4829	469	219961	103161709	21.6564	7.7695
420	176400	74088000	20.4939	7.4889	470	220900	103823000	21.6795	7.7750
421	177241	74618461	20.5183	7.4948	471	221841	104487111	21.7025	7.7805
422	178084	75151448	20.5426	7.5007	472	222784	105154048	21.7256	7.7860
423	178929	75686967	20.5670	7.5067	473	223729	105823817	21.7486	7.7915
424	179776	76225024	20.5913	7.5126	474	224676	106496424	21.7715	7.7970
425	180625	76765625	20.6155	7.5185	475	225625	107171875	21.7945	7.8025
426	181476	77308776	20.6398	7.5244	476	226576	107850176	21.8174	7.8079
427	182329	77854483	20.6640	7.5302	477	227529	108531333	21.8403	7.8134
428	183184	78402752	20.6882	7.5361	478	228484	109215352	21.8632	7.8188
429	184041	78953589	20.7123	7.5420	479	229441	109902239	21.8861	7.8243
430	184900	79507000	20.7364	7.5478	480	230400	110592000	21.9089	7.8297
431	185761	80062991	20.7605	7.5537	481	231361	111284641	21.9317	7.8352
432	286624	80621568	20.7846	7.5595	482	232324	111980168	21.9545	7.8406
433	187489	81182737	20.8087	7.5674	483	233289	112678587	21.9773	7.8460
434	188356	81746504	20.8327	7.5712	484	234256	113379904	22.0000	7.8514
435	189225	82312875	20.8567	7.5770	485	235225	114084125	22.0227	7.8568
436	190096	82881856	20.8806	7.5828	486	236196	114791256	22.0454	7.8622
437	190969	83453453	20.9045	7.5886	487	237169	115501303	22.0681	7.8676
438	191844	84027672	20.9284	7.5944	488	238144	116214272	22.0907	7.8730
439	192721	84604519	20.9523	7.6001	489	239121	116930169	22.1133	7.8784
440	193600	85184000	20.9762	7.6059	490	240100	117649000	22.1359	7.8837
441	194481	85766121	21.0000	7.6117	491	241081	118370771	22.1585	7.8891
442	195364	86350888	21.0238	7.6174	492	242064	119095488	22.1811	7.8944
443	196249	86938307	21.0476	7.6232	493	243049	119823157	22.2036	7.8998
444	197136	87528384	21.0713	7.6289	494	244036	120553784	22.2261	7.9051
445	198025	88121125	21.0950	7.6346	495	245025	121287375	22.2486	7.9105
446	198916	88716536	21.1187	7.6403	496	246016	122023936	22.2711	7.9158
447	199809	89314623	21.1424	7.6460	497	247009	122763473	22.2935	7.9211
448	200704	89915392	21.1660	7.6517	498	248004	123505992	22.3159	7.9264
449	201601	90518849	21.1896	7.6574	499	249001	124251499	22.3383	7.9317
450	202500	91125000	21.2132	7.6631	500	250000	125000000	22.3607	7.9370

TABLE 42.—TABLE OF THREE-HALVES ($\frac{3}{2}$) POWER OF NUMBERS*

No.	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0000	0.0316	0.0894	0.1643	0.2530	0.3536	0.4648	0.5857	0.7155	0.8538
1	1.0000	1.1537	1.3145	1.4822	1.6565	1.8371	2.0238	2.2165	2.4150	2.6190
2	2.8284	3.0432	3.2631	3.4881	3.7181	3.9529	4.1924	4.4366	4.6853	4.9385
3	5.1962	5.4581	5.7243	5.9947	6.2693	6.5479	6.8305	7.1171	7.4076	7.7019
4	8.0000	8.3019	8.6074	8.9167	9.2295	9.5459	9.8659	10.1894	10.5163	10.8466
5	11.1803	11.5174	11.8578	12.2015	12.5485	12.8986	13.2520	13.6086	13.9682	14.3311
6	14.6969	15.0659	15.4379	15.8129	16.1909	16.5718	16.9557	17.3425	17.7322	18.1248
7	18.5203	18.9185	19.3196	19.7235	20.1302	20.5396	20.9518	21.3666	21.7842	22.2045
8	22.6274	23.0530	23.4812	23.9121	24.3455	24.7815	25.2202	25.6613	26.1050	26.5523
9	27.0000	27.4512	27.9050	28.3612	28.8199	29.2810	29.7445	30.2105	30.6789	31.1496
10	31.6228	32.0983	32.5762	33.0564	33.5390	34.0239	34.5111	35.0006	35.4924	35.9865
11	36.4829	36.9815	37.4824	37.9855	38.4908	38.9984	39.5082	40.0202	40.5343	41.0507
12	41.5692	42.0910	42.6128	43.1388	43.6648	44.1952	44.7256	45.2600	45.7914	46.3332
13	46.8720	47.4148	47.9576	48.5048	49.0520	49.6032	50.1544	50.7096	51.2618	51.8240
14	52.3832	52.9464	53.5096	51.0768	54.6440	55.2152	55.7864	56.3616	56.9368	57.5156
15	58.0944	58.6776	59.2608	59.8472	60.4336	61.0244	61.6152	62.2096	62.8040	63.4020
16	64.0000	64.6020	65.2040	65.8096	66.4152	67.0244	67.6336	68.2464	68.8592	69.4760
17	70.0928	70.7132	71.3336	71.9572	75.5808	73.2084	73.8360	74.4672	75.0984	75.7328
18	76.3672	77.0056	77.6440	78.2856	78.9272	79.5724	80.2176	80.8664	81.5152	82.1672
19	82.8192	83.4748	84.1304	84.7892	85.4480	86.1104	86.7728	87.4384	88.1040	88.7732
20	89.4424	90.1152	90.7880	91.4636	92.1392	92.8184	93.4976	94.1800	94.8624	95.5484
21	96.2344	96.9232	97.6120	98.3044	98.9968	99.6924	100.3880	101.0868	101.7856	102.4872
22	103.1883	103.8940	104.6008	105.3076	106.0160	106.7276	107.4392	108.1540	108.8688	109.5864
23	110.3040	111.0248	111.7564	112.4700	113.1944	113.9216	114.6488	115.3788	116.1088	116.8420
24	117.5752	118.3128	119.0496	119.7876	120.5272	121.2696	122.0120	122.7576	123.5032	124.2516
25	125.0000	125.7516	126.5032	127.2576	128.0120	128.7706	129.5292	130.2876	131.0480	131.8112
26	132.5744	133.3408	134.1072	134.8764	135.6456	136.4180	137.1904	137.9652	138.7400	139.5180
27	140.2960	141.0768	141.8576	142.6416	143.4256	144.2120	144.9984	145.7880	146.5776	147.3700
28	148.1624	148.9572	149.7520	150.5500	151.3480	152.1488	152.9496	153.7532	154.5568	155.3632
29	156.1696	156.9788	157.7880	158.6000	159.4120	160.2268	161.0416	161.8588	162.6760	163.4964
30	164.3168	165.1396	165.9624	166.7884	167.6144	168.4428	169.2712	170.1020	170.9328	171.7668
31	172.6008	173.4372	174.2736	175.1128	175.9520	176.7940	177.6360	178.4804	179.3248	180.1720
32	181.0192	181.8692	182.7192	183.5716	184.4240	185.2792	186.1344	186.9920	187.8496	188.7100
33	189.5704	190.4336	191.2968	192.1624	193.0280	193.8960	194.7640	195.6348	196.5056	197.3788
34	198.2520	199.1460	200.0400	200.9008	201.7616	202.6424	203.5232	204.4068	205.2904	206.1764
35	207.0624	207.9512	208.8400	209.7312	210.6224	211.5204	212.4184	213.3014	214.2024	215.1012
36	216.0000	216.9012	217.8024	218.7060	219.6096	220.5760	221.4224	222.3312	223.2400	224.1512
37	225.0624	225.9760	226.8896	227.8056	228.7216	299.6406	230.5592	231.4800	232.4008	233.3244
38	234.2480	235.1736	236.0992	237.0276	237.9560	238.8868	239.8176	240.7508	241.6840	242.6196
39	243.5552	244.4932	245.4312	246.3712	247.3112	248.2540	249.1968	250.1428	251.0872	252.0348
40	252.9824	253.9320	254.8816	255.8340	256.7864	257.7412	258.6960	259.6528	260.6096	261.5688
41	262.5280	263.4896	264.4512	265.4152	266.3792	267.3456	268.3120	269.2804	270.2488	271.2200
42	272.1912	273.1644	274.1376	275.1132	276.0888	277.0672	278.0456	279.6252	280.0048	280.9872
43	281.9696	282.9544	283.9392	284.9264	285.9136	286.9028	287.8920	288.8836	289.8752	290.8692
44	291.8632	292.8597	293.8552	294.8536	295.8520	296.8528	297.8536	298.8564	299.8592	300.8640
45	301.8688	302.8764	303.8840	304.8936	305.9032	306.9148	307.9264	308.9404	309.9544	310.9708
46	311.9872	313.0056	314.0240	315.0448	316.0656	317.0877	318.1112	319.0556	320.0000	321.1080
47	322.2160	323.2452	324.2744	325.3060	326.3376	327.3716	328.4056	329.4416	330.4776	331.5156
48	332.5536	333.5927	334.6333	335.4753	336.7188	337.7588	338.8051	349.8529	340.8972	341.9479
49	343.0000	344.0486	345.0986	346.1500	347.2079	348.2622	349.3179	350.3750	351.4336	352.4886
50	353.5500	354.6128	355.6720	356.7376	357.7996	358.8681	359.9329	360.9992	362.0719	363.1409

*From Water-Supply and Irrigation Paper No. 180.

TABLE 42.—TABLE OF THREE-HALVES ($\frac{3}{2}$) POWER OF NUMBERS.—*Concluded*

No.	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
51	364.2114	365.2832	366.3564	367.4311	368.5020	369.5794	370.6582	371.7333	372.8149	373.8927
52	374.9772	376.0578	377.1397	378.2331	379.3078	380.3940	381.4815	382.5703	383.6606	384.7522
53	385.8453	386.9343	388.0301	389.1219	390.2205	391.3150	392.4163	393.5136	394.6122	395.7122
54	396.8136	397.9163	399.0204	400.1258	401.2326	402.3408	403.4448	404.5557	405.6679	406.7759
55	407.8855	409.0017	410.1139	411.2273	412.3477	413.4639	414.5814	415.7002	416.8204	417.9419
56	419.0648	420.1833	421.3089	422.4257	423.5583	424.6879	425.8131	426.9453	428.0732	429.2080
57	430.3386	431.4704	432.6036	433.7380	434.8738	436.0110	437.1494	438.2892	439.4302	440.5726
58	441.7106	442.8556	443.9961	445.1438	446.2869	447.4372	448.5830	449.7300	450.8842	452.0359
59	454.0849	455.3271	455.4907	456.6455	457.8017	458.9592	460.1179	461.2720	462.4334	463.5960
60	464.7540	465.9192	467.0797	468.2475	469.4106	470.5750	471.7467	472.9137	474.0819	475.2514
61	476.4222	477.3942	478.7676	479.9422	481.1181	482.2891	483.4676	484.6473	485.8222	487.0044
62	488.1880	489.3666	490.5465	491.7339	492.9163	494.1000	495.2912	496.4774	497.6648	498.8536
63	500.0436	501.2348	502.4273	503.6211	504.8161	506.0061	507.2036	508.4024	509.5961	510.7974
64	512.0000	513.1974	514.3960	15.6024	516.8035	518.0059	519.2160	520.4209	521.6270	522.8344
65	524.0430	525.2528	526.4639	527.6762	528.8898	530.1046	531.3120	534.5313	533.7498	534.9630
66	536.1840	537.2996	538.6230	539.8411	541.0670	542.2875	543.5092	544.7389	545.9630	547.1884
67	548.4151	549.6429	550.8720	552.1022	553.3337	554.5665	555.6179	557.0356	558.2652	559.5027
68	560.7416	561.9748	563.2160	564.4516	565.6953	566.9334	568.1795	569.4199	570.6616	571.9113
69	573.1554	574.4006	575.6473	576.8947	578.1436	579.3937	580.6449	581.8974	583.1510	584.4059
70	585.6620	586.9122	588.1707	589.4303	590.6841	591.9462	593.2023	594.4668	595.7253	596.9921
71	598.2531	590.5152	600.7856	602.0500	603.3157	604.5825	605.8505	607.1197	608.3901	609.6616
72	610.9344	612.2083	613.4340	614.7596	616.0371	617.3085	618.5883	619.8692	621.0841	622.4274
73	623.7120	624.0903	626.2699	627.5579	628.8398	630.1302	631.4144	632.6997	633.9862	635.2813
74	636.5702	637.8602	639.1513	640.4437	641.7372	643.0318	644.3276	645.6246	646.9152	648.2145
75	649.5150	650.8166	652.1118	653.4157	654.7208	656.0195	657.3268	658.6278	659.9375	661.2408
76	662.5452	663.8583	665.1650	666.4728	667.7894	669.0996	670.4108	671.7131	673.0368	674.3514
77	675.6673	676.9842	677.2043	679.6216	680.9419	682.2635	683.5784	684.9021	686.2271	687.5454
78	688.8726	690.2009	691.5226	692.8532	694.1771	695.5100	696.8361	698.8361	699.1713	700.8292
79	702.1509	703.4995	704.8324	706.1665	707.5016	708.8379	710.1752	711.5137	712.8531	714.1941
80	715.5360	716.8789	718.2230	719.5683	720.9146	722.2540	723.6026	724.9523	726.2950	727.6496
81	729.0000	730.3460	731.7613	733.0495	734.3989	735.7575	737.1091	738.4699	739.8237	741.1876
82	742.5346	743.8998	745.2580	746.6173	747.9776	749.3392	750.7018	752.0655	753.4303	754.7962
83	756.1632	757.5312	758.9014	760.2624	761.6338	763.0063	764.3798	765.7461	767.1219	768.4904
84	769.8684	771.2474	772.6192	774.0004	775.3743	776.7493	778.1338	779.5110	780.8892	782.2770
85	783.6575	785.0389	686.4215	787.8052	789.1984	790.5843	791.9712	793.3591	794.7482	796.1383
86	797.5296	798.9219	800.3066	801.7011	803.0966	804.4932	805.8909	807.2810	808.6808	810.0833
87	811.4751	812.8781	814.2736	815.6788	817.0763	818.4837	819.8834	821.2929	822.6947	824.1064
88	825.5704	826.9154	828.3214	829.7374	831.1456	832.5549	833.9652	835.3766	836.7890	838.2025
89	839.6171	841.0327	842.4404	843.8671	845.2859	846.7058	848.1267	849.5487	850.9627	852.3868
90	853.8120	855.2382	856.6564	858.0847	859.5051	860.9355	862.3670	863.7905	865.2241	866.6496
91	868.0763	869.5130	870.9417	872.3806	873.8114	875.2432	876.6761	878.1192	879.5541	880.9901
92	882.4272	883.8652	885.3041	886.7445	888.1857	889.6280	891.0712	892.5156	893.9609	895.4073
93	896.8548	898.3932	899.7528	901.1946	902.6456	904.0982	905.5519	906.9972	908.4530	909.9097
94	911.3582	912.8170	914.2675	915.7284	917.1809	918.6439	920.0985	921.5541	923.0202	924.4778
5	925.9365	927.4056	928.8664	930.3281	931.7908	933.2642	934.7290	936.1948	937.6616	939.1295
96	940.5984	942.0683	943.5392	945.0111	946.4841	947.9581	949.4331	950.9091	952.3764	953.8545
97	955.3336	956.8136	958.2948	959.7672	961.2503	962.7345	964.2099	965.6961	967.1735	968.6617
98	970.1412	971.6314	973.1129	974.6051	976.0886	977.5829	979.0684	980.5548	982.0522	983.5407
99	985.0302	986.5206	988.0220	989.5145	991.0080	992.5025	993.9980	995.4945	996.9920	998.4905
100	1000.0000	....	....	....	....	....	....	....	....	....

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